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**Enseñanza y aprendizaje de las matemáticas:
diferentes enfoques y niveles educativos**

*“Teaching and learning process of Mathematics:
different approaches and educational levels”*

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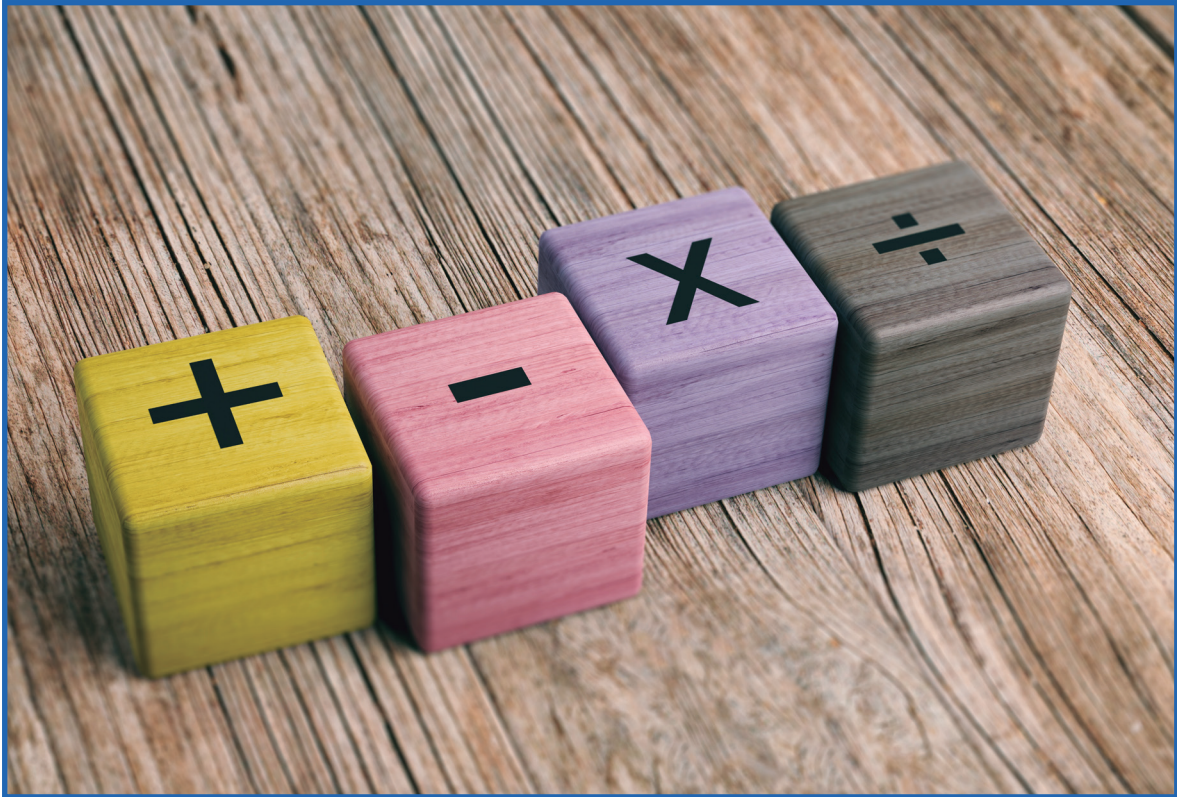
Teaching and learning process of Mathematics: different approaches and educational levels

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Learning mathematics

Mathematics has been studied for thousands of years, since it has contributed to finding answers to many phenomena, not only in the scientific and technological fields, but also in everyday life. Consequently, the teaching and learning of mathematics is an essential component of academic programs; however, the accelerated changes of this era pose new challenges to education.

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This Monographic Section of the Journal “Alteridad” presents research related to the processes of teaching and learning mathematics, carried out in different contexts and educational levels and with different theoretical approaches. The objective, besides sharing the possibilities and limitations of experiences developed in the field of teaching and learning mathematics in different contexts, is to contribute to the analysis and discussion of theoretical and practical aspects related to the processes of teaching and learning mathematics, as well as research results of processes used in different contexts, from the most elementary educational levels to those of Higher Education and Graduate Studies.

Among the papers presented, the first two articles are focused on the analysis of teaching and learning processes in the field of Early Childhood Education in the Spanish context; the next two refer to the analysis of mathematics teachers’ knowledge, one with future mathematics teachers in the Brazilian context and the other with effective mathematics teachers in the Colombian context. The last article seeks to study the teaching and learning processes of a group of High School students in Portugal.

The first article of the Monographic Section “Influence of the teaching context on pattern representation in early childhood education” analyzed, by means of the Mathematics Teaching Itineraries Approach, how the teaching context influences the tasks with repeating patterns in a group of preschool students in Spain. The conclusion of the study is that the teaching context influences the understanding of repetition patterns and a teaching of the patterns from the situational to the formal level is recommended.

At the same educational level of Early Childhood Education, the manuscript “Mathematical representations of 5–6-year-old children when solving an open problem” sought to describe the representations and forms of solution posed by a group of children in a Catalan school, when solving an open arithmetic problem. The results indicate that all participating students elaborate iconic representations, and some combine iconic and symbolic representations to solve the problem.

Regarding the knowledge that mathematics teachers should have for an adequate teaching of mathematics, the article “Didactic-mathematical knowledge mobilized by future mathematics teachers”, by means of the Didactic-Mathematical Knowledge model of the Ontosemiotic Approach, analyzes the knowledge mobilized by future mathematics teachers in Brazil when solving tasks focused on topics related to Financial Education, concluding that the mobilization of didactic knowledge was satisfactory only in the cognitive aspect, and was partial in the other aspects of the model. Likewise and using the same model of teacher knowledge, the article titled “Didactic-mathematical

knowledge of some teachers about prime numbers” analyzes the didactic-mathematical knowledge of Colombian teachers when developing teaching and learning situations about prime numbers, concluding that, although there are elements that evidence the teachers’ knowledge of prime numbers, the teachers fail to give students a broad vision of the meaning of these numbers because they do not know the cognitive and epistemic elements that make possible their management in the classroom.

Finally, based on a qualitative methodological approach, the article “Learning of logarithmic functions of 12th grade students with modeling tasks” characterizes and identifies the difficulties of Portuguese High School students when solving mathematical modeling tasks related to the logarithmic function. It is concluded that, although the students have performed the modeling tasks understanding the application of a complete modeling cycle, they presented difficulties in the properties and characteristics of the logarithmic function, as well as in the use of the graphic calculator.

This monograph is an excellent contribution for researchers and teachers interested in looking for tools to analyze and assess the teaching and learning processes of mathematics in different subjects, educational levels and contexts.

The articles in the Miscellaneous Section address various topics related to creativity and its variables linked to education, academic performance related to self-efficacy and procrastination, positive parenting and self-regulation of learning; and finally, the situation of the Masters in Esmeraldas, Ecuador.

The first article offers a mapping on “Creativity and related variables according to educational stage”: pre-school, elementary, high school, and university studies. Creativity is a topic increasingly addressed in the educational field. Guided by the PRISMA statement, the search was conducted using Dialnet Plus and Web of Science databases. The findings show that intelligence and academic performance are the most studied variables at all educational levels; however, pre-school and elementary school levels focus on the control of emotions and personality, while high school level analyzes problem solving; and the university level focuses on thinking studies.

The following article “Creative education and social justice” is a systematic review that is new and relevant to the Latin American context. The authors, following the criteria of Sánchez-Meca (2010) and Prisma (2009), conducted a search in Web of Science, Scopus, SciELO and Dialnet. In terms of originality, the authors highlight the low number of publications on the subject, particularly in the Latin American context. The article is the first step to raise an emerging issue, the problem of education based on creativity from the perspective of social justice.

Non-cognitive factors of academic success are at the forefront of educational research. In this sense, the third manuscript “Self-efficacy, procrastination and academic performance in university students in Ecuador”, is an interesting topic with great potential: it is undoubtedly important in the contexts of high school and university education, since it is in this evolutionary moment of the person in which self-perception problems are most frequent, and in which procrastination becomes a defense mechanism. The authors show that both self-efficacy and academic procrastination have an impact on academic performance, specially at the beginning of professional university training, and especially in private institutions. The study also highlights the social inequalities, sometimes unattainable for the poor, with certain opportunities offered by private institutions.

The manuscript “Positive parenting and self-regulation of learning in adolescents”, conducted in the city of Cuenca (Ecuador), determines the role played by parental support for autonomy and self-regulation of learning. The results reveal that positive parenting is a factor that promotes self-regulation of learning, with satisfactory academic results. The conclusion shows “the need for close family-school collaboration”. The Self-Determination Theory (SDT) could be new and relevant to propose new studies with other variables, such as, for example, the use and dependence on ICTs.



The study on “Multigrade teachers in Esmeraldas (Ecuador)” is not only interesting but also worrisome. First, in addition to teaching activities, it highlights the fact that teachers must conduct administrative and logistical activities for the operation of the school. The core of the concern is the political and institutional neglect, in addition to racial, regional and gender discrimination, even by the rural community. The commitment to education, the solidarity of teachers, and the self-management for their professional updating seem to be insufficient to achieve the ideals of good living.

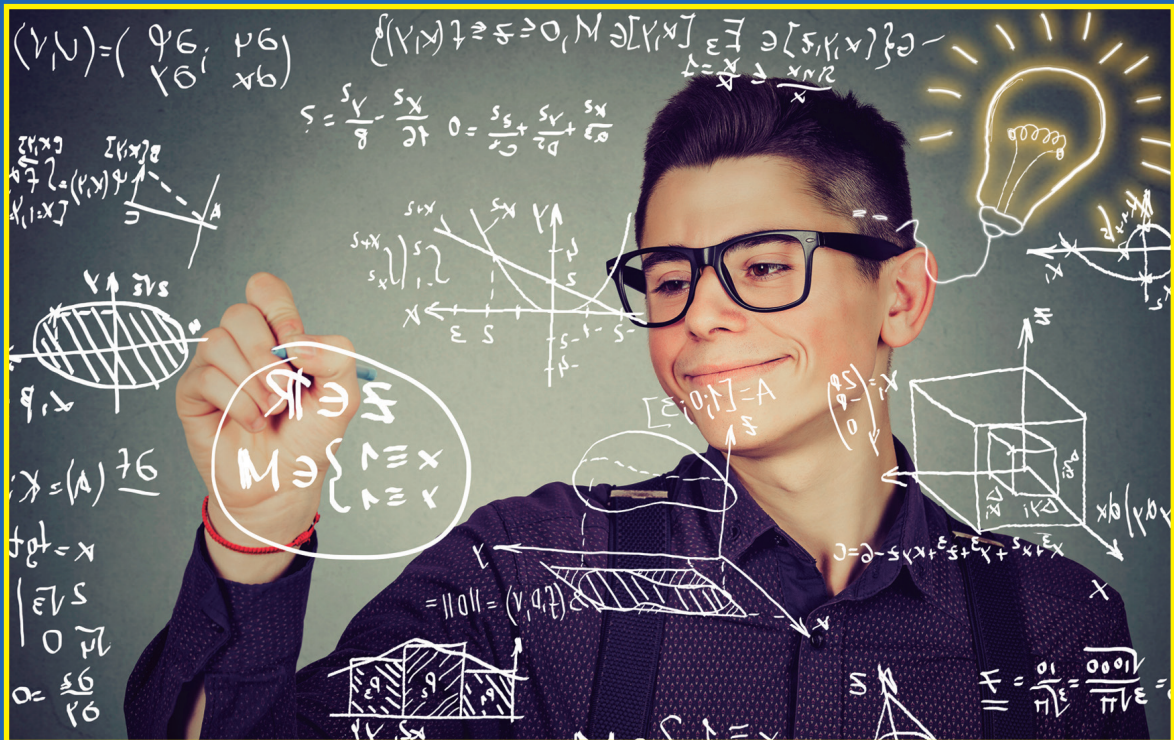
The use of ICT in education, although it can bring out inequities due to the availability or not of technological resources and Internet connection, is characterized by being flexible, adjusting to the diverse realities and circumstances of the students, allowing online and asynchronous learning; thus, solving the commute problem to the school at least for those who work and study; however, its use also involves risks and possible challenges, such as its attachment and dependence to the point of being most of the day using social networks, entertainment platforms, videos, among others, which can be detrimental not only to meaningful learning, but also to the integral formation of young people. Precisely, the Call for Papers for the next issue is about “Responses of educational institutions to the addiction to technologies”. Authors are invited to send their manuscripts to *Alteridad*.



