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Mathematical model of a resistive oven for thermoforming polypropylene sheets

Modelo matemático de un horno resistivo para termoformado de láminas de polipropileno

Lidia Castro-Cepeda[1,*], José Cortés-Llanganate[1,2]

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Abstract

A mathematical model of a resistive oven for the production of thermoformed sheets is developed in this paper; such oven is located in a production plant in the city of Riobamba. The objective of the research is to achieve temperature stability and that the plates have a homogeneous dimension when going through the thermoforming process, to guarantee customer satisfaction. For this purpose, the physical variables that govern the heat transfer phenomena, namely radiation, convection and conduction, are analyzed, to obtain a mathematical model that predicts the temperature profile of the oven in the thermoforming process, from which a controller is designed using various control techniques that are efficiently coupled to the system. A theoretical study of the physical phenomena and of the mathematical equations that represent them is proposed in the first stage of the research. Then, they are solved through computational techniques using Simulink to obtain the temperature profile. Finally, this model is validated by comparing it with those obtained in previous works through statistical techniques, and a new controller that guarantees minimum temperature variability is proposed. As a result of the simulation, a variation of ±1 mm in the width of the plate is achieved.

Keywords: heat transfer, modelling, polypropylene, temperature, thermoforming

Resumen

En el presente trabajo se desarrolla un modelo matemático de un horno resistivo para la producción de planchas termoformadas, localizado en una planta de producción de la ciudad de Riobamba. El objetivo de la investigación es conseguir la estabilidad de la temperatura y que, al pasar por el proceso de termoformado, las planchas tengan una dimensión homogénea que garantice la satisfacción del cliente. Para ello se analizan las variables físicas que rigen los fenómenos de transferencia de calor; radiación, convención y conducción, y así obtener un modelo matemático que prediga el perfil de temperatura del horno en el proceso de termoformado, a partir del cual se diseña un controlador utilizando varias técnicas de control que se acoplen al sistema de forma eficiente. En la primera etapa de la investigación se plantea un estudio teórico de los fenómenos físicos y las ecuaciones matemáticas que los representan. Luego son resueltas a través de técnicas computacionales usando Simulink para conseguir el perfil de temperatura. Por último, se valida este modelo comparándolo con aquellos ya obtenidos en trabajos anteriores a través de técnicas estadísticas; finalmente, se propone un nuevo controlador que garantice la variabilidad mínima de la temperatura. Como resultado de la simulación se consigue una variación de ±1 mm del ancho de la plancha.

Palabras clave: modelado, polipropileno, temperatura, termoformado, transferencia de calor

[1,*]Facultad de Mecánica, Escuela Superior Politécnica de Chimborazo - Ecuador. Corresponding author ✉: lidia.castro@espoch.edu.ec.

[2]Jefe de Planta Prefabricados Riobamba, Unión Cementera Nacional UCEM S.A - Ecuador.

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1. Introduction

The worldwide plastics industry has grown in recent years. Besides the virgin raw material production data, this is also evidenced in the percentage of plastic that is currently recycled in the countries of the world. Regarding recycling, the plastic had an increase of 80 % in 10 years in European countries [1].

In Ecuador, the plastic is used in various production fields, such as: food industry (drinks packages, snacks, etc.), construction industry (translucent roofs, plastic tiles, curtains, etc.), kitchen utensils in general [2]. Specifically, there is an industry in the city of Riobamba devoted to manufacturing translucent polypropylene sheets that are used as a complement of fiber cement roofs.

The plant has three production lines that have been assembled with recycled technology, coming from countries such as Spain; they are acquired at lower prices for reuse after they have completed their useful life cycle. Then, it is repowered in local industries making some adjustments and even mechanizing parts and pieces that are coupled to the system so that they operate with the greatest possible efficiency. The process for transforming the polypropylene is summarized in Figure 1.

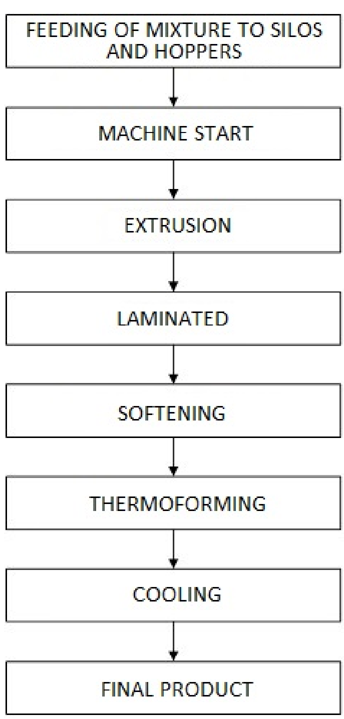

Figure 1. Process for transforming the polypropylene

The main machine of this line is a thermoforming machine that produces type P7 polypropylene sheets. This research work has been carried out on the input oven of such plastic thermoforming machine. Previous works were conducted to control and improve the production of the thermoforming oven.

The first work consisted in setting up a SCADA system for temperature control. It was based on a hysteresis control system, where the resistances of the machine were turned on and then turned off when they reached the desired value of temperature [3]. At present, the described system is being used in the area of extrusion, distribution and output ovens. Figure 2 shows the architecture of the network used in the SCADA system.

