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Effectiveness of Grey coding in an AWGN digital channel data transmission

Efectividad de la codificación grey en la transmisión de datos en un canal digital AWGN

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Resumen

El presente artículo presenta el análisis de la probabilidad de error en la transmisión de datos en un canal digital con ruido blanco gaussiano aditivo (AWGN), considerando una cadena de bits codificada con el código Grey, para diferentes valores de umbral de decisión y de voltajes de transmisión. El análisis se desarrolla a través de la comparación de la evaluación numérica de la probabilidad de error, a través de fórmulas matemáticas exactas, con la frecuencia estadística de error a través del conteo de errores.

Palabras clave: Ruido blanco gaussiano aditivo (AWGN), código de Grey, tasa de bits errados (BER), tasa de símbolos errados.

Abstract

This paper presents the analysis of the error probabilities in a digital AWGN channel data transmission, for a line-coded string of bits, using the Gray code, for different values of thresholds and voltage levels in the transmission. The analysis is performed throughout the comparison of the numeric evaluation of exact formulas with the error probability estimation obtained by statistical frequency repetition and error counting.

Keywords: Additive white gaussian noise (AWGN) channel, Gray code, bit error rate (BER), symbol error rate.

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1. Introduction

The Gray code has been employed many years for line coding and error correction in digital transmissions [1]. For instance, some popular uses of Gray coding are in digital TV transmission [2], logical data flux control [3], optical communications [4], visible light communications [5], among others.

The Gray code is a NRZ four level code, based on the coding of a couple of bits in the next way: 00 = -3A, 01 = -A, 11 = A, 10 = 3A [6]. For any chosen voltage level A, and its reception through the comparison of the received signal with three threshold voltage levels as illustrated in Figure 1.

The four symbols have the same probability 0.25, and distance of one bit between two neighbor symbols. Therefore, if any symbol, due to noise in the channel, becomes into its neighbor symbol, only one bit is affected.

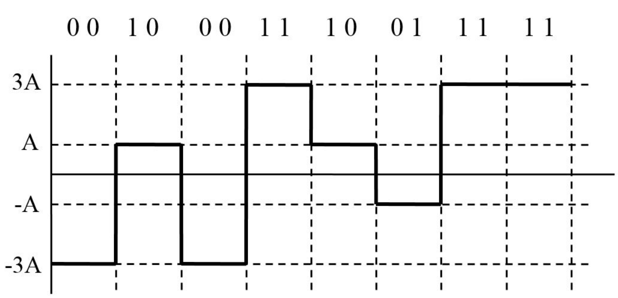

Figure 1. Grey codification example.

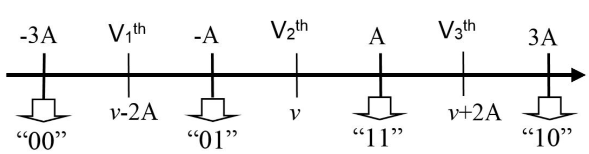

Figure 2. Decision thresholds at the receiver.

Another popular application of the Gray codes is in control systems, due to its unique cyclic property of maintaining only one bit of distance between neighboring symbols, that render this code very suitable for digital logic design of genetic algorithms [7].

The remaining of this paper is organized as follows: Chapter 2 describes the data codification setup employed for the analysis, Chapter 3 presents the main results of the study and Chapter 4 concludes this paper.

2. Channel coding setup

Since the AWGN channel corrupts the signal by the addition of Gaussian noise n whit variance σ^2, and mean μ, the received signal r is equal to the transmitted signal α plus the noise n in the channel, $r = \alpha + n$. The receiver decides which of the four possible levels was transmitted through the comparison with the fixed decision thresholds (Figure 2) $V_1^{th} = v\breve{\ }2A, V_2^{th} = v, V_3^{th} = v + 2A$. Where v is a constant, assuming values in [-A,A].

This paper covers two cases: In the first case, “A” assumes different values in order to span the symbol error probability, $\mathrm{P_s(e)}$, from 10^{-2} to 10^{-4} in function of the ratio A^2/σ^2, with $v = 0$, (optimal threshold). And, in the second case A^2/σ^2 is fixed to the value that gives a symbol probability equal to 10^{-4} and v is variable in the range [-A,A]. For simplicity, the randomly generated white Gaussian noise has mean equal to zero ($\mu = 0$) and variance equal to one (σ^2=1).

2.1. Error-probability based on statistical frequency estimation

For the symbol error rate, it is accounted the number of times the transmitted signal is received wrong. For the bit error rate, the analysis consider also which is the symbol that is received as “wrong received symbol”, in order to consider how many bits were changed and keep account of such events.

