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Use of paradoxes as didactic resources that develop critical thinking in students

Uso de las paradojas como recursos didácticos que desarrollan el pensamiento crítico en los estudiantes

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Abstract

Education has been challenged by the digital context that benefits virtual platforms that contain creations of youtubers or influencers, which focus on entertaining rather than comprehensively educating the student. In this sense, this article considers paradoxes as didactic resources that can help the development of students' critical thinking during their education. This research is documentary and is based on the consultation of written sources and the Internet. It begins by clarifying the concepts of fallacy and *reductio ad absurdum*, since paradoxes have been seen as very subtle fallacies by some scholars such as Bertrand Russell and, in addition, there are those who use paradoxes to make deductions, as occurs in the *reductio ad absurdum* applied in the ontological argument of St. Anselm of Canterbury. Next, a list of paradoxes is analyzed, but with the objective that they can be used in a classroom. Thus, some paradoxes such as the paradox of Achilles and the Tortoise, Galileo's paradox, Hilbert's hotel paradox, Tristam Shandy's paradox, Protagoras' paradox, etc. are discussed. This work closes by trying to make explicit the affective and emotional aspect that a student experiences when dealing with this kind of problems.

Keywords

Paradox, fallacy, fractals, didactics, criticism, education.

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Resumen

Últimamente, la educación se ha visto desafiada por el contexto digital que beneficia a las plataformas virtuales que contienen creaciones de youtubers o influencers, los cuales se enfocan en entretener más que en formar integralmente al estudiante. En este sentido, este artículo considera a las paradojas como recursos didácticos que pueden ayudar al desarrollo del pensamiento crítico del estudiante durante su formación. Esta investigación es documental y se basa en la consulta de fuentes escritas y de internet. Comienza aclarando los conceptos de falacia y reducción al absurdo, pues las paradojas han sido vistas como falacias muy sutiles por algunos estudiosos como Bertrand Russell y, además, hay quienes utilizan las paradojas para realizar deducciones, como ocurre en la reducción al absurdo que se aplica en el argumento ontológico de San Anselmo de Canterbury. Enseguida, se analiza una lista de paradojas, pero con el objetivo de que puedan ser utilizadas en un aula de clase. Así, se trata sobre algunas paradojas como la paradoja de Aquiles y la Tortuga, la de Galileo, la del hotel de Hilbert, la de Tristam Shandy, la de Protágoras, etc. Este trabajo se cierra tratando de explicitar el aspecto afectivo y emocional que un estudiante experimenta cuando trata con esta clase de problemas.

Palabras clave

Paradoja, falacia, fractales, didáctica, crítica, educación.

Introduction

Education cannot compete with the advancement of technology. What can be done is to use technology as a complement to optimize the achievement of goals related to the teaching-learning process. However, technology will not be able to teach humans to think, this can only be done by a teacher who uses strategies to stimulate thinking properly. Precisely, if a teacher uses paradoxes as educational resources in his class, he will achieve something that the technology of our time cannot yet, i.e., provoke debate, enrich thought and generate different points of view that aim to solve some controversial issue.

The objective of this research is to provide a teaching methodology to teachers that guides them to look for paradoxes so that they can improve the contents they disseminate in their class sessions.

The problem to be solved is related to the poor educational level that the current society has. How should paradoxes be used in class sessions to improve the teaching-learning process?

Against this issue, this paper defends the idea that paradoxes serve as educational triggers that motivate students to give their opinion or perspective on the problematic issue presented in class. In this sense, the use of paradoxes in class sessions is more than necessary to achieve the objectives set by the teacher.

Nowadays, technology presents a series of visual and auditory stimuli with which the teacher cannot compete because he is clearly at a disadvantage. With a simple smartphone, a young person can search on the

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internet for any doubt he may have and, in addition, can be entertained by video games. Therefore, it is essential to provide the teacher with cognitive tools that help stimulate the thought of his students so that he can make teaching a more interesting event than the one offered by technology today.

The methodology used in this work is based on a qualitative approach (Sampieri *et al.,* 2014). The philosophical method of logical-linguistic analysis within the limits of analytical philosophy has been used (Salazar Bondy, 2000). The criteria for clarity in terminology will thus be respected. As far as argumentation is concerned, all relevant statements at the philosophical level are proved. However, in order to achieve this research, the technique of reading the sources of documentary collection and documentary analysis is used, i.e., different bibliographic bases are reviewed, and the philosophical analysis has been performed. As for the instruments, research sheets (textual, summary, paraphrases and mixed) have been used to select the relevant quotations regarding our topic. Also, information found on the web has been saved to a USB and the electronic means available have been used to type information, as well as to facilitate communication between researchers. Finally, the procedure that has been followed is the following: bibliographic sources were reviewed, several authors were chosen (which make up the theoretical framework of this work), the search for electronic files related to the subject under study was started, the data was classified, i.e., distinguished between books, journal articles and newspaper publication, among others that allow us to defend our research, finally, the publications that were selected were interpreted properly and it was proceeded to make this paper.

As for the structure of this document, it should be mentioned that this paper begins by clarifying the concepts of fallacy and reduction to absurdity. Then, a list of paradoxes is analyzed, but with the aim that they can be used in a classroom. Thus, it is about some paradoxes such as the Achilles and the Turtle paradox, Galileo's paradox, Hilbert's hotel paradox, Tristam Shandy's paradox, Protagoras paradox, etc. Finally, this work tries to make explicit the emotional aspect that a student experiences when dealing with these paradoxes.

An educational strategy

Education in Peru has long faced serious difficulties. No one is surprised by the low level of education reflected in the PISA (*Program for International Student Assessment*) tests (Gestión, December 3, 2019). Moreo-

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ver, the plagiarism scandal detected in the dissertations of various media personages has raised questions about whether the Peruvian education system is truly good.

The average student in the country has a general culture of *youtubers*, *influencers* and fashion reggaetoneros. Most people prefer to watch *Netflix*, a show or a meme from a youth social network like *TikTok*. Very few value culture, genius, creativity, reading, books, great classic films, national theater, etc. For this reason, whoever wants to be an educator in these circumstances should be awarded if they really want to change this calamitous state of affairs.

Being educated does not mean just knowing or memorizing some data; this should be called "being instructed". An educated person knows the subjects he or she researches, but he or she is also someone who is trained in values. Having values means being a person who is willing to live with dignity, who wants to help others understand the importance of social justice and who understands that solidarity is not a trait of weakness, but a gesture of nobility towards our fellow human beings.