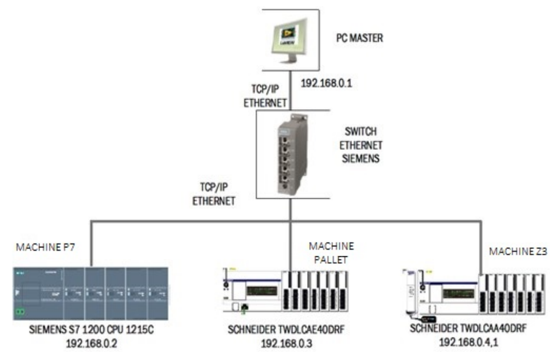

Figure 2. Architecture of the local area network of the machines [3]

The second work consisted in the design of a controller for the input oven, based on system identification. This enabled obtaining the transfer function of the oven. Once such function was identified, a PI controller was calculated for its control [4]. This system is being currently used in the production plant to control the input oven.

Up to this moment, the system has not been modeled mathematically, and thus the real behavior of its variables is unknown. Therefore, it has been considered necessary to find this mathematical model, which will be compared with the existing models. Based on them, modifications that improve the product quality will be suggested in the controllers. At present, the product meets the current regulations of the country, but it is observed that its width is not completely uniform. When it is stacked for distribution and sale, the customer has the sensation and perception that the plates do not meet the regulation, due to the variability of their dimensions. Therefore, the main problem of this oven is that it produces plates with a very variable width that causes that some clients do not accept the product.

Among the research works analyzed, [5–8], Neacă and others use the heat transfer phenomena to quantify the temperature through the mathematical model found, which is similar for this case study. However, each publication is distinguished because its entire system is in accordance with the particular features of each oven. In contrast, the study by Throne [9], [10] is based on two variables, namely the molar absorptivity and the emissivity, emphasizing that they are fundamental in the development of the model. Following the recommendation of this author, the appropriate data will be used in the radiation analysis, which are detailed in the solution statement.

Meanwhile, Khan [11], Erdogan [12] and Chy [13] include in their research work specific properties such as density, thermal diffusivity, thickness, specific heat and thermal conductivity, data that are also considered in the development of this research work, for the conduction and convection analysis, considering the accessibility to the equipment and the features detailed in the description of the oven structure.

On the other hand, Schmidt [14] and Ajersch [15] use infrared sensors and thus their study is aimed at radiation analysis; these publications reach results that fit the reality, but due to the experimental conditions of the oven under study it is not possible to replicate this technique. However, it will be useful to compare and discuss the results at the end of this research. This is similar to Chy and Boulet [16], who divide in layers the material to be heated, and then interpret the results as a single element through numerical techniques; it is not possible to carry out this condition in this study, but it will be useful to contrast such results when formulating the conclusions.

Even though all these researchers developed acceptable mathematical models of the oven, especially for the heating phase, there are still some discrepancies between the simulation, the experimental results and the variables used by each author in his/her research work; in addition, such mathematical models fit the particular features of each oven.

Therefore, this research is intended to develop an improved model for the oven under study that fits its features. It is taken into account that the process is mainly executed through trial and error, based on the intuition and experience of the machine operator. Consequently, it arises the initiative of understanding the oven heating process, specially emphasizing on the features typical of its construction knowing that it is an oven manufactured empirically. With this mathematical model it is sought to establish a more precise way to predict the temperature profile inside the oven, in order to use it in the future for a better control of the process.

There is a study carried out previously about the oven under analysis. In 2017, Cortés implemented a temperature control system for the thermoforming oven of the machine known as P7. It briefly describes the oven, stating that «it consists of metallic plates, that constitute a chamber. Such chamber has a glass wool insulation to reduce heat irradiation to the outside. S-type and U-type resistances (known as such due to their shape) of different electric power are located inside» [3]. The author obtains a model of the oven through system identification graphical techniques, as a step prior to the design of the corresponding controller. Various techniques are applied, and compared with each other. At the end of the corresponding validation, it is chosen the Analytical without Delay graphical method, which yielded the highest percentage of 90.95%.

Regarding the model obtained by means of system identification, Cortés [4] performed the associated calculations to obtain a system controller, which yielded acceptable results with respect to the temperature variability. Nevertheless, there is still the issue of the width of the plates. The purpose of obtaining the mathematical model of the oven of interest seems simple, but putting it into practice is much more complex, since it largely depends on the data available. The aim is to provide a system model that shows the real dynamic behavior of the oven, thus enabling to calculate different controllers that finally optimize its operation.

2. Materials and methods

The methodology to develop this research work is summarized in the diagram of Figure 3.