Monographic section

(Sección Monográfica)

Teaching and learning process of Mathematics:
different approaches and educational levels
*“Enseñanza y aprendizaje de las matemáticas:
diferentes enfoques y niveles educativos”*



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Influence of the teaching context on pattern representation in early childhood education

Influencia del contexto de enseñanza en la representación de patrones en educación infantil

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Abstract

According to the Mathematics Teaching Itineraries Approach (EIEM), which proposes intentional teaching sequences from the concrete to the symbolic, we analyse how the teaching context influences the repetition pattern tasks in a group of 24 children during two consecutive school years (4-6 years old). For this purpose, repetition pattern tasks have been implemented in the two extreme contexts of a previously designed and validated itinerary: real situations and graphic contexts, respectively. The data have been analysed from ethnographic methodological schemes of participant observation (field diary); pedagogical documentation (audiovisual record); and written productions of the patterns (representations). The main results obtained show that: a) in pupils aged 4-5 years, a positive difference of 32.9% of real situations versus graphic resources has been identified; b) in pupils aged 5-6 years, although the difference between the two contexts decreases slightly, it continues to be above 30%. It is concluded that the teaching context influences the understanding of repetition patterns, so that it is necessary to teach patterns from the situational to the formal level.

Keywords: Repetition patterns, representation in mathematics, real situations, graphical resources, early childhood education.

Resumen

Con base en el Enfoque de los Itinerarios de Enseñanza de las Matemáticas (EIEM), que propone secuencias de enseñanza intencionadas desde lo concreto hasta lo simbólico, se analiza cómo influye el contexto de enseñanza en las tareas con patrones de repetición en un grupo de 24 escolares españoles durante dos cursos académicos consecutivos (4-6 años). Para ello, se han implementado tareas de patrones de repetición de los dos contextos extremos de un itinerario previamente diseñado y validado: situaciones reales y contextos gráficos, respectivamente. Los datos se han analizado a partir de esquemas metodológicos etnográficos de observación participante (diario de campo); la documentación pedagógica (registro audiovisual); y las producciones escritas de los patrones (representaciones). Los principales resultados obtenidos muestran que: a) en el alumnado de 4-5 años se ha identificado una diferencia positiva del 32,9 % de las situaciones reales frente a los recursos gráficos; b) en el alumnado de 5-6 años, si bien desciende ligeramente dicha diferencia entre ambos contextos, continúa estando por encima del 30 %. Se concluye que el contexto de enseñanza influye en la comprensión de los patrones de repetición, por lo que es necesaria una enseñanza de los patrones desde el nivel situacional hasta el formal.

Descriptores: Patrones de repetición, representación en matemáticas, situaciones reales, recursos gráficos, educación infantil.

1 Introduction and state-of-the-art

Attention has been addressed to children in recent years for the integral development of the person. Bowman *et al.* (2001, p. 23), state that “[...] young children are capable students and that the educational experience during the preschool years can have a positive impact on school learning”. In this sense, research suggests that early mathematical competencies (basically 4 to 6 years old) can be an indicator of academic success in later stages (Nguyen *et al.*, 2016; Rittle-Johnson *et al.*, 2017; Wijns *et al.*, 2021, among others). Therefore, effective planning and implementation is necessary to promote the development of mathematical competence from the earliest ages, diversifying the use of educational scenarios and leading to good practices in the mathematics classes (Alsina, 2019, *National Council of Teachers of Mathematics* [NCTM], 2000, 2014).

From this perspective, this article assumes the focus of the Itineraries of Teaching Mathematics in Alsina (2019, 2020), henceforth EIEM, that states that the teaching of mathematics in the early ages should be proposed as a journey from the concrete to the abstract, through teaching sequences that contemplate different teaching contexts to promote a consolidated acquisition of mathematical competencies. This approach establishes an intentional sequence that includes three teaching levels that go from the particular to the general, offering a hierarchical orientation of the contexts that make it up. At the first levels are the informal contexts that should be “consumed” on a daily basis: real situations, manipulative resources, and the use of recreational proposals; in the next level, reference is made to intermediate contexts that should sometimes be used, such as literary and technological resources; and finally, formal contexts that should occasionally be implemented, such as tabs and textbooks.

This article applies this approach to the teaching of repetition patterns, as it has been shown that knowledge of patterns and their structure positively influences the early development of mathematical thought (Clements and Sarama, 2015; Lüken and Kampmann, 2018; Mulligan *et al.*, 2020; Papic *et al.*, 2011; Rittle-Johnson *et al.*, 2018; Tirosh *et al.*, 2018; Wijns *et al.*, 2021). Therefore, pattern exploration can be seen as a gateway to promoting generalization (Vanluydt *et al.*, 2021), anticipation, justification, and the representation and precise use of mathematical language (Acosta and Alsina, 2020). When speaking of mathematical patterns, it is necessary to distinguish between pattern as an ordered sequence and between pattern structure, i.e., organization or rule underlying the pattern (Mulligan and Mitchelmore, 2009). These Australian authors point out that patterns comprise two components: one cognitive, related to knowledge of structure; and another meta-cognitive, associated with the ability to search and analyze patterns. Mulligan *et al.* (2020) say that the lack of consciousness of the pattern and its structure may become a predictor of future mathematical difficulties. However, little is known about how the teaching context influences the understanding and representation of patterns. In fact, authors such as Wijns *et al.* (2019) mention the need to study whether tasks with patterns that are implemented optimally promote their full potential, thus fostering the development of algebraic perceptions of schoolchildren. As Alsina (2020) states, the textbook is sometimes seen as a preponderant resource that leaves no room for addressing mathematical concepts and procedures from other scenarios that are more realistic, concrete, and meaningful to schoolchildren.

Hence, the aim is to provide data that allow an approximation to the way in which children from 4 to 6 years old perform repetition patterns and exteriorize their representation in different teaching contexts, in order to address the development of the understanding of pat-



terns in a contextualized and longitudinal way, considering the approaches of the EIEM.

From this perspective, we raised the following research questions:

- How does the teaching context influence tasks with repetition patterns?
- What is the relationship between concrete teaching contexts (real situations) and abstract teaching contexts (graphic resources) during the understanding and representation of repetition patterns?

The objectives are:

- Analyze the relationship between understanding and representing patterns of repetition.
- Demonstrate the influence of the learning context on the success of representing repetition patterns

1.1 Teaching and representation of repetition patterns from the approach of didactic itineraries

The baseline of EIEM as a theoretical framework of our study are discussed; it is defined what is understood by pattern, the importance of its teaching and the representation as a mathematical process that promotes understanding.

Acosta and Alsina (2021) point out that the learning of patterns begins in concrete situations until it is consolidated into abstract experiences. Therefore, the EIEM (Alsina, 2019, 2020) is taken as a reference, based on three interrelated bases: a) the sociocultural perspective of human learning (Vygotsky, 1978), understanding education as a social and cultural phenomenon that sees language and interaction as essential tools to promote learning; b) The realistic model of teacher training (Korthagen, 2001), which considers that teachers should be familiar with various ways of intervening and exercising them in practice, i.e., they should have criteria

for knowing when, what and why a situation is likely to reflect systematically; and c) Realistic Mathematical Education (Freudenthal, 1991), which promotes the use of contextualized problems in real situations as the beginning of the teaching-learning process of mathematics.

Based on these bases, the EIEM (Alsina, 2019, 2020) considers the teaching of mathematics through didactic sequences that include the following three levels:

- Informal level: The teaching of mathematical contents is prioritized from contexts of real situations and close to the students, using manipulative and recreational materials, relying, in turn, on informal knowledge, common sense and experience. In these contexts, the cognitive demands that are used are: exploration, manipulation, or experimentation, conforming as requirements to visualize and understand mathematical ideas in a concrete way.
- Intermediate level: The teaching of content continues into contexts that are formed as a bridge between the contexts of the previous phase and the formal contexts of the later phase. In this level are literary (stories and songs) and technological resources (Applets, programmable educational robots, etc.). In these contexts, cognitive demands are focused on exploration and reflection, which progressively facilitate the schematization and generalization of mathematical knowledge.
- Formal level: The teaching of content ends in graphic and symbolic contexts, where the representation and formalization of mathematical knowledge is encouraged, using conventional procedures and notations to promote the learning of the concrete to the symbolic. Therefore, cognitive demands in these contexts are mainly focus on abstraction and generalization.



From this approach, the target is more in heuristic activities rather than pure exercise, and critical mathematical thought more than repetition (Alsina, 2019).

As indicated in the introduction, this study focuses on the teaching of patterns. When speaking of pattern, we refer to a sequence of elements ordered according to a given norm, rule, nucleus, or periodic unit. Clements and Sarama (2015) explain that the teaching of patterns pursues the search for regularities and mathematical structures. Recognizing patterns is set as a fundamental capacity for many domains of knowledge such as reading, mathematics, or arts, since patterns provide meaning and cohesion (Björklund and Pramling, 2014). For this reason, Papic (2015) suggests the need to promote awareness among boys and girls about patterns to stimulate structural development, relational understanding, and generalization from an early age, and lay the baselines of mathematical thinking in general and algebraic. A consolidated algebraic thought requires the capacity to symbolize and generalize (Sibgatullin *et al.*, 2022).

Our study assumes that patterns can vary according to their regularity and content; based on this statement, patterns can present units that are repeated, that are arranged in a structural or symmetrical way or that grow (Bock *et al.*, 2018). The typology of patterns addressed in our teaching itinerary is repetition, i.e., patterns that through iterative sequences show regularities or repetitions of specific qualitative and/or quantitative characteristics (color, shapes, size, sounds, or numbers, e.g., “green, green, yellow, green, green, yellow” or “■○■○”).

Prestigious authors and institutions point out that the teaching of repetition patterns and the understanding of their structure positively influences early mathematical development, as it promotes a truthful baseline for algebraic thinking (Mulligan *et al.*, 2020; Rittle-Johnson *et al.*, 2018; Wijns *et al.*, 2019). Developing the concept of pattern involves perceiving the underlying rule and consciously and functionally identifying the unit

of repetition. According to Wijns *et al.* (2019), it is necessary to implement tasks with patterns that give children the opportunity to transition from recursive to functional thinking, i.e., to observe the relationship of elements that lies in a series to abstract and represent the internal structure in a guided way.

What does representation in mathematics imply? For Freudenthal (1991) the progressive development of the representation of mathematical ideas and procedures goes from the concrete to the abstract, so that it can take diverse forms through physical objects, natural language, drawings, and conventional symbols. Reed says (2001, p. 215), “drawing can be a window into a child’s mind”. Therefore, it is necessary to respect and encourage the process of representation to learn (and to understand) the symbol that represents an object, a situation, or a mathematical idea. For this reason, Duval (1995, p. 15) considers that “there is no knowledge that can be transferred by an individual without a meaningful activity”. Also, in relation to representation as a mathematical process that externalizes student understanding, Pino-Fan *et al.* (2017) says that such a process plays an essential role in acquiring and treating an individual’s knowledge. From this perspective, the NCTM (2014) is committed to teaching mathematics in a way that enables connections between representations to be established to effectively link conceptual and procedural understanding.

Considering the latter, we conceptualize the representation in mathematics as an interconnected process that allows to express in a concrete way the knowledge and mathematical procedures that students possess using different signs, graphics and/or natural language. In this way it is possible to organize, understand and communicate the mathematical nature of actions previously carried out in the educational and social spheres.

2 Methodology

A qualitative approach is used to test the opportunities for understanding provided by the most



concrete context (real situations) and the most abstract (graphic resources) of the EIEM (Alsina, 2019, 2020) when teaching repetition patterns to early childhood education students (4-6 years old). According to Maldonado (2018), this approach relies on the interpretation, description, analysis and comprehension of qualitative information obtained through recordings, observations, interview, etc. In keeping with this contribution, our design facilitates a descriptive and interpretative analysis that allows to show through the representation in mathematics of the students, the results obtained in a longitudinal way at the informal level, specifically in the context of real situations by comparing the results collected at the formal level (graphical resources).

2.1 Design and procedure

As indicated, the activities within the context of real situations and graphical resources that make up the IEEM have been selected. The six proposals have been submitted to experts who have

assessed: a) didactic aspects, b) organizational aspects, c) methodological aspects, and d) pedagogic aspects of the teaching process. This procedure, along with the reflexive practice developed after each session, has favored the articulation of continuous and retrospective analyzes that inform the design and facilitate its improvement longitudinally. This procedure can be complemented by reading Acosta and Alsina (2020), who validated and applied an itinerary of repetition patterns with 3-year-old school children.

The implementation has been carried out in a longitudinal way, with 24 Spanish students all belonging to the same class of a public school. The sample consists of 12 boys and 12 girls. The average age of the sample was 4.8 years and 5.8 years old for the two school years. This group was selected because of the ease of access; because of the continuity and longitudinal follow-up of the tutoring teacher; and because it is considered a school with low enrollment rate in preschool grades.

Table 1

Proposals developed according to the teaching context

4-5 years old	
Real situations	A1. Google Maps is used on the whiteboard to display different streets in our city in search of mathematical patterns. Through questions, children are told to look at the facades of houses, buildings, and shops. Once the patterns have been identified, they are reproduced together using colored cardboards.
	A2. An image of a garden is shown, and students are invited to describe how the bushes are placed. Students are asked if they believe the bushes follow a sequence and are proposed to recreate a series with play dough.
Graphic Resources	Through a pre-written task designed with different types of awnings, children are told to expand the series.
5-6 years old	
Real situations	A1. Students are presented a basket with socks and sweaters with various designs; chess set, toy piano; card set, pictures of tiles; pieces of fabric with skin drawings of some animals, pictures of awnings...and they are invited to "hunt" and identify the patterns present in the objects inside the basket.
	A2. Walk through the school yard to get photographically the patterns existing in this educational space. Next, the students are presented with the challenge of playing some of the series found in a role.
Graphic Resources	Through previously designed forms, students are told to observe, identify, analyze, and read the series proposed to recognize the elements that make up the minimum unit of the pattern and to complete the series.