Many factors come together in education, as it is a complex activity. However, we can focus on the question of how to organize the teaching-learning process. What distinguishes a teacher from another is the way to use the teaching resources in order for his students to obtain some specific knowledge, associated with some moral value.

The ideal would be to get students to develop their critical thinking; this ability is seen in various activities such as asking relevant questions, making distinctions, seeking counterexamples, suggesting classifications, analyzing statements, proposing hypotheses, defining concepts, discovering not so obvious options, exposing assumptions, searching for causes, ordering the reasons that support a given thesis, detecting the relations between the parts and the whole, connecting ideas, arguing consistently without falling into fallacies, appreciating the importance of the context to study some social fact, etc. (Rosas *et al.,* 2018). What is more than interesting is that the student can develop a coherent, solid and clear argument using logic to do so (Torres da Silva, 2016).

Getting students to improve their critical thinking so that they have an interest in cultivating it should be the central objective of education.

In this regard, there are two ways of educating that have been overcome. The traditional style of education, in which the teacher was considered the source of knowledge that the student should seek to know, is considered insufficient today. The other style, the behaviorist, which considered that the student should follow the instructions of his teacher and imitate

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him in everything, is also considered incomplete. At this time, the most appropriate method to teach is based on constructivist theories of learning represented by Piaget, Ausubel, Bruner and Vigotsky (Solé and Coll, 1995). We affirm that this method is more appropriate because it considers the student as the center of learning (as suggested by Gutiérrez-Pozo, 2023) and, furthermore, because the concept of "competence" that has been used to design syllabi and curricular plans is compatible with the theoretical framework of constructivism, since it requires awareness of the student about his learning and also a critical position about what he is learning.

Constructivism holds "that people form or build much of what they learn and understand" (Schunk, 2012, p. 229). According to this position, the student can build his own knowledge if he really intends to learn. The teacher's task must be to design the right conditions for the creative and autonomous activity of the student to begin. Schunk writes:

> Another assumption of constructivism is that teachers should not teach in the traditional sense of giving instruction to a group of students, but rather should structure situations in which students actively engage with content through material manipulation and social interaction (Schunk, 2012, p. 231).

Likewise, the student cannot learn alone, but rather must have a collaborative spirit so that he can work in community with his fellow students. The idea is that knowledge should be obtained through a shared activity (Schunk, 2012). The following table provides an idea of how constructivist learning environments are created.

Table 1
Principles governing constructivist learning environments

• Pose important problems to students
• Structure learning around important concepts
• Explore and value the point of view of students
• Adapt the academic program to consider the ideas and thoughts of the students
• Evaluate students´learning in the teaching context

Source: Brooks and Brooks, 1999 (cited in Schunk, 2012, p. 261).

A constructivist strategy is problem-based learning (PBL) (Escribano and Del Valle, 2008). These are the fundamental characteristics of this method:

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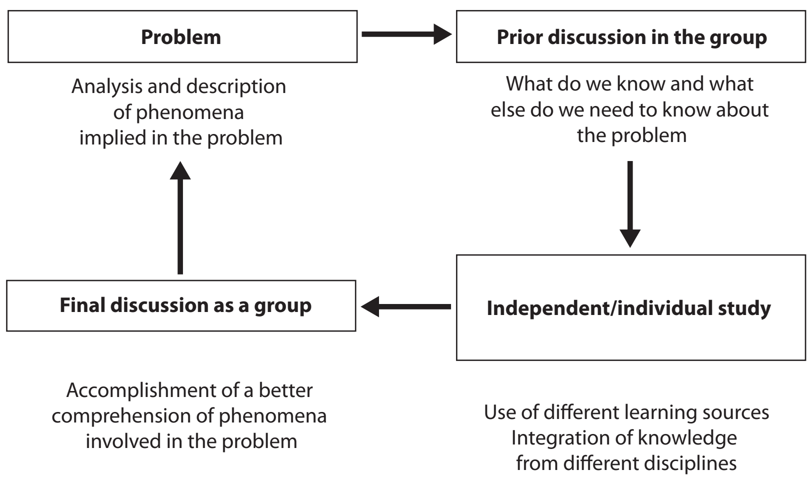
- Learning is student-centered.
- Learning occurs in small groups.
- Teachers are facilitators or guides of this process.
- Problems are the focus of organization and encouragement for learning.
- Problems are a vehicle for developing problem-solving skills.
- New information is acquired through self-directed learning (Manzanares, 2008, p. 15).

From this strategy, the teacher as tutor or guide presents a problem to the student to solve it with his work group. Together they are responsible for researching, reading and consulting in order to arrive at a solution or at least to achieve a better understanding of the question analyzed. Then, they meet to discuss their results and thus fulfill the activity asked by the teacher. This is an effective way to develop critical thinking. Figure 1 illustrates this.

Now the idea is that the most interesting kind of problem, which has not yet been explored sufficiently by teachers, is the problem created by finding and exposing a paradox. Paradoxes can be allied in the teaching-learning process and their use can benefit the educational activity both in the aspect of the achievement of competences and in the moral training of the educator. This research has been based on a documentary study of written sources as well as the Internet.

Figure 1
Understanding the PBL Process from the Student



Source: Manzanares, 2008, p. 20.

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In this sense, the paradoxes will be exposed as educational resources. Thus, in principle, a distinction is made between fallacy and paradox to show how it is possible that a paradox can be "solved" when it is discovered that, in reality, it was an argument based on a barely perceptible error of reasoning. In addition, it is considered important to approach paradoxes as contradictory elements that can be used in tests for reduction to absurdity. Then, a list of paradoxes is developed, partly overcome by tradition, to reinforce our perspective that the paradoxes can eventually be solved and also to reveal the educational and didactic aspect of them.

Fallacies and paradoxes

Both paradoxes and fallacies take the form of arguments. However, while paradoxes are arguments that proceed logically and lead to an unexpected contradiction, fallacies are arguments that are logically invalid but persuasive on a psychological level (Copi and Cohen, 2001). Fallacies occur when an argument seems acceptable, but actually hides some error that is not detectable by the eye. For example, the *ad ignorantiam* fallacy may seem right to some. This fallacy arises when someone states cases similar to the following: "since no one has conclusively proven that God does not exist, then we have to accept that God does exist." The truth is that, in the absence of evidence, nothing can be said or denied about any given matter. Another example is the scarecrow fallacy. Next, the following case is analyzed. Two congressional candidates debate. One says he is vegan because he loves animals; the other takes advantage of that and says he loves animals too, but above all he loves the poor, single mothers and homeless children, implying that his opponent only loves animals, but despises everything else. When noticing that a statement ascribes to the opponent a series of ideas that he has not explicitly mentioned, it is a case of scarecrow fallacy. This is very common in politics.