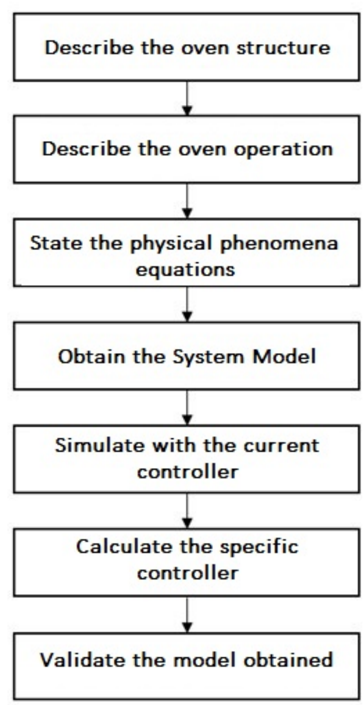

Figure 3. Working methodology

2.1. Description of the oven structure

The oven under study, which is shown in Figure 4, was constructed locally, with ohmic electric resistances that heat up by Joule effect and release their heat. The oven has been constructed with recycled elements, machine pieces and equipment, which are coupled to each other to fulfill the requirements of the production process. For this specific case, the industrial production of thermoformed plates is used nationally in different applications. The oven is constituted by metallic plates. It is parallelepiped-shaped, constituted by two boxes, one inside and another outside made up of 2 mm galvanized steel sheets. Such boxes are separated 5 cm with respect to each other, and insulation is placed between plates. For this design it was used fiberglass manufactured from a mixture of sands, borates and silicates, thus fulfilling the parameters recommended by various authors mentioned in the state of the art. For this reason, from this point on it will be supposed that the oven has the insulation necessary to avoid significant losses that modify the temperature profile of this oven.

Figure 4. Thermoforming oven of machine P7

2.2. Description of the oven physical phenomena

To take measurements about the real operation of the oven under study, two J type thermocouples were placed with the purpose of measuring the temperature at the center of the oven and in a lateral wall every second, during the time necessary to reach stability, approximately 5000 seconds. The oven has a built-in data acquisition system, and these data were stored in a computer connected to the equipment, and will be used to obtain the mathematical model. To make the oven study simpler and more practical, the oven operation has been divided into the three stages that are detailed in Figure 5.

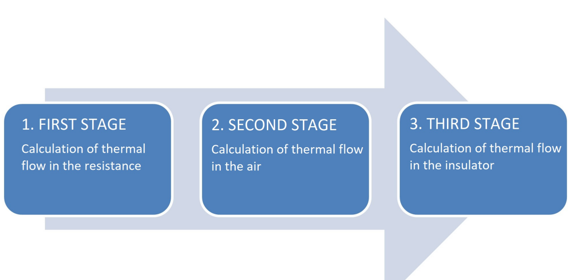

Figure 5. Oven operation diagram

Table 1 shows the nomenclatures to be used in the different sections of the present paper.

Table 1. Nomenclature used in the different equations

Abreviatura	Significado
$i(t)$	Electric current that circulates through the conductor.
$u(t)$	Voltage to which the electrical resistance is connected.
$R(T)$	Resistance of the conducting cable as a function of the temperature.
ρ	Resistivity of the resistance as a function of temperature.
L	Length of the resistance cable.
A	Area of the conductor cross section.
$p(t)$	Electric power.
$\dot{Q}_{sto}$	Thermal flow stored in the resistance.
$\dot{Q}_{rad}$	Heat thermal flow by radiation.
$\dot{Q}_{conv}$	Heat thermal flow by convection.
m	Mass.
$C_c(T)$	Specific heat of the conductor as a function of temperature.
$\triangle T_w$	Heating of the resistance (temperature difference).
T_0	Initial temperature (Ambient temperature of the place, initially $T_w = T_0$).
ε	Emissivity constant.
C_n	Stefan-Boltzmann constant.
S_w	Surface of the resistance cable.
T_1	Temperature inside the oven surface.
T_w	Temperature of the cable.
α	Convection coefficient of the air inside the resistance.
T_a	Temperature of the air inside the oven.
$\dot{Q}_{conv\ t-a}$	Total thermal flow produced by the resistances in the air.
$\dot{Q}_{conv\ a-p}$	Thermal flow produced by the air on the wall.
$\dot{Q}_{sto\ a}$	Thermal flow stored in the air.
$a_p(T)$	Convection coefficient of the air and the oven wall as a function of the temperature.
S_p	Internal surface of the oven walls.
m_{a1}	Mass of the air inside the oven.
$c_a(T)$	Specific heat of the air as a function of temperature.
T_a	Temperature of the air inside the oven with no load.
$\dot{Q}_{out\ cond}$	Thermal flow transferred to the outside by conduction.
$\dot{Q}_{sto\ insul}$	Thermal flow stored in the insulation.
k	Thermal conductivity of the material.
A	Area through which the heat flows.
$\triangle X$	Insulation thickness.
$\triangle T$	Temperature difference.
P	Density.
V	Volume.
y_{model}	Values obtained from the model.
$y_{observed}$	Values observed in the experiments.
N	Number of data.