The activity is carried out in a small group (12 boys and girls) to ease individualized attention and the collection of specific and personalized evidence. The distribution of participants is done at random and the two subgroups of 12 students are maintained throughout the activity. Thus, in a longitudinal way, a total of eight direct intervention sessions are allocated for real situations and four sessions for graphical resources, each lasting 50 minutes. It should be mentioned that informed consent was obtained from all families before the intervention.

The sessions were divided into three phases: a) introduction of the proposal, b) interaction and development, and c) representation and reflection. It is important to emphasize that in the final phase the students represent the pattern that they identified in the activity without having the model in front of them. The role of the teacher is to guide and encourage learning through intentional questions (NCTM, 2014) that promote knowledge and share it with the peer group. Questions that do not involve reasoning or argumentation by students and that are answered with a “yes” or “no” should be avoided.

2.2 Data collection

Data collection includes three tools: I) ethnographic methodological schemes of participant observation using the field diary as a tool to record spontaneous expressions of children during the performance of tasks; II) pedagogical documentation through the audiovisual record, fixed and mobile, of all the sessions; and III) written productions in drawing format of all the representations of students as a sign of the formalization of the knowledge acquired.

Kawulich (2006) considers participating observation as a skill that enables researchers to reflect on and learn about proposals that are developed with participants in a natural con-

text, using observation and active participation as facilitating tools for direct, non-interference interaction. On the other hand, the pedagogical documentation adopts a reflexive character that gives voice to the thought of the child, recognizing the observer as an active agent that co-constructs meaning in a reflexive, active and reciprocal way with the purpose of creating a plural and transformative space (Mitchelmore, 2018). We cannot ignore that verbal and non-verbal expressions are key to interpreting the knowledge and skills of younger schoolchildren (Björklund *et al.*, 2020).

2.3 Analysis of the data

Children's drawings have been categorized according to the diagram below with the intention of eliminating the bias generated by a hierarchical presence of proposals according to the model proposed by the IEEM (Alsina, 2019, 2020). It is considered the “correct” category when the representation is error-free, and “incorrect” when the production is error-free in its structure.

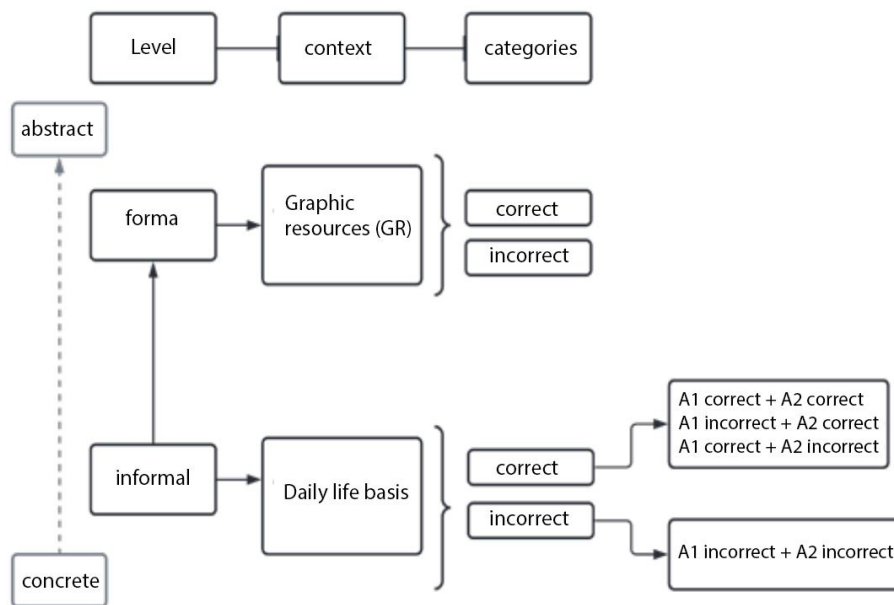
Based on the information shown in Figure 1, a quantitative analysis was performed describing the categorical variables according to the number and percentage of cases in each category.

To support the latter, the audiovisual evidence is transcribed and discussed doing an interpretative analysis of the discourse. This analysis of the most relevant parts allows to review the relationships between texts and reality by presenting the discourse used by the child, his/her point of origin, how it flows, and what accompanies it (Leeuwen, 2008). This information is contrasted with the quantitative data, with the recorded field notes and with the written productions of the boys and girls to also show the role of the teacher.



Figure 1

Flowchart with the categorization process of the representations obtained



3 Results

Considering the aim of the study, the results obtained longitudinally are analyzed in contexts

of real situations and graphic resources, with the intention of checking how the teaching context influences the understanding and representation of tasks with repetition patterns.

Table 2

Results achieved in 4-5 years old students

Real Situations	Frequency	Percent	Pct. Valid
Correct	17	70,8	85,0
Incorrect	3	12,5	15,0
Total Valid	20	83,3	100
Invalid	4	16,7	
Total	24	100	
Graphics Resources	Frequency	Percent	Pct. Valid
Correct	11	45,8	45,8
Incorrect	13	54,2	54,2
Total Valid	24	100	100
Invalid	0	0,0	
Total	24	100	



As shown in Table 2, 85% of valid cases represented correctly the pattern identified in the activities carried out in the context of real situations, compared with 15% who failed to perform the task successfully. However, there is

a significant increase in errors in the context of graphical resources, located in 54.2%. The degree of success in this context is only 45.8%.

Below are the results corresponding to 5-6-year-old students.

Table 3

Results obtained for 5-6-year-old students

Real Situations	Frequency	Percent	Pct. Valid
Correct	23	95,8	100,0
Incorrect	0	0,0	0,0
Total Valid	23	95,8	100,0
Invalid	1	4,2	
Total	24	100	
Graphics Resources	Frequency	Percent	Pct. Valid
Correct	16	66,7	69,6
Incorrect	7	29,2	30,4
Total Valid	23	95,8	100,0
Invalid	1	4,2	
Total	24	100	



According to the information shown in Table 3, 100% of valid cases represented without errors the patterns identified in the context of real situations, while only 69.6% did so in the context of graphical resources. We observed that

incorrect representations decreased by 23.8 % compared to previous year.



In Table 4 we show some examples of the implementation. One is selected for each context and age due to the space.

Table 4

Evidence of the conduction of activities in each context according to the age

Context	4-5 years	5-6 years
Real Situations		
	Plays series Using colored cardboards (A1)	Discovering series with yard elements (A2)




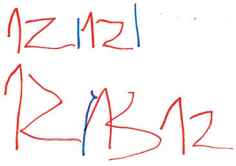
Context	4-5 years	5-6 years
Graphics Resources		
	Expand series	Complete missing elements of the series

Below are some examples of representations and transcripts of dialogs obtained during the pedagogical implementation to illustrate the

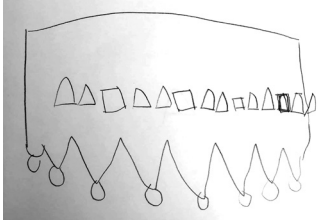

role of the teacher as a generator and promoter of learning.

Table 5

Examples of correct representations obtained in each context according to the age

Context	4-5 years	5-6 years
Real Situations	 <p>Teacher: What does your drawing represent? Student: The awning we saw. Teacher: Why do you use two colors? Student: Because it was yellow-brown, yellow-brown. Teacher: And the awning that had white and blue stripes, is it the same as this one? Student: Yes, because it also has two different colors.</p>	 <p>Student: I painted 1Z and blue line, 1Z and blue line. Teacher: What object of the basket of treasures have you represented? Student: The pink-lilac-white striped jumper. Teacher: Can you explain why you have a 1, a Z and a blue line? Student: Because pink, lilac and white are different colors. Teacher: So, you have assigned number 1 to pink and Z to lilac? Student: Yes, and the blue line to white.</p>



Context	4-5 years	5-6 years
Graphics Resources		
	<p>Student: Look, I painted triangles and squares</p> <p>Teacher: Can you explain what does your drawing represent?</p> <p>Student: These are two triangles and a square, two triangles and a square and so on to infinity.</p> <p>Teacher: Why did you paint two triangles and a square?</p> <p>Student: Because my birthday crown was like that.</p>	<p>Teacher: Can you explain your drawing?</p> <p>Student: On the card, there was a pattern with letters and also numbers.</p> <p>Teacher: Then, have you done the same of the file?</p> <p>Student: No, because I had the C and I used number one.</p> <p>Teacher: The pattern of the file was ABCAB-CABC, yours is AB1AB1AB1, are they the same or different?</p> <p>Student: They are different.</p> <p>Teacher: But if you look at them, they have three different elements (ABC) and (AB1). Then we could consider them as the same because they have the same repetition structure.</p>

From the examples shown in Table 5, it can be seen how the student justification is more elaborated and consolidated in the context of real situations than in the context of graphic resources. In the same way, it is seen how the teacher, through good questions, i.e., open questions and using a precise mathematical language, motivates the students to communicate, justify and reason their answers. This scenario enables a constructive discussion from a parallel perspective lived by another participant, that favors enrichment and conceptualization in those students who have not succeeded in the task (Vygotsky, 2004).

4 Discussion and conclusions

This study investigated how the teaching context influences the performance of tasks with repetition patterns. To this end, students' written productions were analyzed to determine whether they were able to correctly represent repetition patterns in the most concrete context (real situations) and in the most abstract

(graphic resources) of a previously designed and validated pattern teaching process. Based on this analysis, a positive difference of 32.9% of real situations versus graphic resources was identified in the 4-5-year-old students; in the 5-6-year-old, although the difference between the two contexts decreased slightly, it was still over 30%.

If the results are analyzed longitudinally, 15% of 4-5-year-old students have incorrectly represented patterns in tasks posed from real-world situations, while the percentage of incorrect pattern drops to 0% in 5-6-year-old students. In the context of graphical resources, incorrect representations decreased by 23.8%, 54.2% for 4-5-year-old students and 30.4% for 5-6-year-old students. However, despite this decrease, participants generally show difficulty in representing the pattern without prior interaction or manipulation of specific elements that make up the repetition. In other words, the data obtained have shown that understanding is more accurate in contexts where teaching is prioritized from informal situations of exploration of a



daily environment close to boys and girls, where it is easier to establish relationships with their previous knowledge. In this sense, Castro and Castro (2016) point out that they learn through concrete experiences with materials and through intentional and previously planned recreational interactions. Likewise, Zhong and Xia (2020) say that children need opportunities for exploration, manipulation, and experimentation to promote learning from a playful and concrete perspective.

Focused on representations of patterns, we agree with Alsina (2016) when mentioning that the representation of mathematical ideas and procedures is an essential process for learning, and therefore, if there is no representation there is no understanding, and thus there can be no learning of mathematics. Therefore, it can be assumed that, from an early age, children must represent to learn mathematics and thus be able to organize, understand and communicate the mathematical nature of the actions previously carried out at the educational and social level using signs, graphics and/or natural language. This is the main reason why we have used representations in mathematics as an interconnected process that allows to: a) concretely embody the knowledge and procedures of children about repeating patterns; b) assess progress in understanding these patterns; and c) rebalance the process of teaching patterns through the design of contextualized tasks that encourage and extend learning.

From this perspective, Laski and Siegler (2014) show that concrete learning materials are only effective to the extent that the activities designed are aligned with the desired mental representation process. It is for this reason that educational proposals must be related with the aim of increasing the codification of the structural characteristics that make up the pattern to facilitate representation. Carruthers and Worthington (2005) conclude that when teachers encourage children from 3 to 8 years old to play their mathematical ideas on paper, they encourage an understanding of abstract symbolism.

This study has provided relevant data showing how the degree of success of understanding through representation is conditioned by the abstraction level of the context in which the proposal is presented. In this regard, it is considered necessary to plan and structure tasks that include different teaching contexts, in order to offer an educational intervention respectful to the needs of the students, in which it is essential to encourage the use of concrete and informal contexts that allow progress toward the generalization and formalization of knowledge, avoiding patterns that exclusively use paper and pencil. This approach requires, on the one hand, disciplinary knowledge about what is wanted to teach (NCTM, 2014; Pincheira and Alsina, 2021); and, on the other, didactic and methodological skills to deal with a particular concept or procedure from different teaching contexts (Alsina, 2022). Villalpando *et al.* (2020) point out that the professors must transfer the official program into a real setting that allows to give meaning to the teaching practice, in order to bring the academic contents to the students in a reflective, competent and experiential way.