In relation to paradoxes, there has been an attempt by scholars to reduce paradoxes to fallacies in order to prove that they hid some error. Russell himself tried to prove that Cantor's paradox was nothing more than a fallacy, but failed in that endeavor. However, by studying this paradox he was able to digest sufficient inputs to propose his own paradox so or more destabilizing than the paradox of the maximum cardinal number of Cantor (Garciadiego, 1992).

The revelation of this or that paradox has been thwarted as a mere fallacy, it has been the subject of controversy. Yet, while a paradox has

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managed to be unmasked as a fallacy, no one doubts its prophetic value. The paradox, as a problem that attracts our attention, teaches us that there are limits in our understanding of some phenomenon. Those limits can always be overcome, but the learning we have gained in trying to respond to the challenge posed by the paradox has been invaluable. It is precisely this educational aspect of the paradox that could be rescued, despite the epistemic and cognitive overcoming of it.

Paradox and reduction to absurdity

Although this is not always the case, one of the common elements of paradoxes is contradiction. In logic, the contradiction has been used in a convenient way especially in those reasonings classified as "reduction to absurdity".

The structure of the reduction to absurdity starts with an assumption. Then, if both P and P's denial follow from that assumption, then the initial assumption can be denied. For example, this test was successfully used by Euclid to prove that there is no last prime number. Also, by reducing to absurd it is possible to prove that the root of two is not a rational number. Here a simpler case. Suppose you claim that the spider is an insect. If it were, it would have six legs. But after an examination, we noticed that it does not have six legs, but eight legs.

So, we can deduce that it is not true that the spider is an insect. This is a simple case of reduction to absurdity. The logical scheme of the reduction to absurdity would be the following: $[P \rightarrow (Q \wedge \sim Q)] \rightarrow \sim P$. The above can be understood as follows. If someone claims that P and this leads to contradictions of the type $Q \wedge \sim Q$, then what has to be accepted is that $\sim P$.

This is the way some paradoxes have been used by intellectual tradition. The demonstration of A was carried out in this way. If A's falsehood led to paradoxes, then this proved that A was not false, but rather true. It is necessary to review an example used by St. Anselm of Canterbury (trad. in 1998) to prove the existence of God. According to this thinker, when it is said that God is the greatest thing that can be thought, anyone who hears this definition can agree and, by that very fact, God would come to exist as a concept in his mind. The question is whether God can exist only as a concept within the human mind and not as a reality outside the human mind. We're going to take a test for reduction to absurdity. If it is false that God can exist outside the mind, that would pose a problem. On the one hand, God is the greatest by definition, but, on the other hand, if

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God did not exist outside the mind, then he would not be as great as was initially proposed as he would have a limited existence. This is a paradoxical contradiction. Therefore, it is true that God can also exist outside the mind. This is known as an ontological argument.

Paradoxes not only have the objective of puzzling the audience, they can also be used to reinforce an idea or to discuss fundamental concepts of some discipline. In this sense, paradoxes can be used as teaching resources. The following paradoxes are examined: that of Achilles and the Turtle, that of Galileo, that of Hilbert's hotel, that of Tristam Shandy, that of Protagoras, that of Monty Hall, that of God and stone, that of Epicurus, that of time travel, that of the egg and the hen and some geometric paradoxes. This research ends up trying to explain what a student feels when his or her teacher presents him or her with a paradox. Next, we will study the Achilles and the turtle paradox.

Paradox of Achilles and the Turtle

This paradox appears in chapter 9 of Book VI of *Physics* of Aristotle (trad. in 1995) and was raised by Zenon of Elea to prove that the movement is absurd. To achieve this goal, the disciple of Parmenides imagines a supposed race between Achilles and a turtle. As Achilles is an experienced runner, he gives a 10-meter lead to the turtle. The Achilles velocity and the turtle are 10 m/s and 1 m/s, respectively. During the first second of this race Achilles has arrived where the turtle was, but by that same time the turtle will have advanced one meter. As soon as Achilles travels that subway, the turtle will have traveled 0.1 meter. When Achilles goes that .1 meter, the turtle will have gone 0.01 meter and so on. Since space is infinitely divisible, there will always be a small amount of space that the turtle will have advanced to Achilles and therefore Achilles will never be able to reach it. However, on the other hand, it is obvious that Achilles will reach it, because the fastest always reaches the slowest and Achilles is the fastest. This is the paradox.

Some consider that the solution to this problem is based on the idea that an addition of infinite sums does not necessarily give an infinite result. In fact, if we add up the amounts that Achilles travels, namely $10 + 1 + 0.1 + 0.01 + 0.001 + \ldots$ the result we get is $11 + 1/9$, i.e., $11, \bar{1}$. This amount is less than 11.12. Thus, we can say that Achilles reaches the turtle when it travels almost 11.12 meters (Sthal, 1971). This solution is controversial, but acceptable to a large number of people. However, the

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teaching that this paradox provides can be used in a classroom to motivate learning. It has also been proposed that this paradox is based on a fallacy, namely the fallacy of the continuum. This fallacy holds that many small cumulative differences are not relevant to determining whether or not there is a change. For example, since it is unknown exactly how much money must be taken away from a rich person in order for him/her to become poor, then there is no difference between being rich and being poor. This is incorrect because the fact that we do not know when changes occur does not imply that things do not change. However, because the Achilles paradox makes a pernicious use of the expression "although it will not have reached it will be close to achieve it" (Mora, 2019), it can be said that falls into the fallacy of the continuum as it assumes that it will never go from "will be close to reach it" to "reached it". Then, Galileo's paradox will be reviewed.

Galileo Paradox

This paradox was raised by Galileo (1945) when he reflected on the relationship between natural numbers and square numbers. If we think about the first ten natural numbers, we will only find three square numbers. If we think about the first 100 natural numbers, we will only find ten square numbers. If we think about 1,000 natural numbers, we will find 31 square numbers. Therefore, there will always be more natural numbers than square numbers and the idea is maintained that the whole is larger than the part, the whole being made up of natural numbers and the part, by square numbers. Now, what if we consider all natural numbers, which are infinite? If the natural numbers are infinite, then the squares are also infinite. But if this is so, then the idea that the whole is greater than the part would no longer be maintained because the whole and the part would be just as infinite. It is also often said that what Galileo concludes is that the relations of greater, lesser and equal, do not have the same sense between finite quantities as between infinite quantities.