The number of repetitions is a very important issue because the accuracy of the experiment increases as the number of repetitions increases. For the current analysis, the number of repetitions is established in function the most critic case, which is the estimation of the symbol error probability $\mathrm{P_s(e)}$ is equal to 10^{-4}. Other error probabilities (from 10^{-2} to 10^{-4}) require a lower number of repetitions to get the desired accuracy.

Since a probability of 10^{-4} means that it may occur one error within 10000 transmitted symbols, then, with 10000 repetitions, we can get for instance, one, two, or zero errors if we repeat the process three times. Therefore, we might get $\mathrm{P_s(e)} = 10^{-4}, \mathrm{P_s(e)} = 0$ and $\mathrm{P_s(e)} = 2 \times 10^{-4}$, for the same system.

For this reason, in order to make the statistical determination of the error probability less sensible to the inherent variations of the possible successes (errors occurred in this case), it is necessary to increase the number of repetitions. In this paper, the statistical estimation is performed employing 106 repetitions in order to span 100 possible successes and get a good accuracy for a symbol error probability around 10^{-4}.

2.2. Numerical evaluation of probabilities

For the symbol error rate numerical estimation, it must be considered the average of the error probabilities:

$$P_s(e) = \frac{1}{4}\Big(Ps\,(e|00Tx) + Ps\,(e|01Tx) + Ps\,(e|10Tx) + Ps\,(e|“11”Tx)\Big) \tag{1}$$

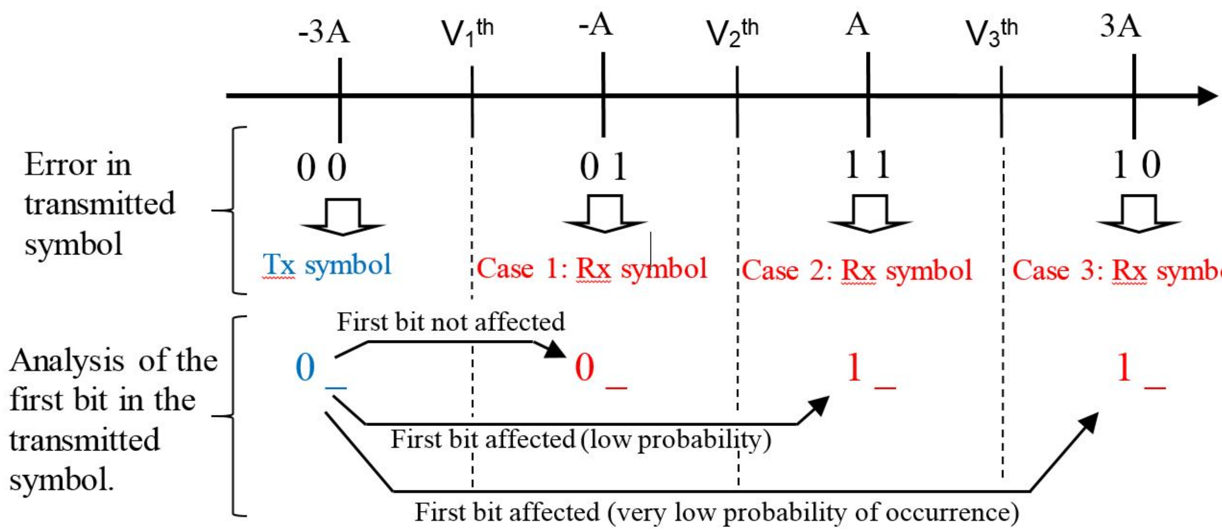

Figure 3. BER estimation (example of analysis for the first bit of the symbol 00).

The $P_s(e)$ for a given symbol, is related to the probability that the received signal $r(i)$ (for any given transmitted signal $i = -3A, -A, A, 3A$), be greater or less than the respective decision thresholds (see Figure 2):

$$\begin{aligned} P_s(e) = \frac{1}{4}\Big(& Ps\left(r_{(-3A)} > V_1^{th}\right) \\ & + Ps\left(r_{(-A)} < V_1^{th} \vee r_{(-A)} > V_2^{th}\right) \\ & + Ps\left(r_{(A)} < V_2^{th} \vee r_{(A)} > V_3^{th}\right) \\ & + Ps\left(r_{(3A)} < V_3^{th}\right)\Big) \end{aligned} \tag{2}$$

Replacing in (2) the correspondent signal values for r and V_i^{th} we have:

$$\begin{aligned} P_s(e) = \frac{1}{4}\Big(& Ps\left(-3A + n > v - 2A\right) \\ & + Ps\left(-A + n < v - 2A \vee -A + n > v\right) \\ & + Ps\left(A + n < v \vee A + n > v + 2A\right) \\ & + Ps\left(3A + n < v + 2A\right)\Big) \end{aligned} \tag{3}$$