At this point, we think that educational situations are context-sensitive by showing that the success of the representation has been closely related to the understanding of the pattern, and that understanding has been most successful at the most concrete level of the EIEM, in which the teaching of mathematical content is prioritized for situations that are real or close to children. It has also been demonstrated that shared knowledge is promoted, generated, and consolidated by using good questions. For this reason, we encourage teachers to support the teaching of repetition patterns from a dialogic and multimodal vision that includes various educational scenarios that move progressively from concrete to abstract contexts. Therefore, our aim is that these real experiences accompany, through reflection, future teaching action (Radford and Sabena, 2015) and that our conclusions could be a source of inspiration, without



being directly generalized to other realities, since the small number of our sample is a limitation. In this regard, we also assume as a limitation the deferred use, through images, of real situations in the first stage of the didactic process, since it may have influenced the responses of the students, and we cannot know for sure whether errors would have been reduced with a live deployment. As future lines of research, we propose to continue to demonstrate how the other contexts of the EIEM influence the teaching of repetition patterns, analyzing the relationship established between the mathematical knowledge of the students and the ability to justify and argue their responses.

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Mathematical representations of 5 and 6-year-old children when solving an open-ended problem

Representaciones matemáticas de niños y niñas de 5-6 años cuando resuelven un problema abierto

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Abstract

Problem solving and representation are two fundamental processes of mathematical activity. Their development provides a key basis for learning mathematics at all school levels. Hence the importance of promoting these processes from an early age. The aim of this article is to describe the representations and ways of solution posed by a group of children in pre-school education (5-6 years), in a Catalan school, when solving an open-ended arithmetic problem. The study follows a descriptive-interpretative methodology. A school task is designed and implemented from which individual written productions are obtained. In addition, interviews were conducted with each of the students and the corresponding video recordings were made. The data are systematised and a two-phase analysis is carried out: initially the types of representation are characterised and then the calculation methods used by the children. The results indicate that all the participating pupils produce representations to solve the problem. All the children make iconic representations, and a few combine iconic and symbolic representations. As for the ways of solving the problem, continuous counting predominates, although in some cases proposals are made in which more complex reasoning is evident. In these cases, the children propose groupings which are expressed by means of drawings and symbols.

Keywords: Problem solving, representation, reasoning, calculation methods, mathematical activity, early ages.

Resumen

La resolución de problemas y la representación son dos procesos fundamentales de la actividad matemática. Su desarrollo proporciona una base clave para el aprendizaje de las matemáticas en todos los niveles escolares. Por ello, la importancia de la promoción de estos procesos desde las primeras edades. El objetivo de este artículo es describir las representaciones y formas de solución planteadas por un grupo de 23 niñas y niños de educación infantil (5-6 años), de una escuela catalana, cuando resuelven un problema aritmético abierto. El estudio sigue una metodología descriptiva-interpretativa. Se diseña e implementa una tarea escolar de la que se obtienen producciones escritas individuales. Se realizan además entrevistas a cada uno de los alumnos y se cuenta con los registros en video correspondientes. Los datos se sistematizan y se realiza un análisis en dos fases: inicialmente se caracterizan los tipos de representación y luego los métodos de cálculo planteados por los niños. Los resultados indican que los alumnos participantes elaboran representaciones para resolver el problema. Todos los niños y las niñas realizan representaciones icónicas, y algunos pocos combinan representaciones icónicas y simbólicas. En cuanto a las formas de solución del problema predomina el conteo continuo, aunque en algunos casos se realizan propuestas en las que se evidencian razonamientos más complejos. En estos casos, los niños¹ plantean agrupaciones las cuales se expresan mediante dibujos y símbolos.

Descriptores: Resolución de problemas, representación, razonamiento, métodos de cálculo, actividad matemática, edades iniciales.

1 Introduction and state-of-the-art

Current academic approaches to early childhood education mention the importance of globalized approaches, interdisciplinarity and the need for competency development (National Council of Teachers of Mathematics-NCTM, 2000; 2014; Ministerio de Educación y Formación Profesional, 2022; National Association for the Education of Young Children-NAEYC, 2020). Authors such as Clements and Sarama (2016); De Castro *et al.* (2012); Vanegas and Giménez (2018), among others, highlight the role of mathematical processes in the acquisition of competencies and point out that these are essential to promoting the ability to use mathematics in a comprehensive and effective way in different contexts. Supporting and enriching these processes and promoting the development of children's mathematical thinking is one of the challenges of early childhood education (Baroody, 2003; Cheeseman, 2019; Clements and Sarama, 2021; Ginsburg and Amit, 2008; Lopes *et al.*, 2017). As a result, raising and solving problems, analyzing different strategies and solutions, and reflecting on them should be main activities in the teaching and learning processes of mathematics at each school level (NAEYC and NCTM, 2013; Mason, 2016; Schoenfeld, 2016).

According to Edo (2005), math learning is a socially mediated construction process. It is especially relevant when thinking about early childhood education, as it involves assuming that children do not learn by receiving and passively accumulating information from the environment, but they do so through an active meaning-making and sense-making process, where problem-solving, communication and representation are essential processes (Battista, 2016). If mathematics is considered as the result of certain actions carried out by people and as a changing phenomenon, mathematical activity must be characterized by the desire to find something: data, processes, relationships, results, a way of communicating, etc.

Therefore, early mathematics education should focus on helping children live mathematical activity situations, i.e., search situations where the focus is the practices of children.

As Baroody (1993), Saundry and Nicole (2006) and Carruthers and Worthington (2010) mention, representations and drawing are fundamental tools for solving problems in early ages. These authors argue that representations are essential in the construction of meanings because representations help children to concretize the problems and decide the procedure to use in their solution. Carruthers and Worthington (2009) also stress the importance that teachers recognize representations made by children while solving problems. In this way they will be better identify the ideas and ways of reasoning. For this reason, it is necessary to explore the type of representations and how children in early education solve problems. The purpose of this article is to describe the type of representations and strategies used by a group of children (5-6 years old) when involved in an open problem-solving task.

This study is based on two main aspects. The first refers to the solution of open problems at an early age and the second relates to the use of representations and their importance in solving and communicating problems.

Ramírez and de Castro (2014) say that it is essential to introduce problem solving in early childhood education, since it not only encourages the development of informal strategies but also because it helps children to give meaning to arithmetic operations and certain procedures they will learn as they advance in their schooling. We agree with Alsina (2012) who, following the NCTM (2000) approaches, proposes that there are four aspects concerning the solution of problems that should be worked from the early childhood education: a) to construct mathematical knowledge through problem solving, to propose a variety of contexts; b) solve problems arising from mathematics and different contexts, everyday situations, daily routines, experimentation situations, among others; c) apply and adapt a variety



of appropriate strategies to solve problems, such as asking good questions; encourage interaction, negotiation and dialog in the classroom; etc.; and, d) regulate and reflect on the process of solving mathematical problems.

While it is true that there are different types of mathematical problems (realistic, authentic, open, among others), teachers are the ones who choose to use one or the other regarding the objectives proposed. According to Pehkonen (1997), an open problem is one where the starting situation is opened, as opposed to closed problems where the beginning and end are exactly explicit. In this group, Baroody (1988) refers to routine and non-routine verbal statement problems. Non-routine problems are those that involve different procedures for their solution, and may have different answers. Routine or non-routine problems are those of division, which involve the action of separating the total parts into units or as wholes. In the investigation of Saundry and Nicole (2006) two types of non-routine division problems are presented: a) arithmetic problems arising from a grouping (set of elements), which must then be distributed; and b) arithmetic problems that also start from a grouping (set of elements), but where their division involves more than one operation to solve it (a set of elements must be divided into subsets).

In the early ages, representations serve both to build new mathematical knowledge and to express mathematical ideas (NCTM, 2000). In this sense, Burgués and Sarrañana (2013) argue that it is desirable for mathematical language to become a natural form of expression in the classroom among teachers and children. To achieve this objective, the conversation about mathematics must be promoted, first through verbal language, and progressively introduce the terms and forms proper to mathematical language (oral and written). It is not about children memorizing symbols, but learning to communicate their mathematical ideas with meaning, hence the importance to explore their representations.

Teachers must analyze the representations of their students and their discussions (their mathematical communication) to evaluate the development of their mathematical thought and thus offer them the necessary elements to connect their own languages to the conventional mathematical language (NCTM, 2000; Edo *et al.*, 2009). The truth is that children naturally represent cognitive ideas through paper, objects, play, etc., in short, they use the representations to shape their schemes and make them meaningful on paper (Carruthers and Worthington, 2006; Worthington, 2009).

Various authors have studied the mathematical representations of children in the early ages. Thus, Deliyianni *et al.* (2009) studied the ways of representation generated by the students of pre-school and elementary school by examining the compliance with the norms during the didactic process in the solution of problems. While Saundry and Nicole (2006) investigated how children of the early ages think mathematically and represent an arithmetic problem, Smith (2003) and Woleck (2001) argue that drawings perform two fundamental functions: a) they serve to model problems and b) they are the support of mathematical activity that allows them to be solved. In addition, they point to the importance of listening to the students' explanations of their drawings to understand the mathematical activity they perform. In the case of Carruthers and Worthington (2006), from the analysis of mathematical drawings and writings by children up to the age of eight, they identify five types of graphics: dynamic, pictographic, iconic, symbolic, and written. For Carruthers and Worthington (2005, 2006), children reach the mathematical meaning of abstract symbols from their own representations and by constructing their own meaning.

Carruthers and Worthington (2006) propose two dimensions for interpreting the mathematical graphs of boys and girls aged from 0 to 8 years old. The first concentrates on *written representations of numbers and quantities* and the second focuses on the *written calculation methods* devised by children. These authors also produce

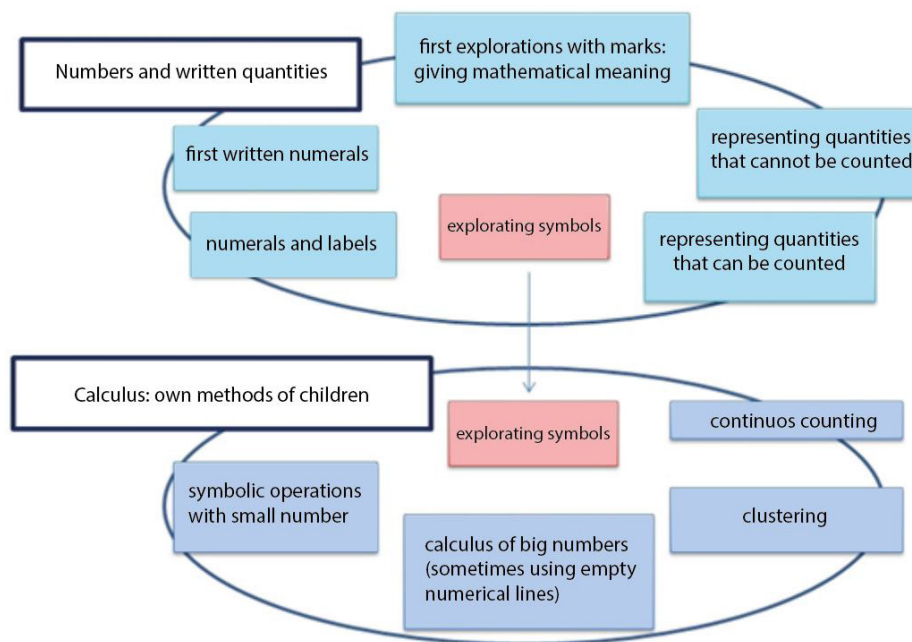


a non-hierarchical taxonomy (Figure 1) illustrating the categories considered in each of the dimensions identified for the characterization of

children's mathematical graphs (Carruthers and Worthington, 2013).

Figure 1

Taxonomy: Children's mathematical graphics



Note. Carruthers and Worthington (2013).

In the first dimension: Written numbers and quantities, five categories are considered:

- First explorations with marks, sometimes these first representations are seen by adults as simple scribbles, but they are an important step for children on their way to multi-dimensional representations of their world.
- *Early written numerals* or when children refer to their marks as numbers. At this point children understand which numbers and letters have meaning and begin to make a general differentiation between them: "this is a number", even though they are not yet recognized as numbers, but they may have numerical qualities.
- *Numerals as labels*, in this case children identify numbers and letters in their

surroundings and show interest in using them; they observe the function of written numbers in a social sense. There is a time when children move from identifying these symbols in their surroundings to write them for their own purposes. This is a significant change because by choosing to write these numbers they convert what they read into a standard symbolic language and choose to use them in meaningful contexts.

- *Representing quantities that are not counted*, these are made by younger children, and are live and not very accurate representations. For example, the case of a three-year-old boy who represents an eight-legged spider, but the representation shows a spider with many legs, more than eight. It is clear



that the child represents his or her personal sense of quantity, not a concrete amount.

- *Representing quantities that are counted*, in this case children make representations of amounts and count them, for example, by drawing vertical lines and saying that they are “rain drops” and counting them at the end. For Carruthers and Worthington (2010), uncounted quantities precede counted quantities, but there may be an overlap between these two aspects. In turn, this developmental aspect leads children directly to the beginning of written calculations.