This problem would be reviewed later by Georg Cantor, and the solution he proposes is based on the use of the two-way correspondence concept. The idea is that, indeed, the number of natural numbers is the same as the number of square numbers, and this can be checked because each natural number corresponds to a square number, and yes, they are the same kind of infinity, namely, zero $\aleph_0$ alif. Now, the surprise is that there is not a single type of infinity, but a whole succession of transfinite

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numbers. For example, the number of real numbers is larger than the number of natural numbers. It can be concluded that what Galileo found was a paradox from the point of view of the mathematics of his time. However, from another theoretical framework, namely that of mathematics of the nineteenth century, this situation is no longer a paradox but rather a fact. However, again the teaching that this paradox leaves us is very valuable, as it reveals that what we call "knowledge" is relative to a stage of scientific development. Then, we will study the paradox of Hilbert's hotel.

Paradox of Hilbert's hotel

The mathematician David Hilbert (2013) raised some curious counterintuitive ideas about infinity. In the following, an attempt will be made to simulate a class session of a math teacher. Imagine a hotel with endless rooms. Infinite guests arrive at any given time. The hotel is full and all the rooms are occupied. However, a tourist arrives at the moment and asks for a room. At this moment the apprentice is asked, what can be done to bring this new guest in? We need to let him think for a moment. After a time, he is told that the interesting thing is that, despite being full, the hotel could be organized in such a way that this new guest could be admitted. Then, under the manager's orders, all visitors will be moved to the next room so that the new visitor can get their room. This problem was solved, however, after a while another problem occurred, as an excursion arrived with countless guests. Again, the apprentice is asked, what can be done to bring this excursion of infinite guests? Once again, it is necessary to let him reason for a short time and then comment to him that, although it seems that no more visitors can be admitted, this is not the case. Again, under the manager's orders the guests of the *n* rooms *were* moved into a *2n* room. Thus, the odd rooms were free and since the odd ones are infinite, the tour could find accommodation.

David Hilbert was already aware of the antics of infinity. These paradoxes, in reality, are only ways to spread in a more didactic way the strange nature of infinity. However, it is clear that the way to learn through paradoxes involves trying to solve these mental challenges. Accepting these challenges will make the student more prepared for decision-making in his daily life, as he will have exercised critical thinking in a convenient way. Then, the paradox of Tristam Shandy is analyzed.

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Tristam Shandy's Paradox

Tristam Shandy (a character created by Lawrence Stern) argued that he would not be able to write about his experiences because he realized that it took two years to write about his first two days of life. However, Bertrand Russell (1983) stated that there is a way he can write about his whole life. What would be this way? We must let the pupil think about this. Russell argued that, if Tristam Shandy lived infinite years with high intensity each of his days, the writer could write whatever he wanted about each of his days of life. So the day 1000 would take him to write 1000 years and the day 1600 would take him to write 1600 years. According to Clark:

> This is so because every couple of days of life can correspond to a successive couple of years that it takes to write those days, although his memory will need to be pushed back more and more, without limit. For example, he will have to write the 101st and 102nd days about a century later, in the 101st and 102nd years, and the 100th and 102nd days will write them almost a millennium later (2009, p. 239).

At this point, it should be clear that there was a time in history when mathematicians cared very much about the nature of infinity. The professor can also play with his students by proposing these puzzles to them so that they can, little by little, get used to thinking freely and creatively. Next, we will know the paradox of Protagoras.

Paradox of Protagoras

This paradox is also known as the paradox of lawyers. The first to present it was Aulo Gelio (about 150), but Diógenes Laercio (1985) would write it again later. Protágoras taught a man named Euatlo a lawyer in exchange for half his pay, on condition that he would complete the payment of the teaching when he won his first trial. Obviously, if he lost his first trial, he did not have to pay Protagoras. They both accepted this pact. However, after a certain time Protagoras still did not receive his money for having taught his student, then he asked him the reason and Eulato replied that he had not been able to pay him because he had not yet defended any lawsuit, as he had dedicated himself to other activities. Protagoras then decided to sue him for payment. The two men presented their arguments to the judge. Protágoras pointed out that, despite the result of the trial, Eulato must pay him, since if Protágoras wins the trial, Eulato must pay because the judge's ruling would oblige him and if Protágoras loses the

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trial, Eulato must pay him because according to the pact, Eulato would have won his first trial and that implied that he complied with canceling him financially. Eulato also wanted to take the floor. He claimed that he should not pay his teacher because equally, despite the outcome of the trial, he would not be obliged to do so. If Eulato wins the trial, then Protagoras' lawsuit would not take place and therefore the judge could not force him to pay. If Eulato loses his trial, then he would have lost his first trial, and according to the covenant, he should not pay him. The puzzling question is: who is right? Clearly, they cannot both be.

This paradox is very appropriate to formulate it to law students of early years of university, since it allows to know basic concepts of the career such as demand, plea, trial, pact and others. However, it is also useful to be able to differentiate between morality and law and, specifically, between moral norms and legal norms. Thus, the fact that a student must pay his teacher for what he learned could be considered a case of moral norm, however, the fact that the covenants and/or contracts must be fulfilled, can be considered as a case of legal norm. One way to solve this paradox is to determine which type of norm has the highest hierarchy. Leibniz proposed another solution in his doctoral thesis entitled *Disputatio Inauguralis de Casibus perplexis in Jure* de 1666 (published in Artosi *et al.,* 2013). He said Eulato will win the trial, but Protagoras could sue him again. In the latter case, the judge's ruling would be in favor of Protágoras since the condition of winning his first trial would already have been met by Eulato. The Monty Hall paradox will be exposed shortly.

Monty Hall paradox

In 1975 *The American Statistician* published Steve Selvin's letter (1975) where this paradox appears. It can be discussed concepts related to the probability theory. In a competition show, the driver named Monty Hall, offers contestants a car if they correctly choose a door of three on offer. If they lose, they will be given a comfort gift, namely a goat. A contestant chooses a door and the presenter does not open the chosen door, but opens another door behind which it is discovered that there is no prize. In that case, Monty Hall offers the opportunity to change doors or stay with the chosen one. The contestant begins to doubt that the situation seems to be used by the presenter to somehow persuade him to change doors. But should or should not the contestant change doors? What is the option that will make it more likely to win?