Next, considering the fact that the events are statistically independent (thus, the "or" condition, v, implies the sum of related probabilities), (3) can be simplified as:

$$P_s(e) = \frac{3}{4}\left(Ps\left(n > v + A\right) + Ps\left(n < v - A\right)\right) \tag{4}$$

Then, since n is a Gaussian (with μ=0), $P_s(e)$ can be expressed in terms of the Complementary Error Function, erfc:

$$P_s(e) = \frac{3}{4}\left(\frac{1}{2}erfc\left(\frac{(v+A)-\mu}{\sqrt{2}\sigma}\right) + \frac{1}{2}erfc\left(\frac{\mu-(v-A)}{\sqrt{2}\sigma}\right)\right) \tag{5}$$

$$P_s(e) = \frac{3}{8}\left(erfc\left(\frac{A+v}{\sqrt{2}\sigma}\right) + erfc\left(\frac{A-v}{\sqrt{2}\sigma}\right)\right) \tag{6}$$

In order to evaluate the bit error rate (Pb(e) or BER), it is necessary to split the symbols in order to consider the probability of each individual bit, which gets to eight cases instead of the four cases found in the symbol error analysis (see Figure 3).

Therefore, the BER is equal to the sum of the probability of considering the first or the second bit in each symbol, times the error probability of the first and second bit respectively:

$$\begin{aligned} P_b(e) = & P\left(first\ bit\right) \times P\left(e|first\ bit\right) + \\ & + P\left(second\ bit\right) \times P\left(e|second\ bit\right) \end{aligned} \tag{7}$$

Given that, the most useful assumption to be considered is assuming or channels is a symmetric binary channel, then:

$$\left(P\left(first\ bit\right) + P\left(second\ bit\right)\right) \tag{8}$$

Therefore,

$$P_b(e) = \frac{1}{2}\left(P\left(e|first\ bit\right) + P\left(e|second\ bit\right)\right) \tag{9}$$

This way, the individual probabilities can be determined considering the appropriate thresholds to be compared when a bit is wrong (Figure 3). Thus, with the same resolution as for P_s, in the end for the BER we get,

$$\begin{aligned} P_s(e) = & \frac{1}{8}\left(erfc\left(\frac{3A+v}{\sqrt{2}\sigma}\right) + erfc\left(\frac{3A-v}{\sqrt{2}\sigma}\right)\right) + \\ & + \frac{3}{16}\left(erfc\left(\frac{A+v}{\sqrt{2}\sigma}\right) + erfc\left(\frac{A-v}{\sqrt{2}\sigma}\right)\right) \end{aligned} \tag{10}$$

Equations 6 and 10 are directly applicable to the case when A is fixed and v is variable. Nevertheless, when A is variable, and $v = 0$, the formulas for symbol error rate and bit error rate would be,

$$P_s(e) = \frac{3}{4} erfc\left(\frac{1}{\sqrt{2}} \times \sqrt{\frac{A^2}{\sigma^2}}\right) \tag{11}$$

$$P_s(e) = \frac{1}{4} erfc\left(\frac{3}{\sqrt{2}} \times \sqrt{\frac{A^2}{\sigma^2}}\right) + \frac{3}{8} erfc\left(\frac{1}{\sqrt{2}} \times \sqrt{\frac{A^2}{\sigma^2}}\right) \tag{12}$$

3. Main results

Figure 4, shows the curves for the first case of analysis: "A" with variable values and $v = 0$ (which correspond to the optimal threshold). Notice the values of the signal to noise ratio (SNR), A^2/σ^2, are extremely determinant on the error probability because lower SNR values mean greater noise level and, consequently, a greater degradation in the signal quality.

Notice that the use of 106 repetitions permit to get very accurate results so that there is a good approximation of the statistical curves to the numerical curves.

In the second case of analysis, "A" is fixed to 3.6455 (which is the value that gets a BER=10^{-4}), and v variable in the range [-A,A]. This case considers a variation of the three decision thresholds through sweeping the v parameter, around the optimal value given by $v = 0$. Therefore, it can be expected that the number of errors grows as the value of v moves away from zero. In fact, it is exactly the behavior of the curves of symbol error rate and BER in Figure 5.

Notice that for $v = 0$, the value of the probability in the curves of statistical and numerical estimated BER is 10^-4, which is precisely what was set through the value A = 3.6455 ($A^2/\sigma^2 = 13.29$). Again, thanks to the use of a high number of repetitions, the approximation of the statistical curves to the numerical curves is also good in this case. It is very notorious the difference for the numerically evaluated and statistically estimated probabilities for values of v around zero (lower probabilities needs more repetitions to be more accurate). Table 1 shows a set of interesting values and their probabilities, numerically calculated and estimated by means of statistical frequency repetitions.