2.2.1. Calculation of heat flow in the resistance

It starts with the transformation of electric energy into thermal energy in the resistance. It is fundamentally based on the basic Ohm's law given by equation (1).

$$u\,(t) = i\,(t) * R(T) \tag{1}$$

The only element in the oven that takes part in this energy transformation is the resistance, whose calculation is fundamental because it is unknown when constructed manually. Equation (2) is used for this purpose, which involves the resistivity, a function of temperature, the length and the cross section of the resistance, data that can be easily obtained through measurements, tables and knowing the material of which it is made of.

$$R = \rho * \frac{L}{A} \tag{2}$$

Then, it is applied the energy conservation law in thermodynamics, which states that the amount of heat received by the system is transformed and performs work against external forces, as shown in equation (3).

$$p\left(t\right) = \dot{Q}_{sto} + \dot{Q}_{rad} + \dot{Q}_{conv} \tag{3}$$

The thermal flow stored in the resistance is expressed through equation (4).

$$\dot{Q}_{sto} = \frac{d\left(Q_{sto}\right)}{dt} = \frac{d\left(m * C_C\left(T\right) * \Delta T_w\right)}{dt} \tag{4}$$

The heating by radiation is verified in equation 5.

$$\dot{Q}_{rad} = \varepsilon * C_n * S_w * \left(T_w^4 - T_1^4\right) = K_1 * \left(T_w^4 - T_1^4\right) \tag{5}$$

2.2.2. Calculation of thermal flow in the air

After carrying out the corresponding calculation of resistance, it is continued with the equations for the air inside the oven, with the purpose of relating the heat transfer phenomena and developed the desired mathematical model. The total thermal flow, presented in equation (6), is obtained from the equation that relates the thermal flow produced by the resistance and the thermal flow by the air on the wall.

$$\dot{Q}_{conv\ t-a} = \dot{Q}_{conv\ a-p} + \dot{Q}_{sto\ a} \tag{6}$$

The thermal flow produced by the air on the wall is expressed using again the convection phenomenon. Afterwards it is calculated the thermal flow stored in the air, which is expressed in (7).

$$\dot{Q}_{sto\ a} = \frac{d}{dt}\left(Q_{sto\ a}\right) = \frac{d}{dt}(m_{a1}c_a(T)(T_a - T_0)) \tag{7}$$

2.2.3. Calculation of thermal flow in the insulator

After the study of heat transfer in air has been completed, it is carried out the insulation analysis. Equation (8) is used for this purpose, which defines the heat flow on the wall due to the air movement, which is equal to the sum of the thermal flow transferred to the outside by conduction and the heat stored in the insulator itself.

$$\dot{Q}_{conv\ a-p} = Qconduction\ ext + Qsto\ insul \tag{8}$$

The oven has an insulating wall made of fiberglass, and hence it is necessary to quantify it, and the thermal conduction phenomenon is used for this purpose. The mathematical expressions and definitions that will be used in the solution of the mathematical model has been explained in a general manner, and the following section details the most important calculations that will be fed to the system.

2.3. Solution statement

After presenting in detail the physical phenomena supported on classical theory that appear during the heating of the oven for the thermoforming production process, it is proceeded with the mathematical calculation with the purpose of obtaining the differential equations that will enable finding the desired mathematical model. The Simulink mathematical tool was used with the aim of implementing computationally the solution to the problem, since it is visual programming environment that operates on Matlab.

The data in the technical sheet of the material is used to start with the calculations of resistance. These data were found in the company catalogs to specifically verify that the correct information supplied by the provider is used. The conductor used in this case is Nikrothal 70 [17]. Appropriate calculations are carried out that lead to equation (9).

$$T_w = \int \left(\frac{\left(\frac{u^2(t)}{R} - K_1 * \left(T_w^4 - T_1^4\right) - \alpha * S_w * (T_w - T_a) \right)}{m * Cc} \right) dt + T_0 \tag{9}$$

This equation is implemented in Simulink, and the scheme of inputs and outputs may be visualized in Figure 6.

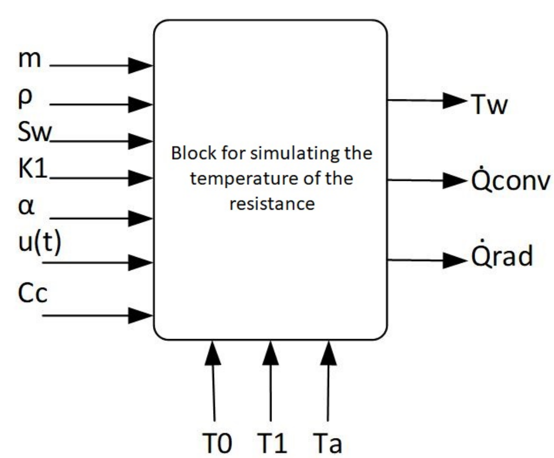

Figure 6. Scheme of inputs and outputs for calculating the temperature of the resistance

Then, the behavior of the resistance as a function of the temperature is obtained, and with it the heat that transforms through the physical phenomena already described. Such implementation may be observed in detail in Figure 7, which corresponds to equation (9).

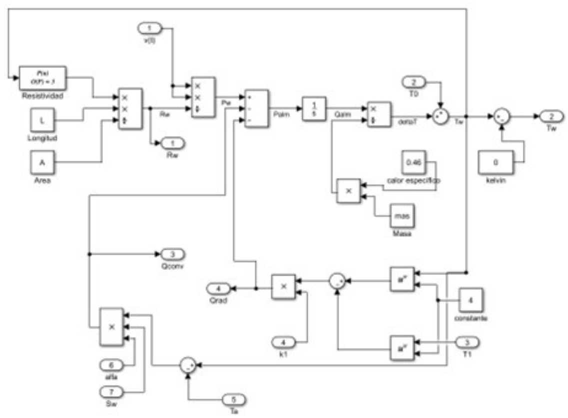

Figure 7. Implementation of equation (9)

Figure 8 shows the response of the resistance temperature as a function of time when it is connected to a voltage of 440 VCA; such temperature profile will be used in the model of the thermoforming oven.