The analysis of this paradox allows to study in more detail the fundamental concepts of probability. In fact, this has been investigated in

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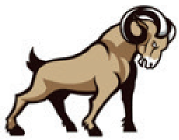
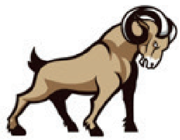
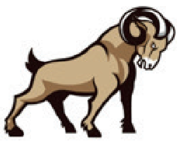
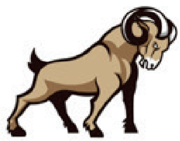
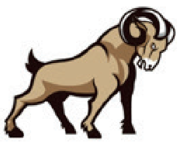
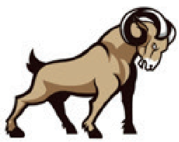
Gea *et al.* (2017). If considering that the prize is only behind one of the three doors presented, then it can be said that there is a 1/3 chance of winning. The show host opens one of the empty doors and asks if the person wants to change his/her choice. This could be interpreted as the probability of winning now being 1/2, but in fact it is not. What actually happens is that when you choose a door, you can have three situations, namely, the car can be behind the first or second or third doors.

Table 2
Outline of the Monty Hall Paradox

	DOOR 1	DOOR 2	DOOR 3
SITUATION 1			
SITUATION 2			
SITUATION 3			

Source: Own production.

First situation: if we have chosen the first winning door in which the car is, the presenter will show us an empty door. In that case, if we change, we lose.

Second situation: if we have chosen the first losing door, the presenter will show us an empty door because he knows that the car is in the second door. In that case, if we change, we win.

Third situation: if we have chosen the first losing door, the presenter will show us an empty door because he knows that the car is in the third door. In that case, if we change, we win.

This indicates that whenever we change, we have a better chance of winning than losing, specifically, twice out of three altogether. It is therefore advisable, despite appearances, to change doors. What is interesting about

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this paradox is that it is useful for exploring mathematical concepts associated with probability. Then, the paradox of God and the stone will be studied.

Paradox of God and the Stone

This paradox, which has medieval roots but appears in Savage (1967), could be taught during philosophy of religion or could even be posed as a challenge for theology students. Can God create a stone so big that he himself cannot lift it? If he can create it, then he can no longer lift it, and thus he could not do everything. If you cannot create it, then you could not do it all. Considering both options, God could not do everything, i.e., he would not be almighty. And if so, can we still call him God? Here the concepts of God and omnipotence are questioned.

A physical version associated with the above is linked to the paradox of the immovable object versus the irresistible force. On the one hand, an immovable object is one that no one or anything can move. On the other hand, an irresistible force is a force that encounters no opposition, i.e., that nothing can offer it resistance. What if an immovable object meets an irresistible force? This is another paradox.

These paradoxes could be replicated under the idea that the expressions "God (who can do everything) creates a stone so big that he cannot carry" and "the irresistible force (to which nothing can be resisted) is resisted by an immovable object", are contradictory phrases at the semantic level and, therefore, leads to paradoxes. The same is true of phrases such as "even number which is also odd", "singles who are married", "square circles", "the smell of blue", etc. All these phrases are nonsense and therefore the objects they allude to do not exist. In the first case, God is conditioned by an action that limits his nature, when in principle, nothing can limit him. In the second case, irresistible force also encounters a limitation (the immovable object) that destroys its own definition. What could be stated is that there are no worlds where God creates a stone that cannot be carried and where an irresistible force meets an immovable object. This could be the subject of discussion in a philosophy classroom or even a physics classroom. Next, we will know the Epicurus paradox.

Epicurus paradox

Lactancio (2014) in *De Ira Dei* attributed to Epicurus a paradox about God (Hickson, 2014). If God exists, why does He allow evil to exist? Maybe he does not know it exists. If so, God would not know everything, and

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this is absurd. Maybe he does know it exists, but he cannot help it. If so, God could not do everything and this is absurd. Maybe he does know that it exists, and he can avoid it, too, but he does not want to. But if it were so, God would not be good and this is absurd. So why does God allow evil to exist? Maybe he will do it to prove us. But this is useless because God, being omniscient, already knows what will happen and if so, does not need to prove us. Perhaps this is the devil's fault, yet if God is omnipotent and supremely good, he would have defeated the devil long ago. Another option may be free will. But is it possible that God can create a world with free will and without evil? If it is not possible, God cannot do everything, which is absurd, and if it is possible, then God also created evil, and that would not make it good, but again, this is absurd.

In *Theodicea,* Leibniz (2013) proposes a way to solve the question of the existence of evil in a world created by a very good God. God knows that pure goodness does not produce variety, but a little evil can generate greater good than goodness alone. Savater writes:

> Consider, for example, a library, and a work as extraordinary as the *Iliad*. An *Iliad* library is enriched by an important book. But let us imagine a library of ten thousand volumes and that they were all the *Iliad*. It would be a lower place, compared to others with the *Iliad* and nine hundred and ninety-nine other, but different, minor books. In other words, what seems to us to be a deficiency —not all works are as good as the *Iliad*— is actually an enrichment, because there is a diversity that otherwise would not exist (Savater, 2008, p. 111).

Here is another analogy elaborated based on Rawls (2006). This can be compared with the idea of allowing billionaires to exist in societies where there is poverty, social injustice, and inequality. Counterintuitively, it is good to allow very wealthy people to exist, because this generates more economic movement, and the economy could develop conveniently if not for everyone, then at least for the vast majority. In a country where there are no such people, there is no one who invests enough capital to move the economy toward progress. By contrast, in countries where free markets are allowed, economic progress is possible to some extent. Similarly, Leibniz reasons that if there were not a little evil, you could not give all the existing variety compared to other worlds where there is no evil. That is why there is evil in this world. This solution, however, is debatable and students could accept or reject this idea. The aim is to provoke discussion and exchange of views. Then, the paradox of time travel is analyzed.

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Paradox of time travel

This paradox (which is also known as the paradox of the grandfather) was raised in the novel *Le Voyageur Imprudent* by René Barjavel (1944) and can be used to discuss the nature of time with physics or philosophy students. Imagine that a person travels back in time to the past. Now, this unwitting traveler ends up killing his own grandfather. This creates a problem because if the grandfather did not live, then neither did his father and, consequently, the traveler would not exist either. But if that traveler did not exist, then the journey did not occur, the grandfather continued alive, he begat the traveler's father who, in turn, begat the traveler himself. This traveler would then visit the past and kill his grandfather, and so the paradox continues.