According to Carruthers and Worthington (2006), as children explore calculations in a variety of ways, their own representations support their mental methods and help them calculate. The count has a strong presence at the beginning of the written calculus (Clements and Sarama, 2013; Baroody *et al.*, 2019)

In the second dimension: *Calculation methods* devised by children, the following categories are described:

- *Counting continuously* refers to the first representations children make for addition and subtraction. Various studies show that young children make simple additions and subtractions with counting strategies, telling everything, all the elements. So, if there are two sets, they count one and when they finish, they continue with the next, without separating the two sets.
- *Separating sets*, in this case children exhibit different strategies to show that two amounts are separated. They make groupings of two or more sets of elements that must be added, placing each on one side of the sheet of paper or leaving a space between them; separating the sets with words; placing a vertical line between the sets, among others.
- *Exploration with symbols*, in this case children begin to make explicit use of symbols

(invented or sometimes using standard symbols). It is also considered when children make marks in their procedures that show that they understand the symbols, even if they do not appear explicitly.

- *Symbolic operations with small numbers*, at this point children already know the standard symbols and understand their role and have developed strategies to solve problems.
- *Calculating with large numbers* (sometimes using annotations or empty number lines). Calculating with large numbers is more difficult, as it is needed to understand what the large numbers involved look like and may need to manipulate several steps. This is where mental methods and some taught supports can be valuable, such as the number line.

It should be mentioned that children's responses to a certain mathematical task involved in representations can be classified into several of these categories, not necessarily one.

There has been a common questioning about how young children can solve mathematical problems, since most of them do not know how to read or write. This type of question reveals a misconception which must be solved (Lopes *et al.*, 2017). It is important to understand that thinking and language are linked, and that representations play a fundamental role in children's ways of thinking and communication. Investigating how children respond when they have a mathematical problem; the types of drawings they do spontaneously; the things they think while drawing; the relationships they establish and express both orally and in writing is key to understanding how they construct their mathematical ideas.

2 Methodology

In this research a descriptive-interpretative methodology was used (Latorre *et al.*, 2003;



Cohen *et al.*, 2018). A descriptive analysis of the data is made, to later relate and interpret these descriptions considering the theoretical references. Specifically, a mathematical task is proposed to explore and describe aspects of the mathematical thinking of the participating children when solving problems.

An open arithmetic problem was selected. This problem was implemented in a group of 23 students in pre-school (5-6 years) at a school in Cerdanyola del Vallès (Cataluña). The problem was solved by the children individually and in written form. In addition, to recognize the strategies and processes followed by the children in solving the problem a semi-structured interview was conducted and a video was recorded.

The problem is an adaptation of the one proposed by López (2015), where a family context is proposed to children, related to food. The following is the statement of the problem:

You want to make a fruit salad. You can buy bananas, pears, oranges and apples. In total, you buy 15 pieces of fruit. Explain how many pieces you buy for each fruit.

Different aspects were considered in the selection and adaptation of the problem, such as those pointed out by Baroody (1993) when characterizing the non-routine problems: the unknown is not obvious, the problem provides information on the total of fruits, but the unknown refers to the number of fruits of each type that could be used to make a fruit salad with that total. It can be solved in different ways and different solutions can be obtained.

The problem was presented to the children orally and the following guidelines were given:

- The problem must be solved individually.
- Different representations can be used: drawings, numbers, letters or several of them at the same time.
- At least two types of fruit should be used to respond to the problem, not all four types are required.

- It must be considered that 15 pieces of fruits should be used

The children had blank sheets of paper to make their proposals. According to Edo and Marín (2017) at the moment of the proposal it is important to select the instructions appropriately so that the graphic representation the child makes shows what he/she thinks and how he/she thinks. In turn, it is desirable to create a climate of confidence and tranquility so that each child can reflect, choose, represent, and explain his or her reasoning. Blank paper marks will show languages and meanings, allowing the teacher to observe each student's learning and thus giving relevance to the student's marks (Carruthers and Worthington, 2006).

Regarding the semi-structured interview, a series of basic questions were set:

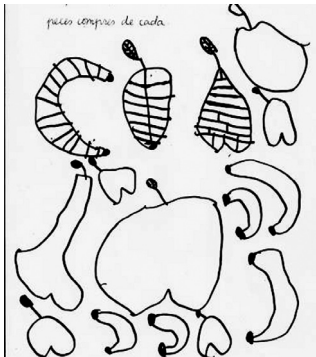
- Can you explain what you did?
- How many fruits of each type have you drawn? Why?
- Are you sure you have 15 fruits? How do you know?
- Have you drawn all kinds of fruit? Why?
- What have you done to know when you should stop drawing?
- Have you tried to use the same number for each type of fruit?
- What do the numbers you used indicate?
- How did you know how many more fruits you should draw while solving the problem?

The data from this research are the written protocols of each of the participants to the proposed task and the transcripts of the dialogs generated in the interviews. This information is initially organized into a data collection tool. As an example, Figure 2 shows an extract of that instrument. This includes the identifier assigned to the student (S1), the written answer and the initial part of the interview.



Figure 2

Extract from the data collection instrument

Student	Written Protocol	Transcription
A1		<p>P: ¿What did you do? T: What have you done? S1: I've drawn them all T: All? S1: Yes T: Very good How many bananas have you made? S1: Four bananas T: What else have you drawn? S1: This round thing are strawberries, there are two T: Two strawberries? And this round, are these oranges? S1: Yes. They're oranges, huh! T: How many have you done? S1: One, two, three, four, five, six, seven! T: Teacher A: Student</p>

Own elaboration.

Data analysis was carried out in two phases. In the first phase, the children's responses to the problem were individually analyzed, focusing this analysis on the *representations*. Following the proposal of Carruthers and Worthington (2005), the *representations* are classified into three cate-

gories: *iconic*, *written*, and *symbolic*. The two initial categories (dynamic and pictorial) proposed by these authors are not considered because of the age of the children participating in the study. Table 1 describes the indicators associated with each of these categories

Table 1

Indicators of the representation category

Categories	Indicators
Iconic	Uses a conceived picture of reality
Written	Uses letters or words to complete the answer
Symbolic	Includes numerals, dots, lines, circles, or signs

In the second phase to complement the study of children's productions, the *calculation methods* that followed in solving the problem were analyzed. For this analysis, continuing with the characterization taxonomy of children's mathematical graphs proposed by Carruthers and Worthington (2013), the categories of the dimension methods of written calculations were considered: *counting continuously*, *separating sets*, *exploring with symbols*, and *symbolic operations with small numbers*.


The *calculating with larger numbers* is not considered as it does not fit the problem conditions.

As mentioned Badillo *et al.* (2014), we also think that the solution and representation strategies raised by children are connected and, therefore, a global look will allow a richer analysis of the mathematical practices developed by children in solving the problem.



An analysis instrument is constructed, combining the aspects analyzed in phases 1 and 2. Then in Figure 3 we illustrate the analysis of the answer provided by student ten (S10).

Figura 3
Analysis of A10

Student	Written Protocol	Transcript of the interview	Analysis
S10		<p>Q: Tell me, what have you done? Have you drawn the fifteen pieces of fruits?</p> <p>S10: Mmm pears, I made a pear, with pears</p> <p>Q: You made many pears, and what else? Is this a banana?</p> <p>S1: A banana</p> <p>Q: I see that you have used numbers, was it to count what number each was until reaching fifteen? If you start with one, two, three, the eight... where is the fifteen?</p> <p>S1: Here</p> <p>Q: Have you gotten fifteen then? Did you stop when you reached fifteen?</p> <p>S1: Yes</p> <p>Q: Good job!</p> <p>Q: Teacher A: Student</p>	<p><i>Representation</i></p> <p>Student 10 performs a representation that can be classified as iconic and symbolic. On the one hand, with the representation the child shows the image he/she has of the fruits, and on the other he/she adds numbers to list each of the pieces.</p> <p><i>Strategies</i></p> <p>The student focuses on drawing to solve the problem. Represents the amounts that counts. The student draws all the fruits and focuses on reaching the final number (15) by counting each piece at a time, i.e., uses the ordinal by extension. It seems that the student recognizes the cardinal, and that uses the numeral with an order function; it is evident that the student is exploring with symbols.</p>
Q: Teacher A: Student			

Own elaboration.

3 Results

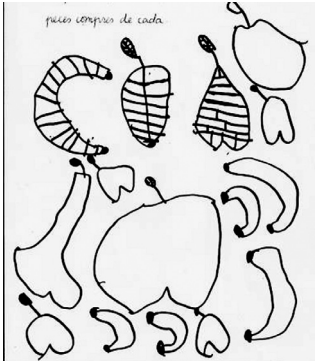

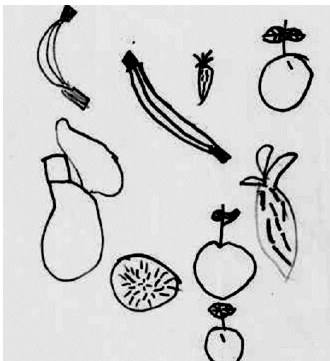
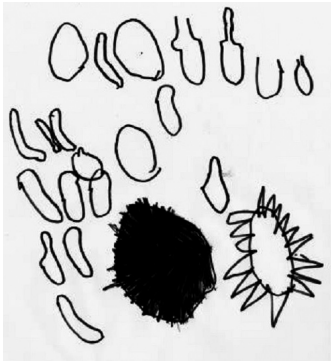
The results are organized into two parts: characterization of the representations used by children and identification of the strategies developed in solving the problem.

3.1 Characterization of the representations used by children

As for the type of representations, 23 students who participate in the study use iconic representations.

In using this type of representation, children rely on drawing to count and control the total amount defined in the proposed problem. At the same time, their drawings indicate the type of fruits chosen by children, the quantity they have considered for each type and in some cases the intentionality of grouping them (distributions). It should be mentioned that, although everyone proposes an iconic representation, not all adequately solve the problem, as in five of the students. Figure 4 shows different examples of responses, showing the above aspects.

Figure 4
Examples of different types of iconic representation responses

Correct problem answer-No grouping	Correct problem answer-With grouping
	
Representation of student 1 (S1)	Representation of student 2 (S2)
Inadequate answers to the problem	
	
Representation of student 14 (S14)	Representation of student 16 (S16)

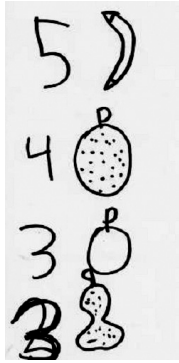

Out of the total of children, three, in addition to using an iconic representation, also use a symbolic representation. In these representations a more complex reasoning is evident. Children who performed iconic-symbolic representations no longer focus only on counting to reach the total, but on the operational, proposing different subgroups to meet the condition of having 15 fruits in macedonia. These representations include, in addition to the drawing of the fruits of each child,

the numerals that indicate the amount they have associated with each type (see figure 5).

Figure 5 shows an example of a response that combines representations. In the representations proposed by S4 and S12, it can be seen that children recognize that the whole (15 fruits) can be separated into discrete sets of various elements (e.g. bananas, oranges, apples and pears), which may (or may not) have different cardinal (e.g. 5, 4, 3, 3 or 5, 4, 3, 2, 1).



Figure 5
Examples of response of S4 and S12

Response with combination of iconic-symbolic representations	
	
Representation of student S4	Representation of student S12

3.2 Identification of calculation methods developed in problem solving

As for the methods used by children to solve the problem according to the dimensions proposed by Carruthers and Worthington (2006), 19 of the children's solution proposals were classified in the category of *counting continuously* and four in the category of *separating sets*. In the first case, representations that express a quantity are classified. Children represent and account for things they choose (in this case fruits) but do not see physically (e.g. S1, see figure 4). Usually, this type of representation is drawn in a horizontal linear layout (e.g. S2, see figure 4), although others can be found. This is considered the first step in the calculation exploration. In our case, most of the children focused more on the goal of having 15 pieces of fruit than on the order in choosing which type of fruit to draw. If we look at the solution given by S10 (Figure 3) the final number of its count represents the total. These different mathematical practices related to continuous counting are key to recognizing the strategies children are developing (Carruthers and Worthington, 2006). Thus, some children,

as the interviews showed, continually counted, starting with one of the fruits, what they considered "the first." Almost all children understand that it is necessary to count everything to get to a total, except those who failed to adequately solve the problem (e.g. S14, S16-Figure 4).

In the second case, the representations indicate separations in fruit subgroups (e.g. S2 - figure 4, S4 and S12 - figure 5). In our study, none of the children used marks (lines, words, circles, etc.) to differentiate the subgroups. But in the four cases in this category, the children represent separate sets (of fruits) that then add up to meet the condition of having 15 pieces in total in the macedonia. Student S2 (Figure 4) uses the space and distribution of the fruits to indicate the sub-groupings he has made of the fruits. The interview corroborates that he has made banana-apple-pear groups. Identifying the set and the elements of the set and repeating it continuously until reaching 15. In the case of S4 (Figure 5), it was possible to see in the interview that he performs the calculation as a narrative in words (Carruthers and Worthington, 2006) when describing what he has done: "I added five bananas and four oranges here, and then I added three apples and three pears to have a total of 15."