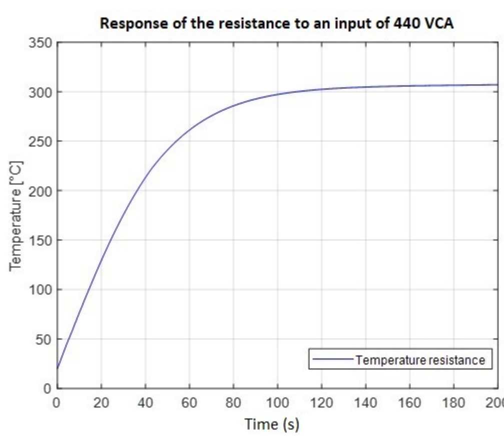

Figure 8. Resistance temperature along time

Finally, the temperature of the oven is calculated, resulting in equation (10) that is implemented in Simulink. Such equation is the result of the mathematical calculations of the fundamental equations of thermodynamics.

$$T_a = \frac{\int (\dot{Q}_{conv\ t-\ a} - a_p\,(T)\,S_p(T_a - T_1))\ \ dt}{m_{a1} c_a(T)} + T_0 m_{a1} c_a(T) \qquad (10)$$

Figure 9 shows the basic scheme that describes the inputs and outputs of the Simulink implementation of equation (10).

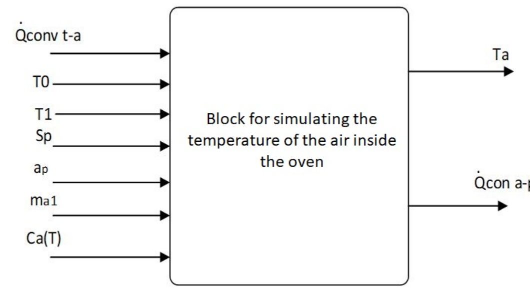

Figure 9. Scheme of inputs and outputs for calculating the temperature of the air inside the oven

The calculation of the temperature of the air inside the oven, equations and data obtained through tables and calculations are implemented in Simulink. Such implementation may be observed in detail in Figure 10, which corresponds to equation (10).

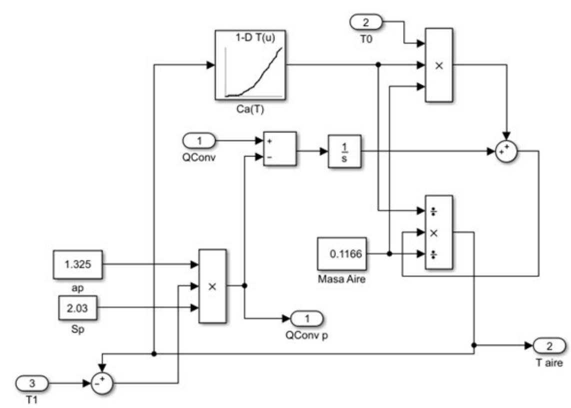

Figure 10. Implementation of equation (10)

After the calculation of the thermal flow of the resistance and the air of the oven is available, the subsequent step is to calculate the temperature lost in the insulation to obtain the temperature of the oven wall. The temperature of the air can be obtained relating all these calculations, data that should be known for its further processing in the calculation of the control systems. The data obtained through the oven data acquisition system is used in the case of the wall temperature. Figure 11 shows the total implementation for calculating the temperature of the air inside the oven. The blocks defined in previous steps have been used for this purpose.

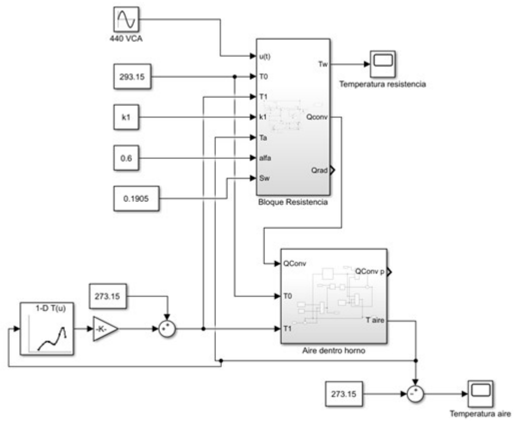

Figure 11. Total implementation of temperature calculation in the air of the oven

Figure 12 shows the response of the mathematical model comparing it with the response of the oven obtained through data acquisition. As it is observed, the model fits very well the real performance curve. This enables to predict that the validation of the mathematical model will be better than the validation of the model obtained by means of graphical identification techniques published by Cortés [4].

A step input has been applied to both the real system and the mathematical model for obtaining Figure 12. For the real system, a voltage of 440 VCA has been applied to the resistances. A similar value of voltage has been applied to the mathematical model. The data acquisition in the oven is carried out reading the type J thermocouple, which is sampled at a period of 1 second.