The issue relates to our understanding of time. Time can be conceived as linear or circular. This interpretation is cultural, although, since the Enlightenment, the West assumes that time advances forward marking the path of progress. As for the paradox, it could be said that once the traveler kills the grandfather, he should not have been affected, since another timeline would have been created where he was not actually born, but it is not the same traveler, but his version in that new timeline created. This question then involves opening the mind to ideas such as alternate timelines and possible worlds. This is precisely the trend of the latest science fiction films such as *2019's Avengers: Endgame* and *2022's Everything Everywhere All at Once*. The topic of possible worlds, in turn, can lead to review basic notions of modal logic. And the idea of alternate timelines allows us to think about whether historical events occur necessarily or contingently. If historical facts are necessary, then there are no alternate timelines; but if historical facts are assumed to be contingent, that means they can occur as not occurring. And the discussion can continue. Then, the paradox of the egg and the hen will be analyzed.

Paradox of the egg and the hen

This paradox appears in Question III of Book II of the *Desktop Talks* of Plutarch (1987) and raises the difficulty of deciding the origin of something. It is said that chickens come from an egg and that chickens also produce eggs. And this creates a vicious circle, because it would leave unexplained the idea of whether it was the egg or the chicken first.

In this regard, Aristotle's theory of act and power (trad. 1994) could be considered in order to attempt to evaluate the issue in a certain way. According to the philosopher, movement is the passage from act to power.

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For example, when a seed grows and becomes a plant, it can be said that the power of "being plant" was updated in the seed and thus reached its perfection (*entelechy*). Now, in relation to the paradox, it can be affirmed that there is the power of "being a chicken" in the egg with which it would reach its perfection. In that sense, the egg would be first because while it is true that the egg turns into a chicken, it is not true that the chicken turns into an egg. The chicken produces eggs, which is very different. However, Aristotle also raised controversial metaphysical ideas. For example, he stated that what is first in time at the physical level is not first in being at the metaphysical level. Thus, it can be seen that there is a preeminence of the intellective over the sensible because the first, being immaterial, is neither corrupt nor contingent. At this point it is seen the influence of his master Plato. Aristotle asserts that for the child to become a man, there must somehow be the power to "be a man" as a precondition, as a kind of plan to follow. From this point of view, the power "to be chicken" would come first. As seen within Aristotle's theory of act and power, the question remains undecided.

However, we can suggest that students pursue up-to-date biological theories on their own, so that they can make a well-founded, scientifically backed judgment, and not just on the basis of free, if interesting, speculation. For example, from evolutionary biology it can be stated that the chicken, as a bird, comes from reptiles. And besides, the reptiles mostly reproduce by eggs. Over hundreds or thousands of years, one of those eggs gave rise to a protochicken, which, when it spawned, gave rise to another egg, from which, through successive breeding processes over hundreds or thousands of years, a chicken emerged as we know it today. If this were true, the egg would come first. But let us remember that science is constantly self-correcting. So in the future this could change. For this reason, it is always urgent to be updated on the progress of science. Next, we will know some geometric paradoxes related to fractals.

Geometric paradoxes. Fractals

Geometry is that part of mathematics that studies space. There are many known geometric theorems, especially those dealing with triangles. For example, the Pythagorean theorem is particularly famous. However, there is one property that is widely recognized by students, namely the sum of internal angles of a triangle equals 180 degrees.

The thing is that this truth is acceptable to some extent because there are other geometries where the sum can be more. Think of a sphere.

Sophia 35: 2023.

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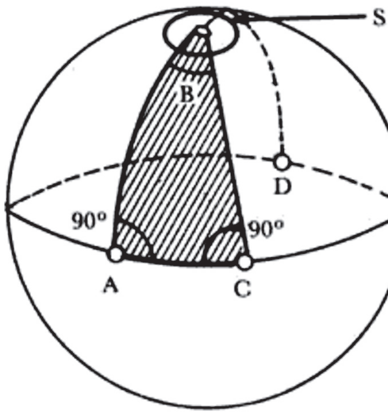
Let us look at the equator and the triangle formed by meridians coming from the same pole as in the following image.

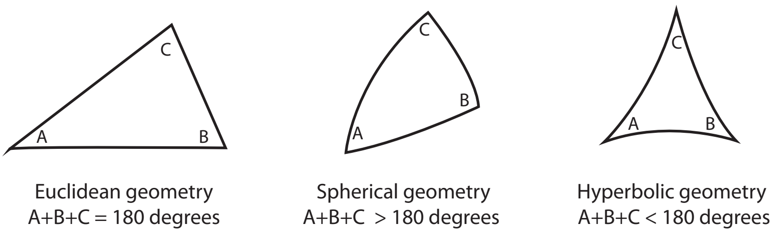
Figure 2
Analyzing a Sphere



Source: Sánchez, 2022.

It is evident that the angles that meridians make with the equator are 90 degrees and if we add the angle B at the top, we would have a triangle whose sum of angles is more than 180 degrees. This geometry is called "spherical." But there is also another called "hyperbolic", where the sum of internal angles is less than 180 degrees.

Figure 3
Three Different Geometries



Source: Own production.

So, when asked, how much do the inner angles of a triangle add up to? The answer should be: "it depends on what geometry we are placing ourselves in."

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Similarly, the question of how many dimensions a figure has depends on the geometry in which we are located. Even the dimension might not be an integer, as occurs between fractals. Precisely, the fractal theory was proposed by Mandelbrot (1983). This is a scientific theory that aims to study the patterns that govern fractures, roughness and cracks. Benoît Mandelbrot writes: "Why is geometry often described as "cold" and "dry"? One reason is their inability to describe the shape of a cloud, a mountain, a shoreline, or a tree. The clouds are not spherical, nor the conical mountains, nor the circular coasts, nor the crust is soft, nor is the ray rectilinear" (Mandelbrot, 1983, p. 15). Fractals are mathematical objects whose basic structure, irregular or fragmented, is repeated at different scales. They have the following traits: they are very irregular, they are self-similar, and their dimensions are given by fractional numbers.

Then, three fractals will be presented and their paradoxical aspects will be analyzed. However, it is important to note that the controversial aspect of these figures disappears when it is understood that they are part of the theoretical framework of a geometry totally different from the usual one.

Cantor powder

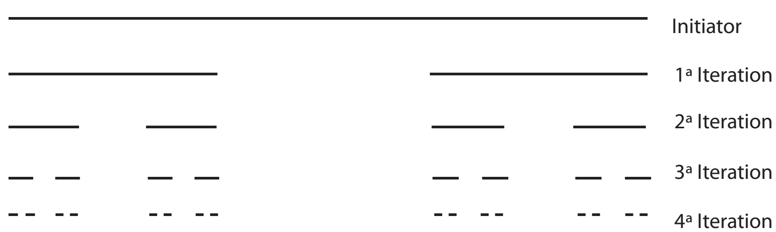
Cantor dust is constructed according to the following steps:

First step. It is a line that has to be divided in three. The center segment is then deleted. This is the first iteration.