Finally, there are three solutions in the dimension: *explorations with symbols*. According to Carruthers and Worthington (2006) children in this category organize their solutions and sometimes represent them leaving a space between the sets to imply that an (operant) is needed in that place, for example, as in the solution given by S4 (Figure 5). They usually use personal or invented symbols or approximations of standard symbols. In our study, the three children use standard numeral symbols to indicate the amount they associate with each type of fruit or to support continuous counting (S10-figure 3). In another case, one of the children (S12-figure 5) used the word “and” to indicate “+”. The combination of drawings, words, numbers and/or personal symbols is also typical of this category, as is the case in the three cases mentioned above.

Children who perform representations combining the iconic and symbolic are those who also use symbol exploration strategies, demonstrating a more complex reasoning. Moving to other types of strategies, such as performing standard symbolic operations with small numbers, requires that children have developed them previously. However, as suggested by Vanegas and Giménez (2018), when children solve problems, the important thing is not to move from one strategy to another, but to use appropriate strategies that show an increasingly complete and adequate interpretation of the problem.

Discussion and conclusions

As suggested by Carruthers and Worthington (2005), we consider that children sometimes use a combination of representations, for example, iconic and symbolic, when they are in a period of transition. It seems that when they do it, they are moving from familiar representations to new ones, although they are not yet ready to dispense with non-essential elements. In our study, this occurs with three out of the 23 participants. This transition period is very important as children move toward more abstract forms of mathematics.

However, it is also important to note that some children return to less developed graphic forms when they find that the mathematics presented is more challenging, because they are based on prior knowledge and ways they feel more confident.

When children move from recognizing numbers as symbols associated with different contexts in their lives to writing them for their own specific purposes, it evidences a significant change, because when they choose to write certain numbers (in our study, to indicate the total number of fruits of each type they would use to make macedonia) they have moved what they read into standard symbolic language and have chosen to use them in meaningful contexts (Worthington and van Oers, 2017). It is important to engage children in play and problem-solving environments that challenge them and allow them to experiment and choose their own methods.

Seeing the different representations children use when facing math tasks will allow teachers to better recognize their ways of thinking and the aspects they give relevance to when working with certain mathematical notions. In addition, the analysis of representations and associated strategies will allow the teacher to better evaluate the development of children's mathematical thinking. Consequently, new schoolwork can be designed to help children develop skills to explain, describe, relate and argue.

The richness of the problems and/or tasks proposed to children is a key element in enhancing the development of their mathematical thinking. Indeed, problems must be posed in a wide range of contexts that have real meaning for children, since it will help them have a personal sense of mathematics. As proposed by NAECY and NCTM (2013), we consider that problem solving, reasoning, communication, connections, and representation make it possible for children to acquire knowledge of the content. These processes develop over time if they are supported by well-designed learning opportunities. Children's development and use of these processes is one of the most enduring and important achievements



of mathematics education. Their intuitive ideas become true mathematics when children reflect on them, represent them in different ways, and connect them to other ideas.

With this research we have been able to show how the analysis of children's representations and strategies in a problem-solving environment can provide important feedback on children's thinking; therefore, relevant elements for reflection on this subject in the initial training of early childhood education teachers. We hope to continue working in this topic, on the one hand, by exploring the representations that children construct when they engage in different mathematical tasks; on the other, by analyzing and using different references to characterize these representations and incorporating these findings into professional tasks in the initial training of teachers. We are interested that teachers understand that it is possible to develop quality mathematics from the early ages (Lee and Ginsburg, 2007) and to identify how research results such as the one described in this article can be useful in their professional context, supporting design, planning and evaluation of school proposals that promote the development of mathematical thinking in early childhood education.

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Didactic-Mathematical Knowledge mobilized by future mathematics teachers

Conocimiento didáctico-matemático movilizado por futuros profesores de matemáticas

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Abstract

From the perspective of mathematics education, several studies focus on the training of mathematics teachers. This article is a part of a master's research. It aims to analyze the didactic-mathematical knowledge mobilized by future mathematics teachers when solving tasks centered on themes related to financial education. It is qualitative research developed in the context of teacher education, consisting of a sample of undergraduate students in mathematics from two public higher education institutions in the North of Minas Gerais (Brazil). For data collection, a workshop was developed with five tasks, four of which were synchronous, carried out by Google Meet, and one asynchronous, posted on Google Classroom, which articulates mathematical content with themes related to Financial Education, namely: reflection on a class of math; reflecting on inflation and price indices; analyzing housing finance; analyzing interest and its applications in credit operations; elaborating activities related to interest applications. Data analysis, carried out from the mathematical and didactic dimensions of the Didactic-Mathematical Knowledge (DMK) model, revealed that future teachers satisfactorily mobilized the common knowledge of the mathematical content and, partially, the expanded knowledge. The mobilization of didactic knowledge was satisfactory only in the cognitive facet and partial in the other facets of the DMK.

Keywords: Mathematics teachers' formation, Didactic-Mathematical Knowledge, financial education.

Resumen

Desde la perspectiva de la Educación Matemática, varios estudios se centran en la formación de profesores de matemáticas. Este artículo forma parte de una investigación de maestría. Tiene como objetivo analizar el conocimiento didáctico-matemático movilizado por los futuros profesores de matemáticas al resolver tareas centradas en temas relacionados con la Educación Financiera. Se trata de una investigación cualitativa desarrollada en el contexto de la formación de profesores, constituida por una muestra de estudiantes de licenciatura en matemáticas de dos instituciones públicas de enseñanza superior del norte de Minas Gerais (Brasil). Para la recolección de datos, se desarrolló un taller con cinco tareas, cuatro de ellas sincrónicas, realizadas por Google Meet, y una asincrónica, publicada en Google Classroom, que articulan contenidos matemáticos con temas articulados con la Educación Financiera, a saber: reflexión sobre una clase de matemáticas; reflexión sobre la inflación y los índices de precios; análisis de la financiación de la vivienda; análisis del interés y sus aplicaciones en las operaciones de crédito; elaboración de actividades relacionadas con las aplicaciones del interés. El análisis de los datos, realizado a partir de las dimensiones matemática y didáctica del modelo de Conocimiento Didáctico-Matemático (CDM), reveló que los futuros profesores movilizaron satisfactoriamente el conocimiento común del contenido matemático, parcialmente, el conocimiento ampliado. La movilización del conocimiento didáctico fue satisfactoria solo en la faceta cognitiva y parcial en las demás facetas del CDM.

Descriptores: Formación de profesores de matemáticas, Conocimiento Didáctico-Matemático, Educación Financiera.

1 Introduction

Teachers play a vital role in the development of the teaching and learning processes of mathematics at different educational levels, which requires training that allows them to relate the knowledge of the content to be taught with didactic and methodological strategies that would overcome possible obstacles derived from educational processes.

In the teaching of Mathematics and Natural Sciences, teacher training is a relevant topic studied in different research (Gellert *et al.*, 2012; Ponte, 2014; Stahnke *et al.*, 2016; Potari and Ponte, 2017; Barros *et al.*, 2014). More specifically, one of the research trends is focused on specialized mathematical content knowledge for teaching. Godino *et al.* (2017), state that such knowledge has been discussed by several authors through different theoretical perspectives and that in the specific literature related to mathematics teacher education is:

It is recognized that the didactic training of teachers is a field of scientific and technological research that requires attention from the Didactics of Mathematics, since the development of students' thinking, and basic mathematical skills depends on such training. (Godino *et al.*, 2017, p. 91)

Considering the relevance of the discussions around the necessary knowledge for the teacher (Breda *et al.*, 2018; Giacomone, 2018; Morales-Maure, 2019), this article aims to analyze the didactic-mathematical knowledge mobilized by future mathematics teachers when solving tasks focused on topics related to Financial Education from the perspective of Mathematics Education, whose relation will be described below.

The teaching practice that seeks the approximation between mathematical contents and reality through a critical and reflective perspective highlights the need to relate mathematics with contextualized topics, which can be enhanced through an approach to specific topics

of Financial Education, especially related to the current economic scenario. Reinforcing this idea, the Survey of Consumer Indebtedness and Delinquency (PEIC), conducted by the National Confederation of Commerce of Goods, Services and Tourism-CNC (2021, p. 1, the translation is ours) in Brazil, shows that “the percentage of families with debts in the country [Brazil] ended in 2020 at a high level, after three consecutive months of reduction”. Similarly, research proposes deepening discussions on Financial Education in all social sectors, especially in schools, given that money management begins in childhood (Teixeira, 2015; Souza, 2018; Assis, 2019; Martins, 2019; Melo, 2019; Ferreira, 2020), which can be achieved through School Financial Education (Silva and Powell, 2013).

We corroborate the idea that highlights the potential of approaching Financial Mathematics contents, contextualized and coherent with the reality of students, both to work School Financial Education from the beginning of Elementary school (Teixeira, 2015) and to instrumentalize financial literacy, which is directly associated with:

The ability to read, analyze and interpret financial situations; knowledge of the essential and necessary elements of financial mathematics relevant to the context of the subjects; ability to take a reasoned critical stance; ability to consider the variables and implications of their actions; make conscious decisions oriented towards individual and social financial well-being. (Sena, 2017, p. 58, the translation is ours)

The research, approved by the Research Ethics Committee of the State University of Montes Claros (Unimontes) through the Certificate of Ethical Appreciation (CAAE) No. 30562920.6.0000.5146 and Opinion No. 4.031.671, consists of qualitative research which, according to Godoy (1995, p. 21, the translation is ours), seeks “to capture the phenomenon studied from the perspective of the people involved,



considering all relevant points of view”. The sample consisted of twelve undergraduate mathematics students from two public institutions in northern Minas Gerais. Data collection was performed by recording meetings in *Google Meet*, joint solutions and discussions of the proposed tasks, and the participants’ productions in *Google Classroom*. The data analysis focused on the categories included in the Didactic-Mathematical Knowledge model, developed by Godino (2009) and Pino-Fan and Godino (2015).

The following sections deal with the synthesis of the theoretical framework, the methodology, the analysis and discussion of the data and the final considerations.

1.1 Theoretical framework

This research focuses on the theoretical tools developed in the context of the onto-semiotic approach to mathematical knowledge and instruction-EOS. (Godino *et al.*, 2007).

According to the EOS, the mathematics teacher must have a mastery of mathematical content far beyond the knowledge that must be mobilized by the students of the educational level in which he/she is going to work. However, knowledge mobilization of teachers does not guarantee a convincing performance in the teaching and learning processes, since these processes are complex, so it is essential to have a deeper knowledge of both mathematics and its didactics, beyond the knowledge manifested/mobilized by students.

To investigate this knowledge in the context of mathematics teacher education, the Didactic-Mathematical Knowledge (DMK) model (Godino, 2009; Pino-Fan and Godino, 2015) of the EOS was developed, which is articulated with other models related to teacher knowledge, such as Pedagogical Content Knowledge (Shulman, 1986, 1987), Mathematical Knowledge for Teaching (Hill *et al.*, 2008) and the notion of Competence (Shoenfeld and Kilpatrick, 2008).

This model interprets and characterizes the teacher’s knowledge, which is approached in this research from two dimensions: mathematical and didactic.

The mathematical dimension refers to the specific knowledge of mathematics, including the subcategories *common knowledge* and *extended content knowledge*, as pointed out by Carvalho (2017). The first refers to what the teacher needs to know about a specific mathematical object to solve problems and tasks proposed in textbooks and other material and technological resources at a given educational level (shared knowledge between teacher and student). On the other hand, the second allows the teacher to establish a relationship between the mathematical object of study and other mathematical notions at the same educational level and higher levels.

The didactic dimension includes the six categories of analysis described below.

Epistemic: related to the didactic-mathematical knowledge of the teacher; it is the understanding of mathematics for its teaching, it allows to perform different representations of a mathematical object, to solve a task by different methods, to establish relationships between the mathematical object of study and other mathematical objects of previous and subsequent educational levels. This integrates both the notions of the proficiency model (Shoenfeld and Kilpatrick, 2008) about knowing mathematics in depth and the ideas of specialized content of the content (Hill *et al.*, 2008).

Cognitive: referred to the knowledge of how students learn mathematics, which allows establishing a relationship between personal meanings (student’s knowledge), institutional meanings (knowledge from the point of view of the educational center) and class planning, taking into account the foresight of possible errors and difficulties.

Affective: inherent to the affective and emotional factors of students with respect to mathematical objects. It has to do with the knowledge necessary to assimilate the students’



moods and the factors that motivate or not the resolution of a mathematical problem. Both the cognitive and affective factors incorporate and expand the notions of knowledge about the student and his or her characteristics (Shulman, 1987), about the knowledge of students as people who think and learn (Schoenfeld and Kilpatrick, 2008), and about the knowledge of the content and of the students (Hill *et al.*, 2008).