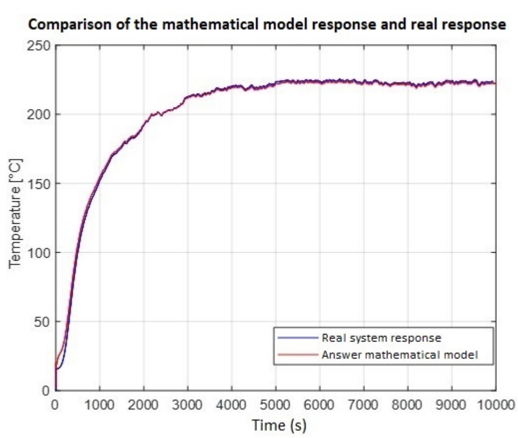

Figure 12. Comparison of the response of the mathematical model *vs* the real response

3. Results and discussion

3.1. Validation of the model obtained

The mathematical obtained is validated using mathematical techniques that enable quantifying it. This section considers various points of view, as explained below.

3.1.1. Comparison of the root mean square error of the mathematical model *vs* the current model

Figure 13 is similar to Figure 12. All the curves shown are the responses to a step input of 440 VCA. The behavior of the real system is shown in blue, whose data were taken experimentally through the data acquisition system and processed in MATLAB. The response of the mathematical model obtained through the calculations described in previous sections is shown in red. The difference is in the green curve, which corresponds to the response of the model obtained by graphical identification to a step input of 440 VCA; this response is currently used in the plant for calculating the controller. It may be visually inferred in this plot that the mathematical model is similar to the profile of real temperatures, and so it is expected that this error is smaller when it is quantified.

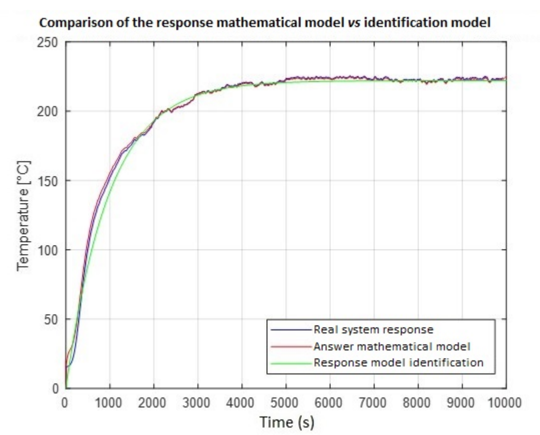

Figure 13. Comparison of the responses of the mathematical model *vs* the model calculated by graphical identification

The root mean square error (RMSE) [18], whose mathematical expression is represented by equation (11), is calculated for the first validation.

$$RMSE = \sqrt{\sum_{I=1}^{N} \frac{\left(y_{model} - y_{observated}\right)^2}{N}} \quad (11)$$

The values of 2 RMSEs of the responses to a step input of 440 VCA were calculated:

- RMSE between the model obtained by graphical system identification, which is currently used for calculating the controller, and the real system, whose result is represented in equation (12).

- RMSE between the mathematical model found in this research work and the real system, whose result is represented in equation (13).

In the statement of the solution, it has been obtained the curve the represents the real mathematical model, but it is necessary to compare it with the experimental data, and at the same time with the model currently used for controlling the thermoforming machine.

$$RMSE_{current-model} = 4,3354\ °C \quad (12)$$

$$RMSE_{mathematical-model} = 2,3395\ °C \quad (13)$$

From the comparison of the errors calculated it is evident that the error of the mathematical model is smaller than the error of the current model; therefore, the mathematical model found in this research work fits the real curve of the thermoforming oven with greater precision.

3.1.2. Comparison of control responses to a step input

Starting from the original control system shown in Figure 14, the mathematical model is inserted in the current control system with the purpose of analyzing its response to a step value (input signal) of 140 °C.

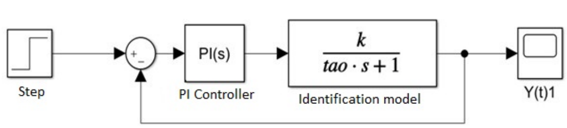

Figure 14. Temperature control system using the identification model

Figure 15 shows the comparison of the response of the control system with the identification model (blue line) *versus* the response of the control system with the mathematical model (red line), and the step of 140 °C (black line). It is observed that the responses are different despite using the same controller, and thus it is interpreted that it is not the appropriate one for the mathematical model and, therefore, it becomes necessary to find one that fits better to the features typical of the mathematical model; in this way, it is expected that there is a greater stability in the temperature and this implicitly leads to improve the final product avoiding the mismatches in the width of the thermoformed plates.