Step two. Divide the other segments into three and delete the middle portion of each of the two segments. This is the second iteration.

Third step. The same must be done for the next remaining segments to infinity.

Figure 4
Iteration of Cantor's powder



Source: Argote, April 6, 2013.

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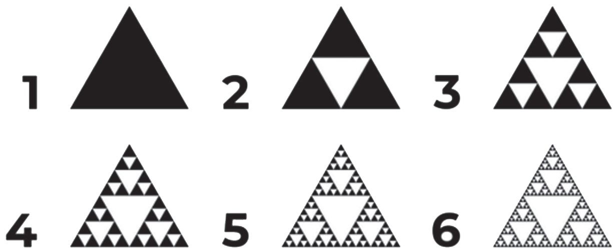
The figure resulting from applying the above process infinitely is the Cantor dust. The problem with this figure lies in placing it in its geometric space. How many dimensions does it have? It is not a point, it is not a straight line, it is not a figure. What is it? It is a type of figure that corresponds to a fractal. A fractal is a geometric construct whose dimensions are not given by whole numbers, but rather by fractional numbers. The dimension of this figure is between 0 and 1, i.e., 0.6309297.

Sierpinski triangle

This triangle is constructed following these steps:

Figure 5
Construction of the Sierpinski Triangle



Source: Olexandrgodomich, 2022.

First step. We start from a normal equilateral triangle. Since the side is 2, its perimeter is 6. In addition, we know that the area of the region shaded with black color of this figure is $\sqrt{3}$ since the formula for finding the area of an equilateral triangle is $\frac{l^2\sqrt{3}}{4}$ (i.e. side-by-side by $\sqrt{3}$ between 4).

Step two. Next, we divide the area into four, erasing the area piece from the center. The perimeter of the 3 triangles is now: 3.3 = 9. Also, since it has been divided between 4 and, in addition, we have left 3 pieces, the area of the shaded region is equal to 3/4 $\sqrt{3}$

Third step. We reapply this process, i.e., we divide each triangle into four parts and erase the piece of central area. The perimeter of the 9 triangles shall be: 3.3.3 (1/2) = 27/2. Meanwhile, because it has been divided between 16 and, in addition, we have kept 9 pieces, the area of the shaded region is $(3/4)^2\sqrt{3}$.

Fourth step. We continue to implement this process and obtain the following results. The perimeter of the 27 triangles continues to increase

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and is: 3.3.3.3.(1/4) = 81/4. And its area, based on similar considerations to the above, equals $(3/4)^3 \sqrt{3}$

Step five. In this fifth phase we apply the same. The new perimeter is: 243/8 and the area will be $(3/4)^4 \sqrt{3}$

As can be seen, this fractal manifests an uncommon relationship between the area and its perimeter. While the perimeter tends towards infinity, the area tends to be zero. In Euclidean geometry it often happens that a figure with an infinite area has an infinite perimeter and, in turn, a figure with an infinite perimeter has an infinite area. In addition, a figure with an area equal to zero should have no graphic existence, which is not the case with the Sierpinski triangle. Also, a figure with perimeter equal to infinity has measures from its sides to infinity or has infinity sides. But, in the triangle analyzed it happens that there are only accumulations of points everywhere. The fractal dimension of this object is 1.58496.

Koch Snowflake

This figure is constructed as follows:

First step. It starts by analyzing the perimeter of the image (a) in Figure 10 that represents an equilateral triangle. If each side is equal to 1, its perimeter is 3. The area is worth $(\sqrt{3})/4$ and we know this when applying the equilateral triangle formula.

Step two. Divide each side into 3 parts and on the middle parts build other equilateral triangles as in the image (b). 6 1/3 segments have been added, but we have deleted 3 1/3 segments. In total we have increased 3 segments of 1/3. The new perimeter is: 3 + 1. Since three new equilateral triangles have been added whose sides are worth 1/3, the new area is now worth $(\sqrt{3})/4 + (\sqrt{3})/12$

Third step. Repeat the process. We start by dividing those 1/3 segments into three parts and then we do everything else. In the end we have to increase 24 segments of 1/9, but we also erase 12 segments of 1/9. In total we have increased 12 segments of 1/9, or 4/3. The new perimeter is: 3 + 1 + 4/3. Because twelve new equilateral triangles have been added whose sides are worth 1/9, the new area is now worth $(\sqrt{3})/4 + (\sqrt{3})/12 + (\sqrt{3})/27$.

Fourth step. Again, we are going through this whole procedure again. The perimeter would be equal to: $3 + 1 + 4/3 + (4/3)^2$. And if we continue like this, this process repeated infinitely will result in the con-

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stant increase of a new power of (4/3). We will therefore have a figure whose perimeter is Z.

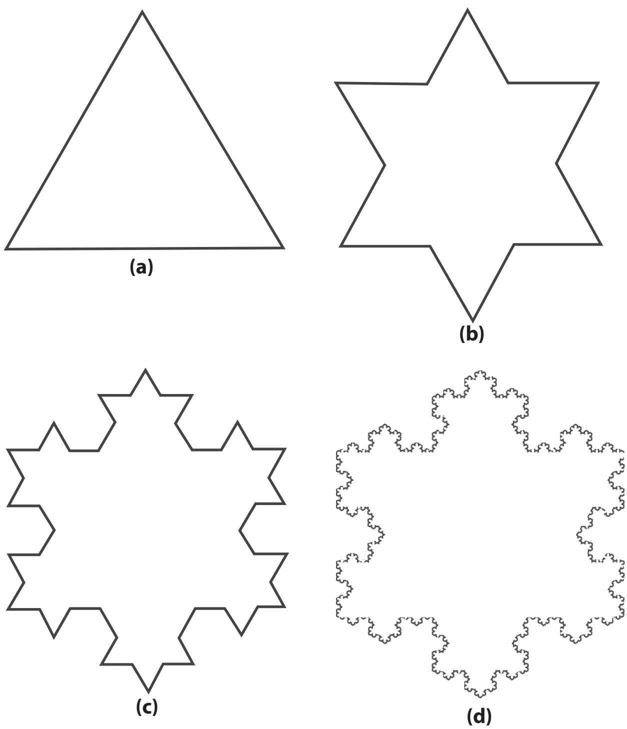
$$Z = 3+1+4/3+(4/3)^2+(4/3)^3+(4/3)^4+ \ldots$$

Well, Z necessarily tends to be infinite. But graphically we observe its finitude. On the other hand, area A measures

$$A= (\sqrt{3}\ )/4 + (\sqrt{3}\ )/12+ (\sqrt{3}\ )/27 + \ldots$$

This means that it tends to be finite, albeit small. This amount does not exceed 0.7 and is approximately 0.6928. Therefore, this figure retains certain properties of the flat figures as the finitude of the area, but distorts others as the infinity of its perimeter.