Interactional: it is related to the knowledge that allows predicting, implementing and evaluating the moments of interaction between teachers and students, among students, among others that occur in the teaching and learning processes. This aspect considers the ideas of building relationships that support learning, proposed by the notion of competence for teaching (Schoenfeld and Kilpatrick, 2008), which involves knowledge to predict, implement and evaluate sequences of interactions carried out by teachers and students involved in the teaching and learning process.

Mediational: refers to the teacher's knowledge regarding the use of material and technological resources, as well as the allocation of time to improve the learning of mathematical content.

Ecological: refers to the teacher's knowledge of interdisciplinarity, the mathematics program and the social, political and economic contexts.

The aspects that make up the didactic dimension make it possible to analyze, describe and develop the knowledge of the teacher or future teacher involved in the different phases of the teaching and learning process. Faced with a mathematical task, the teacher must be able to mobilize the diversity of meanings that arise from it, presenting different solutions from various representations, explanations, and justifications, in addition to promoting adaptations to the students' level of knowledge (Godino *et al.*, 2017).

2. Methodology

This research is qualitative, following the proposal of data collection and analysis. The data

were analyzed through content analysis (Bardin, 2016) based on the categories proposed by DMK.

According to Bardin (2016), content analysis is organized in three stages, pre-analysis, exploration of the material, processing of the results, inference and interpretation.

Pre-analysis aims at choosing and elaborating indicators. In turn, the exploration of the material consists of coding, decomposition or enumeration procedures, based on pre-established rules. Data processing seeks a condensation of the important information from which inferences will be made.

The data were collected from the development of a workshop on the digital platform *Google Meet*, through five remote meetings, lasting four hours each, for a sample of twelve undergraduate students in mathematics, during January 2021.

For the conduction of the workshop, four synchronous tasks and one asynchronous task were proposed, which relate mathematical contents with the following topics related to Financial Education:

- Task 1: Reflection on a mathematics lesson.
- Task 2: Reflection on inflation and the HICP and CPI price indexes.
- Task 3: Analysis of housing finance in the national context.
- Task 4: Analysis of interest and its applications in credit operations.
- Task 5: Development of activities related to the applications of interest rates.

The synchronous tasks (1 to 4) were developed in groups, with subsequent socialization and discussion of answers, while task 5, asynchronous, was carried out individually, with subsequent socialization. Transcriptions of the recordings were prepared and used in the analysis and discussion of results, together with the other activities developed by the participants and published in *Google Classroom*. The data analysis was carried out based on the dimensions and aspects in the DMK and/or adapted from Godino (2009).



3 Results and discussion

The analysis of the data collected in the workshop was carried out based on the mathematical and didactic dimensions of the DMK model. In this sense, we present the analysis of the data through the aspects/indicators presented in the dimensions.

3.1 Mathematical dimension

The mathematical dimension of the DMK will be analyzed through *common knowledge*, referring to the mathematical knowledge mobilized by the future mathematics teachers and related to the mathematical contents presented in the proposed tasks; and *extended knowledge*, referring to the mathematical knowledge mobilized by the future teachers and related to more advanced levels of study, intramathematical relation of contents and generalizations.

To analyze common and extended knowledge, we used as a reference Task 4, which consisted of 21 items, including three problem-sit-

uations that enables to investigate common and extended knowledge. We used the following indicators:

Common knowledge: Mobilizes mathematical knowledge correctly, partially, or inadequately to solve tasks.

Extended knowledge: Generalizes when solving tasks and connects them to other more advanced content.

The common knowledge of the content consists of the mathematical knowledge of the future teacher related to the same level at which he/she would teach (Elementary and High School). It will be analyzed through the situations-problem 1, 2 and 3 of Task 4, the first two being adapted from the National High School Exam (*ENEM: Exame Nacional do Ensino Médio*), which require the mobilization of mathematical knowledge related to Financial Mathematics, whose topics are most frequently presented in activities related to Financial Education (Gaban, 2016; Martins, 2019). The first problem-situation addressed real estate financing, as explained in Figure 1.



Figure 1

Situation-problem 1

(ADAPTED FROM ENEM - 2015) A couple makes a real estate financing of R\$ 180 000.00 to be paid in 360 monthly installments, with an effective interest rate of 1% per month. The first installment is to be paid a month after the disbursement, and the amount of the monthly installment is R\$500.00 plus 1% interest on the remaining balance (amount due before payment). Note that with each payment the remaining balance is reduced by R\$500.00 and consider that there is no installment due. Making the installment payments regularly, we ask for:

- a) The detailed calculation of the amount of the installment paid in the 10th month, using three different strategies.
- b) Name the mathematical concepts used in solving this situation-problem.
- c) Identify the properties and/or theorems used in the different solutions of the situation-problem 1
- d) Justify your answer with a logical argument

Note. Research data.

All groups correctly mobilized the knowledge necessary to find the solution to problem-situation 1. The other items will be discussed later in the epistemic aspect of the didactic dimension.

The first part of problem-situation 2 of Task 4, described in Figure 2, deals with credit operations, and will be discussed below.

Figura 2

Situation-problem 2 of task 4

(ADAPTED FROM ENEM - 2000) Juan wants to buy a car whose cash price is R\$21,000.00 and this value will not be readjusted in the following months.

He has R\$20,000.00 that can be used at a compound interest rate of 2% per month, and he chooses to leave all his money applied until the amount reaches the value of the car.

- a) () two months, and he will have the exact amount
- b) () three months, and he will have the exact amount
- c) () three months, and he will have approximately R\$225.00
- d) () four months, and he will have the exact amount
- e) () four months, and he will have approximately R\$430.00

2.1. Solve the situation-problem 2, presenting all calculations and procedures used.

[Recall the formula $Mn = C \cdot (1+i)^n$, where Mn: amount in period n; C: capital; i: rate; n: period].

Note. Research data.



We note that all groups marked the correct alternative for solving problem 2. However, when analyzing the different solutions presented for item (2.1), we identified that Group 4 made a mistake by presenting a solution using simple

interest, which is not coherent with the solution of the proposed problem-situation related to the compound capitalization system, which can be seen in Figure 3.

Figure 3

Solution of the situation-problem 2 by group 4

$$M = 20.000$$

$$J = C \cdot i \cdot t$$

$$J = 20.000 \cdot 0,02 \cdot 3$$

$$J = 1.200$$

$$\text{depo } 20.000 + 1.200 = 21.200$$

depo com 3 meses de carrefone no valor.

Note. Research data.

As this is an objective topic, the resolution proposed by Group 4, although it used an erroneous procedure, allowed them to find a value close to the correct alternative. We expected the research participants to arrive at the correct result by calculating the amount month by month or by applying the compound interest formula considering time as a variable.

The extended knowledge of the content will be analyzed from items 2.3 of situation-problem 2 and in situation-problem 3 of task 4, which include the deduction of a formula to calculate the amount and the resolution of a problem related to the capital accumulation factor, with the subsequent deduction of the formula, as shown in Figures 4 and 5.

Figure 4

Situation-problem 2 of task 4

Situation-Problem 2 (2nd part)

(ADAPTED FROM ENEM - 2000) Juan wants to buy a car whose cash price is R\$21,000.00 and this value will not be readjusted in the following months.

He has R\$20,000.00 that can be used at a compound interest rate of 2% per month, and he chooses to leave all his money applied until the amount reaches the value of the car. [...]

Note. Research data.



Figure 5
Situation-problem 3 of task 4

(ADAPTED-MATEMÁTICA FINANCEIRA) Determine the value of the amount, at the end of the fifth month, of a series of 5 monthly installments, equal and consecutive, in the amount of R\$ 100.00 each, at a rate of 4% per month, knowing that the first installment is applied at the end of the first month, i.e., 30 days from the date taken as base (zero amount), and that the last one, at the end of the fifth month, agrees with the time when the amount is requested. [...]
3.2 Derive the expression (formula) to calculate the monthly installments.

Note. Research data.

Problem-situation 3 was correctly solved by groups 3 and 4. However, only the students of Group 4 were able to deduce the formula proposed in item 3.2, which shows that future mathematics teachers have difficulties in the deduction of formulas and mathematical generalizations. However, although the prospective teachers mobilized common content knowledge related to simple and compound interest, and the sum of terms of geometric progression, the results of this research show a low mobilization of extended knowledge.

On the other hand, we corroborate the finding of Coutinho and Teixeira (2015) that teachers do not have adequate training in Financial Mathematics, making difficult the insertion of Financial Education in Elementary school.

3.2 Didactic dimension

The didactic dimension, which refers to knowledge for teaching mathematics, will be analyzed through the six aspects of the Didactic-Mathematical Knowledge model: epistemic, cognitive, affective, interactional, mediational, and ecological. Each aspect will be synthesized using the following indicators:

Epistemic: Solves tasks using different representations, uses concepts and properties, and justifies task solutions.

Cognitive: Describes possible learning conflicts when students solve tasks.

Affective: Describes strategies that motivate students to perform the task.

Interactional: Presents methods to implement sequences of interactions among the agents involved in the teaching and learning process.

Mediational: Values the adequacy of time and the contribution of the use of materials and technological resources.

Ecological: Identifies the elements of the program that are addressed in the tasks and the social, material, and other factors that condition their performance.

The epistemic aspect covers the knowledge of the mathematical content required for teaching at the specific school level. This research includes tasks related to the specific objects of study of Financial Mathematics. The analysis of this aspect revealed that all groups were able to find the correct solution for the proposed problem-situation. However, they presented difficulties related to the elaboration of the solutions through different representations, predominating in all groups the solution of Group 2 shown in Figure 6.



Figure 6

Common procedure of all groups

$$\begin{aligned}
 1^{\text{a}} &\rightarrow 500 + 1\% \text{ de } 280.000 \\
 2^{\text{a}} &\rightarrow 500 + 1\% \text{ de } 279.500 \\
 3^{\text{a}} &\rightarrow 500 + 1\% \text{ de } 279.000 \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 10^{\text{a}} &\rightarrow 500 + 1\% \text{ de } 275.500 \\
 &\text{Logo } 500 + \frac{1}{100} \cdot 275.500 \\
 &\Rightarrow 500 + 2755 = 2255
 \end{aligned}$$

Note. Research data.

Items b, c and d are related to the identification of mathematical concepts, properties and theorems, and the logical argumentation that justifies the answer. Although all groups found the correct solution to problem-situation 1, there was a generalized difficulty in these items.

According to Pino-Fan and Godino (2015), the epistemic aspect refers to the teacher's ability to solve a mathematical task through different procedures, identify the mathematical concepts that arise during the solution and relate the mathematical object addressed with others of the same educational level and more advanced. Corroborating these ideas, Carpes and Bisognin (2020) consider the teacher's ability as one of the essential tools for the development of student thinking and learning, since it allows identifying the emergent knowledge or prerequisites necessary to solve a task.

The cognitive aspect allows analyzing the knowledge of the future teacher related to the learning and mathematical understanding of secondary and high school students, being analyzed through tasks 2, 3 and 4. From the answers submitted by the groups, it was possible to identify the predominance of three types of errors or difficulties: (1) lack of understanding of the main

concept addressed by the task and its application; (2) poor prior knowledge of the mathematical concepts necessary to perform the task; and, (3) inadequate use of digital technologies.

We can infer that the possible errors and difficulties expressed by the research participants may be because these tasks were contextualized with everyday issues and required both Financial Education and Financial Mathematics. Task 2 addresses problem-situations related to inflation and indexers; task 3 focuses on simulations through the *Caixa Econômica Federal* website and discussions related to decision making on real estate financing; and task 4 deals with interest and credit operations. We understand that one of the possible reasons why future teachers considered the lack of understanding of the concepts by elementary and high school students as an obstacle may be associated with their low affinity with the topics addressed in the proposed activities. As for the second error/difficulty indicated, all the tasks focus on content related to Financial Mathematics, especially percentages and simple and compound interest, which would require previous knowledge, from elementary and high school students, necessary for the development of the tasks, which may



not be well consolidated. Regarding the third error/difficulty mentioned, all the workshop tasks were developed through distance learning and, in addition, the completion of tasks 2 and 3 required access to the Internet and the manipulation of some technological resources, such as the Broad Consumer Price Index (IPCA) calculator and the *Caixa Econômica Federal* housing simulator, which leads us to interpret that these possible difficulties could be related to the lack of skills of the teachers to access and manipulate specific technological applications and resources.

The affective aspect deals with the teaching knowledge related to the emotional, affective, and attitudinal aspects of students in relation to mathematical objects. Some studies (Machado *et al.*, 2010; Zan *et al.*, 2006) mention the existence of a significant correlation between affectivity and cognition, in the sense that affectivity emanates from cognition, and cognition is part of affectivity. The participants of the research highlighted Financial Education as a “subject” that favors learning, because it allows the student to approach the subject of study with situations of their everyday life, giving mathematical contents a less abstract perspective, highlighting the relevant nature of mathematical knowledge by enabling the applicability of mathematics in situations that allow the individual to understand and solve everyday situations (Seah and Bishop, 2000; Campos *et al.*, 2015, Ribeiro *et al.*, 2018).