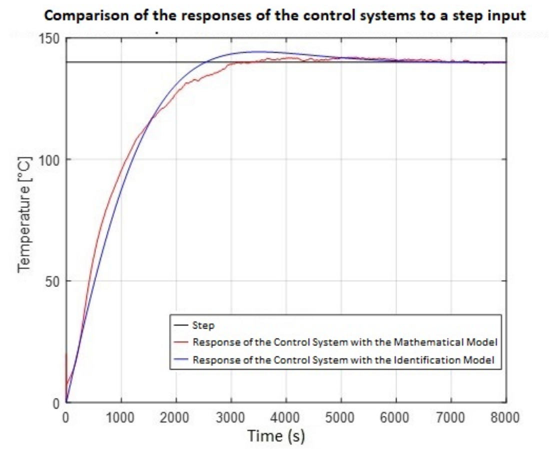

Figure 15. Comparison of the responses of the control systems

3.1.3. Calculation of a new controller that fits to the conditions of the mathematical model, to simulate and verify its response

The classical Ziegler-Nichols control method is applied to calculate the new controller and simulate the response of the system. This is one of the most well-known methods for tuning the parameters of PID controllers, and its rules come from an experimental response according to the dynamics of the process and without assuming any previous knowledge of the plant to be controlled [19–21].

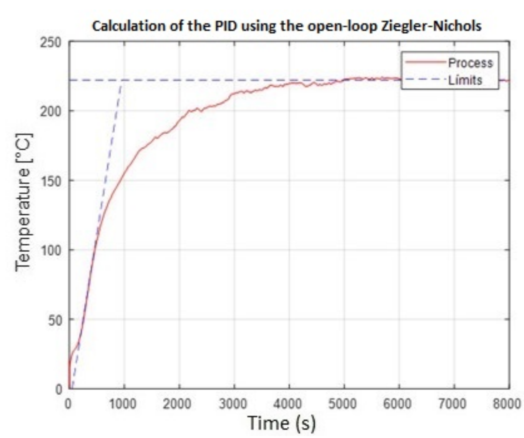

Figure 16. Calculation of the PID parameters by means of the open-loop Z-N for the mathematical model

Figure 16 shows the limits used to obtain the data required, and thus apply the open-loop Ziegler-Nichols method.

As it is observed in Figure 17, the response of the PID controller using the Ziegler-Nichols method does not show improvements compared to the current system. This is because the stabilizing time (time to reach the set-point value) is high and the temperature variation does not decrease. Due to this it is necessary to calculate the PID controller through using method.

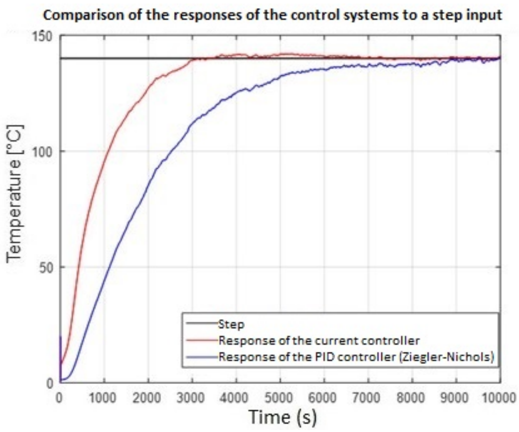

Figure 17. Comparison of the response of the PID controller applying Ziegler-Nichols *vs* the current PI controller

The Simulink PID Tuner tool provides a single-loop PID tuning method of fast and wide application for the PID controller blocks. The parameters of the PID controller may be tuned with this method to achieve a robust design with the desired response time.

Figure 18 shows the responses of the control system, comparing the one obtained through the Simulink PID Tuner tool (shown in blue), with the response of the current controller (shown in red). It is verified that this controller generates a better system response reaching temperature stability in the smallest time possible, it adjusts much better resulting in a smaller temperature variability. It should be taken into account that the data used for the subsequent analysis are the data obtained after 4000 seconds, since it is there when the system remains stable.

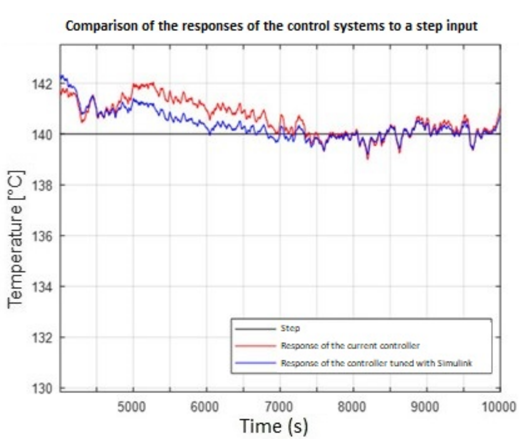

Figure 18. Detailed visualization of the responses of the control systems *vs* controllers tuned

The behavior of the data is verified through statistical analysis techniques to compare them with the current ones, to quantify the improvement that would be produced if this controller is applied in the production process. The mean temperature of the current controller is 140.62 oC, whereas the mean temperature with the controller proposed in this work is 140.35 oC, so it is evidenced quantitatively that there is a greater temperature stability applying this controller that uses the mathematical model developed. In addition, this is reflected in the error of the standard deviation, which is smaller for the new controller. The 72.81% of the data simulated with the new controller are normal with respect to their mean, whereas only 65.55% of the data of the current controller are normal with respect to their mean, but if the data of the current controller are compared with respect to the standard deviation of the new controller, only 54.41% of the data are normal. Hence, it is identified that the new controller provides better results to the system, which guarantees a greater stability of the temperature if it is implemented.