Figure 6
Koch Snowflake



Source: Northrop, 1949, p. 190.

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All two-dimensional flat figures have a finite perimeter because they are bounded by closed lines. However, the figure analyzed has an infinite perimeter despite the fact that it is graphically possible to detect its finitude. With the above, it is stated that this snowflake breaks with one-dimensionality, since it is projected to infinity exceeding the straight segments. But in the two-dimensionality plane the finitude of the area of closed polygons is still respected. Therefore, this snowflake figure is actually a fractal whose dimension is more than that of the line (meters) and less than that of the plane (square meters), i.e., it is between 1 and 2. Exactly, its dimension is 1.26186. Now, there will be a discussion about how a student feels when faced with these paradoxes.

How does a student feel about paradoxes?

When a student understands the true and frightening problematic nature of a paradox, he is astonished, but he may feel some vertigo. The same sensation experienced when watching a very interesting series is repeated, and suddenly the triumphant and heroic protagonist is pierced by a sharp sword that the antagonist manages to use with his last breath.

"Ahhh! Ohhh! ... What?..." Surprised students often express themselves emotionally and affectively in the face of these big problems. Even paradoxes may seem to scare them or annoy them. That is the idea. A class cannot become the exposition of a list of knowledge or knowledge that the teacher coolly transmits to his pupils. In fact, a good educational session should motivate them so that they, on their own, can continue researching. And this consideration implies that the teacher must teach transmitting values, i.e., with love, respect, care and diligence. At the same time, there must be authority in the classroom by trying to make students realize that the teacher is a man or a woman of culture. From this perspective, teachers are guardians of culture and thought.

The teacher must be prepared. The education provided must be based on strategies so that the basic concepts of the student can "artificially" enter into crisis. The teacher must dose the use of these powerful paradoxes so that the pupil can learn that even the safest thing in the world falls under the powerful weight of critical thinking. Teachers must help them overcome despondency, fear, lack of freedom, what they will say, abuse and, finally, everything that endangers our humanity. This shows that both reasoning and emotional intelligence are at stake in the educational process (Bravo and Urquizo, 2016).

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The student must feel that learning is a beautiful thing. He must feel challenged not so much by the teacher as by his own mind. It is necessary that the teacher can master the use of paradoxes as teaching resources in the current situation of our education. Due to this unfortunate situation, the student assumes that going to school hardly helps him to achieve his most practical goals in order to be able to join the work reality. Paradoxes can serve as support to improve the educational reality. The time has come for that old way of teaching that keeps students away from discussion, controversy, debate and the desire to know more every day to disappear.

Conclusions

In this article paradoxes have been considered as didactic resources. Thus, the concepts of fallacy and reduction to absurdity were clarified, since paradoxes have been seen as very subtle fallacies by some scholars, and, in addition, there are those who use paradoxes to make deductions as occurs in the reduction to absurdity.

Then a list of paradoxes is analyzed to use them in the classroom. So, it is about some paradoxes such as the Achilles and the Turtle paradox, the Galileo paradox, the Hilbert hotel paradox, the Tristan Shandy paradox, the Protagoras paradox, the Monty Hall paradox, the God and the stone paradox, the Epicurus paradox, the time travel paradox, the egg and hen paradox, and some geometric paradoxes related to fractals.

This research has been completed trying to make explicit the emotional and affective aspect that a student experiences when dealing with this kind of problems, i.e., it tries to explain what a student feels when his teacher presents him a paradox.

There are findings as well as limitations in this work. This research has some difficulties. First, teachers would have to constantly develop academic works to find paradoxes and thus spread them in their different classes. Secondly, the paradoxes that have been selected in this paper have been somewhat known in other academic fields and, in that sense, do not represent any novelty. Finally, thirdly, the fact that there are so many paradoxes could cultivate in students a skeptical perspective about reality and knowledge in such a way that they choose to remain silent rather than continue investigating.

Finally, we will point out the achievements of this paper. This work has sought to reveal the educational and didactic aspect of the paradoxes. It is recommended that the problem generated by the finding and expo-

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sure of a paradox be exploited didactically. The truth is that paradoxes can be allied in the teaching-learning process and their use can benefit the educational activity. The teaching that paradoxes leave us is very valuable, as it reveals that what we call "knowledge" is something that can always be constantly expanded. Paradoxes have propedeutical value and, as attention-grabbing problems, teach that there are limits to our understanding of some phenomenon.

The way to learn through paradoxes involves trying to solve the mental challenges posed. With paradoxes, students of any career can think properly about the fundamental concepts of their own specialty. Thus, the education provided must be based on strategies so that the basic concepts of the student can "artificially" enter into crisis. With paradoxes, the pupil learns that even the safest thing in the world falls under the powerful weight of critical thinking. The solution (or dissolution) proposals that inspire the paradoxes are questionable and the students could accept or reject them, precisely, the aim is to provoke discussion and exchange of views. When faced with paradoxes, one feels like talking, expressing opinions, or at least thinking about the issue carefully.

If a response is not forthcoming, the teacher can commit students to researching up-to-date theories on their own so that they can make a well-founded, scientifically-supported opinion, not just on the basis of speculation. A class cannot become the exposition of a list of knowledge that the teacher coolly transmits to his pupils. In fact, a good educational session should motivate students so that they, on their own, can continue researching so that they know more about the issue raised. Teachers must help them overcome despondency, fear, lack of freedom, what they will say, abuse and, finally, everything that endangers our humanity. And the path of research is a good path to exercise our freedom without fear and wanting to know more and more.

When a student understands the true and gloomy problematic nature of a paradox, he is astonished in a way much like that experienced by early philosophers at seeing the order and chaos exhibited by the world. Surprised students often express themselves emotionally and affectively in the face of these big problems. When they know the challenge posed by a paradox, there is some concern on their part. Accepting these challenges will make the student more prepared for decision-making in his daily life, as he will have exercised critical thinking in a convenient way. Likewise, a paradox teaches us what we do not know, makes us aware of our limits. In that sense, it makes us more humble. The student must feel that learning is something revitalizing. Life should not be forgotten dur-

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ing the teaching-learning process. The pupil must feel challenged not so much by the teacher as by his own mind.

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