4. Conclusions

This paper demonstrates the increased performance the Grey code provides to a digital signal transmission even in cases when the white noise is very strong and significantly degrades the SNR of the data transmission.

The simplicity of the code constitutes one of the best advantages of employing it in many fields of digital transmission systems and the results show that, statistically, the behavior of the code is practically the same as the mathematically evaluated results.

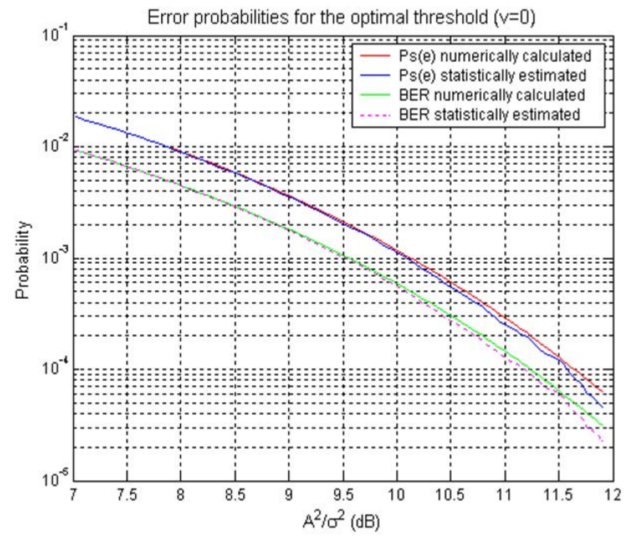

Figure 4. Error-probability curves for v fixed to the optimal threshold ($v = 0$) and signal to noise ratio, A^2/σ^2, variable.

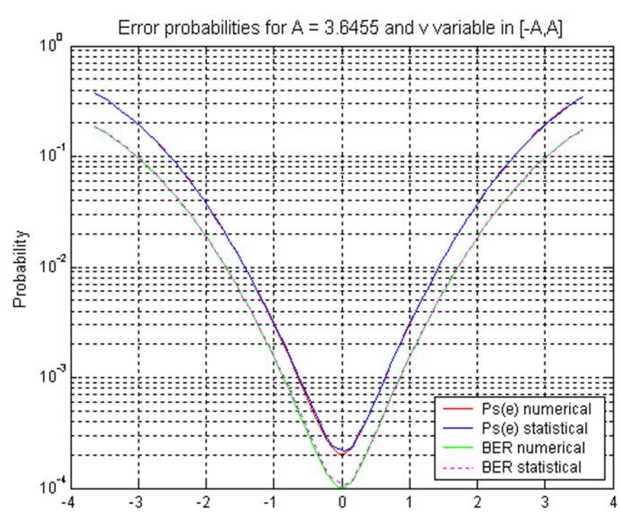

Figure 5. Probability curves for variable thresholds and $A^2/\sigma^2 = 13.29$ (BER = 10^{-4} when optimal threshold used).

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Table 1. Some interesting values for the statistically and numerically estimated probabilities.

Probabilities when A^2/σ^2 variable, v=0						**Probabilities for v variable, A=3.6455 ($\sqrt{13.29}$)**				
		Statistical		**Numerical**			**Statistical**		**Numerical**	
A^2/σ^2 **[dB]**	A^2/σ^2 **(linear)**	**BER**	$\mathbf{P_s(e)}$	**BER**	$\mathbf{P_s(e)}$	v	**BER**	$\mathbf{P_s(e)}$	**BER**	$\mathbf{P_s(e)}$
6.91	4.91	9.99e-3	1.97e-2	1.00e-2	2.00e-2	3	9.78e-2	0.195	9.75e-2	0.194
7.87	6.12	4.97e-3	9.85e-3	5.00e-3	1.00e-2	2	1.86e-2	3.75e-2	1.87e-2	3.74e-2
9.55	9.01	9.96e-4	1.94e-3	1.00e-3	1.99e-3	1	1.54e-4	3.11e-4	1.52e-4	3.05e-4
10.13	10.30	4.67e-4	9.94e-3	5.01e-4	1.00e-3	0	1.10e-4	2.20e-4	1.05e-4	2.00e-4
10.65	11.61	2.24e-4	4.62e-4	2.45e-4	5.00e-4	-1	1.54e-4	3.11e-4	1.52e-4	3.05e-4
11.23	13.29	0.95e-4	1.86e-4	1.00e-4	2.00e-4	-2	1.86e-2	3.75e-2	1.87e-2	3.74e-2
11.64	13.29	4.47e-5	9.05e-5	5.01e-5	1.00e-4	-3	9.78e-2	0.195	9.75e-2	0.194

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