It is conducted a linear regression analysis with data provided by the quality department, with measurements of the temperature and the width of the thermoformed plates, with the purpose of obtaining an equation and verifying the correlation between temperature and width, where A represents the width and T the temperature of the oven, as shown in equation (14). With this mathematical model it is calculated the width of the plates with the temperatures obtained in the system with the current controller, and the corresponding statistical analysis is conducted.

$$A = -1,20 + T + 1088,62 \quad (14)$$

Figure 19 shows the histogram of the plates measured, and there is a variation of ± 2 mm; in addition, it is clearly seen that the data are quite disperse. Hence, the plates are heterogeneous with respect to their width, and when stacked it gives the sensation that the regulation is not met.

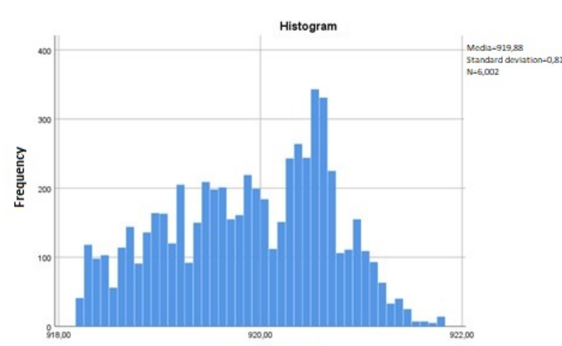

Figure 19. Histogram of the measured width of the thermoformed plates

On the other hand, Figure 20 shows the histogram of the plates calculated with the linear regression model using the system with the proposed controller, and it is clearly observed that there is smaller dispersion of the data and there is a variation of ± 1 mm; thus, it is assumed that they will look more homogeneous when stacked, providing security to the client regarding the product quality. Although it would be important to have experimental data to verify the data obtained in this research work.

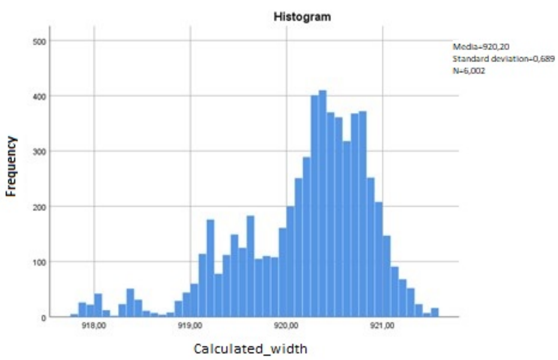

Figure 20. Histogram of the calculated width of the thermoformed plates

In this way, it is validated the mathematical model and in turn the new proposed controller, which is expected to be applied in the production plant to have access to these data; this confirms the effectiveness of what has been detailed in this document.

4. Conclusions

It was obtained the mathematical model of the resistive oven for producing thermoformed sheets, from which it was calculated a new controller for the system. Its behavior was simulated and the variability of the plate width was projected as ± 1 mm, as opposed to the variation of ± 2 mm with the current controller. Such results show the effectiveness of the new controller.

It was found the mathematical model of the oven. For this purpose, its operation was divided in three parts. First, the equations that have influence on resistance heating were formulated. Then, the equations that have incidence on oven heating through radiation and convection were expressed. At last, the equations that have influence on the heating of the oven wall temperature were used. However, the last part was not calculated since the specific information of the insulator on the oven walls is not available. Instead, the data acquired by means of the equipment data acquisition system was used.

It was found the mathematical model that best fits the real behavior of the oven. This is evidenced when calculating the root mean square error of the mathematical model, which is smaller compared to the model obtained by system identification. Therefore, it is concluded that the mathematical model obtained in this work has a better validation than the one obtained with the current control system.

It was simulated a system that uses the current oven controller coupled to the mathematical model found. It was evident that the operation dynamics of the oven is slower with respect to the simulation of the current control system. This is due to the fact that the mathematical model takes into account different phenomena that occur, such as the loss of heat in the oven walls. For this reason, it was necessary to calculate a new controller that fits the mathematical model in a better way.

Another controller was calculated that improved the final behavior of the system. However, it was not obtained by means of traditional calculations of PID controllers. Instead, automatic tuning techniques available in Simulink (PID tuner tool) were used. The main difference is that the controller is not calculated in terms of the initial behavior desired by the user (2% overshoot, stabilization time of 3600 seconds).

In this case, the overshoot is approximately 3.5% and the stabilization time is 4000 seconds. The mathematical model obtained analyzing the data of the polypropylene plates' width was validated. For this purpose, it was necessary to calculate a function that relates the temperature with the width of such plates. After comparing the results of the two systems (current and new), it is concluded that the new control system has a variation of ± 1 mm, whereas the current control system has a variation of ± 2 mm.

To obtain a better validation of the systems with respect to the width of the plates, it is necessary to design better methods for recording information, that store the temperature together with the specific dimension features in real time. In other words, knowing the width of the plate at the instant at which the oven has a particular value of temperature. For this purpose, it would be necessary to implement a more complex data acquisition system that would have direct incidence on the production costs, which for the moment is not feasible in the production plant.

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