The interactional aspect considers the necessary professional knowledge about the interactions between the agents involved in the teaching process. Analyzing this aspect, we found that participants pointed out the relevance of developing activities in groups or in pairs, arguing that discussions among students would enhance learning. In this sense, Brandão and Neres (2018) point out a fundamental role of the teacher in the development of activities that promote cooperation and collaboration of all individuals involved in the educational process to provide a more attractive environment and enhance the cognitive development of the student.

Regarding the use of material and technological resources, as well as the adequacy of time for the development of tasks, aspects included in the mediational aspect, future teachers emphasized the need to adjust the workload in a way that allows to review mathematical contents related to previous knowledge, and the teacher preparation for a satisfactory use of technological resources that should be available to meet school demands.

Below are some excerpts taken from the collective discussion that took place during the second meeting, mentioning some concepts related to the preparation for the use of technological resources:

A10: I do not feel prepared to teach mathematics with technologies; I consider that what I was taught during my training is not enough. Even if the school where I will work provides me the resource, I would not feel prepared.

A1: I agree with A10, I feel prepared only to teach a quality class using GeoGebra or Excel because they were the only *computer programs* we used during my undergraduate studies. I believe that the approach of activities related to technological resources in higher education institutions is insufficient to prepare undergraduate mathematics students to use them adequately in their classes.

Cardoso and Figueira Sampaio (2019) state that, even though technology is part of the daily life of a considerable part of the population, there are still obstacles to its use in the classroom. One of the reasons is the deficiency in the use of technological resources in the initial training of teachers.

The *ecological aspect* considers the knowledge of the program that includes the object of study of mathematics at the corresponding educational level, its relationship with other programs and the social, political, and economic factors (Pino-Fan and Godino, 2015).



To reflect on the knowledge related to this aspect, the participants of this research were asked about the organization of topics related to Financial Mathematics throughout the levels of schooling and the approach to Financial Education in the Mathematics program of elementary and high school. Regarding the knowledge of the mathematical contents included in the mathematics program related to Financial Mathematics, all groups expressed a satisfactory knowledge. However, with respect to the insertion of Financial Education in elementary and high school Mathematics classes, Groups 1 and 4 did not express any opinion, Group 3 was unable to give an opinion and only Group 2 expressed a relatively consistent position.

The analysis of the ecological aspect of the didactic dimension of the DMK reveals that the future mathematics teachers satisfactorily mobilized knowledge about Financial Mathematics. However, they were not able to adequately mobilize knowledge about the insertion of Financial Education in the Mathematics academic program or establish its relationship with other mathematical or interdisciplinary contents. This enables us to identify the need to develop training processes focused on Financial Education in the context of the training of teachers who teach mathematics.

4. Conclusions

This article is part of a research conducted in the context of a master's degree, with the objective of analyzing the Didactic-Mathematical Knowledge (DMK) mobilized by future mathematics teachers when solving tasks focused on topics related to School Financial Education.

In line with the objective, this research allowed us to inquire into a formative process focused on the contents of Financial Mathematics through the contextualized approach of contents related to real situations (Teixeira, 2015) for future mathematics teachers.

Regarding Critical Mathematics Education, both teachers and future teachers assumed a fundamental role in the teaching and learning processes of the concepts of Financial Mathematics included in the different tasks developed from the perspective of School Financial Education, which contributed to the training of future mathematics teachers for the development of Financial Education at school (Campos *et al.*, 2015).

The analysis of the mathematical dimension of DMK revealed that the future mathematics teachers correctly mobilized common content knowledge by presenting correct solutions to all the selected questions, except for Group 4, which partially mobilized mathematical knowledge and made a mistake in the solution of situation-problem 2, and Groups 1 and 2, which presented incorrect solutions to situation-problem 3 of Task 4. As for the extended dimension of mathematical knowledge, its analysis was performed based on the tasks mainly related to formal deductions and generalizations, pointing to low mobilization. The results showed the difficulties of prospective teachers in moving from elementary to advanced mathematical thinking (Tall, 2002).

Regarding the didactic dimension of DMK, the research sought to analyze the didactic knowledge mobilized by future mathematics teachers based on indicators related to the epistemic, cognitive, affective, interactional, mediational, and ecological aspects. Based on the indicators used, we consider that didactic knowledge was mobilized satisfactorily in the cognitive aspect and partially in the other aspects.

The results of this research point out the potential of the DMK to carry out research focused on the analysis, categorization, production and/or mobilization of Didactic-Mathematical Knowledge in the context of the training of future mathematics teachers. Furthermore, they allow considering as perspectives for future research the development of studies related to the phases of the didactic design oriented to the mobilization of DMK of future



mathematics teachers on Financial Education, as well as the study and adaptation of suitability criteria to the complexity of Financial Mathematics objects included in training processes and/or activities focused on Financial Education.

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Didactic-mathematical knowledge of some teachers about prime numbers

Conocimiento didáctico-matemático de algunos docentes sobre los números primos

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Abstract

Studying the teacher's knowledge has become one of the most relevant lines of research nowadays, considering that it encompasses different factors that have a direct implication in the teaching and learning processes. Considering the above, the objective was to establish the mathematics teacher's knowledge of prime numbers through the Didactic-Mathematical Knowledge Model (DMK). For this purpose, a case study was carried out in which five teachers who work in different High Schools in Colombia were taken as the unit of analysis. For the design of instruments and the analysis of the information, the indicators of didactic-mathematical knowledge of the DMK were taken and three situations related to epistemic, cognitive, affective, interactional, mediational, and ecological elements of prime numbers were designed. From the analysis made by the teachers to the situations, concrete elements emerged revealing the teachers' knowledge regarding prime numbers and some factors that evidence their difficulty in handling the connection of this type of numbers with other objects of mathematics. In addition, it is concluded that teachers do not manage to give students a broad vision of the meaning of these numbers by not knowing cognitive and epistemic elements that make possible their management in the classroom.

Keywords: Knowledge, didactics, onto-semiotic approach, educational indicator, prime number, mathematical object.

Resumen

El estudio del conocimiento del profesor se ha convertido en una de las líneas de investigación más relevantes en la actualidad, considerando que engloba diferentes factores que tienen una implicación directa en los procesos de enseñanza y aprendizaje. Teniendo en cuenta lo anterior, el objetivo fue establecer el conocimiento del profesor de matemáticas sobre los números primos a través del Modelo del Conocimiento Didáctico-Matemático (CDM). Para esto se realizó un estudio de caso en el que se tomó como unidad de análisis a cinco docentes que desarrollan su labor en la educación básica secundaria de diversas instituciones educativas de Colombia. Para el diseño de instrumentos y el análisis de la información se tomaron los indicadores del conocimiento didáctico-matemático del modelo CDM y se diseñaron tres situaciones relacionadas con aspectos epistémicos, cognitivos, afectivos, interaccionales, mediacionales y ecológicos de los números primos. Del análisis realizado por docentes a las situaciones, emergieron elementos concretos que revelaron los conocimientos de profesores respecto a los números primos y algunos factores que evidencian la dificultad para manejar la conexión de este tipo de números con otros objetos de la matemática. Se concluye que el profesorado no logra dar al estudiantado una visión amplia del significado de estos números al desconocer elementos cognitivos y epistémicos que posibilitan su manejo en el aula.

Descriptores: Conocimiento, didáctica, enfoque ontosemiótico, indicador educativo, número primo, objeto matemático.

1 Introduction

The didactics of mathematics seeks to understand the different problems that emerge in the teaching and learning of mathematics and how they could be solved (Godino, 2021). In this search, the importance of improving the teacher training process has been highlighted (Ball, 2022; Ball *et al.*, 2008; Rowland *et al.*, 2005; Shulman, 1986), understanding that a competent teacher for the teaching and learning of mathematics must, for example, know multiple ways of conceptualizing and representing mathematical content, understand the key aspects of each topic and see connections with other objects in the intra- and extra-mathematical context (Godino *et al.*, 2018). It is also essential to know the origins, structure, and curricular developments, as well as content directions (Ball, 2000; Kilpatrick *et al.*, 2001).

Given the complexity and breadth of mathematical objects, attention can be focused on learning and teaching concrete objects (D'Amore and Sbaragli, 2019). In this article, the knowledge of mathematics teachers about prime numbers is presented from the view of the Didactic-Mathematical Knowledge Model (DMK). The choice of this mathematical object occurs by considering that there is evidence of the lack of knowledge by some teachers about strategies to show students the importance and usefulness of prime numbers, so they must resort to superficial and anecdotal conceptions that lead to misconceptions (Bernaschini, 2017; Kiss, 2020; Zazkis and Lijedahl, 2004).

1.1. Didactic-Mathematical Knowledge Model (DMK)

DMK has developed in the framework of the Onto-semiotic Approach to Mathematical Knowledge and Instruction (EOS), a theoretical system that has its own epistemological, ontological, and methodological foundations (Godino, 2022). Specifically, in the EOS it is considered that the learning of mathematics occurs through problem solving (Godino *et al.*, 2020). In this sense,

teaching is the process through which the teacher establishes a didactic trajectory for the mathematical object(s) of which the student is expected to acquire knowledge (Godino *et al.*, 2008).

The teaching and learning process of mathematics is full of multiple dimensions (personal, social, political, economic, psychological, cognitive, epistemological, instructional, etc.) that generate the need to raise foundations that cover each of them deeply (Burgos *et al.*, 2018). The DMK has developed considering that teachers of mathematics must know about the six elements that define a process of study (Godino, 2009; Pino-Fan *et al.*, 2014).

The first is the epistemic aspect, which considers the representativeness of the meanings that are developed in the classroom in contrast with the existing reference meaning of each mathematical object (Burgos and Godino, 2021). The cognitive element addresses the proximity of the meanings and their relationship with the cognitive development of the learner; the affective aspect directs to the involvement of the students in the trajectory determined by the teacher (Beltrán-Pellicer and Godino, 2020; Hummes *et al.*, 2019).

The interactional and mediational aspects arise when talking about the knowledge of the mathematics teacher about teaching; the first referring to the identification and resolution of cognitive conflicts that arise in learning and the second to the adequacy of material and temporal resources (Giacomone *et al.*, 2019). Finally, the ecological element links the knowledge that teachers should have regarding the program, its implementation and adjustment to the demands of society and the students' environment (Castro and Pino-Fan, 2021).

The previous aspects become operative through the analysis of practices, actions performed in the mathematical activity; configurations, description of mathematical objects and processes that emerge in the practices; norms and consideration of the rules and habits that condition the practices; and finally suitability, the process of identifying possible improvements in the instructional processes. In the DMK, the study and reflection on each aspect is materialized in the indicators presented in Figure 1.



Figure 1
DMK indicators

Knowledge of the content (common and specialization)		
Epistemic aspect	Indicators	Denotation
Common knowledge	Do the task	FEp1
Specialized knowledge:	Stake in the plausible solutions of the task and other related tasks.	FEp2
Type of problems	Identify the variables of the task; generalizes the statement.	FEp3
Languages	Do the task by using different representations	FEp4
Procedures	Do the tasks by using different procedures (intuitive and formal)	FEp5
Concepts / properties	Identify the concepts and properties in the solutions	FEp6
Arguments	Explain and justify the solutions	FEp7
Broad knowledge:		
Connections	Identify possible generalizations of the task and connections with more advanced topics	FEp8
Knowledge of the content in relation to the students		
Cognitive and affective aspect	Indicators	
Cognitive configurations (strategies, representations, statements, argumentations, ...)	Describe the types of cognitive configurations that learners have developed in solving the proposed task (or tasks).	FCA1
Mistakes, difficulties, learning conflicts, conceptions, etc.	Describe the main types of learning conflicts in the solution of this type of tasks.	FCA2
Learning assessment	Explain students' personal meanings in solving this type of task or content.	FCA3
Attitudes, emotions, beliefs and values	Describe strategies that can be implemented to promote student involvement in solving these tasks.	FCA4
Knowledge of the content in relation to teaching		
Interactional and mediational aspect	Indicators	
Roles of the teacher and students in relation to the task or content. Ways of interaction: teacher-student; students-students. Material resources. Allocated time.	Describe the didactic configuration you would implement using the given mathematical task.	FIM1
Didactic trajectory (sequence of didactic configurations)	Describe other tasks related to the one provided and how to manage the corresponding didactic trajectory	FIM2
Knowledge of the program and intra and inter-disciplinaries connections		
Ecological aspect	Indicators	
Program oriented	Identify the elements of the program that are addressed through the performance of the task(s). Proposal (aims, objectives)	FEc1
Intra-disciplinary connections	Explains connections that can be made to other topics in the syllabus through the completion of the assignment	FEc2
Inter-disciplinary connections	Explains connections that can be made to other topics in the syllabus through the completion of the assignment	FEc3
Other conditioning factors	Identifies social, material, or other factors that condition the performance of the task or the development of the intended or implemented educational project.	FEc4

Note. Taken and adapted from Godino (2009).



2 Methodology

The approach assumed is qualitative, that is a flexible inquiry process that intends to understand the meaning of the actions of the subjects who experience the phenomenon of interest (Bejarano, 2016). In addition, the scope of this research is descriptive, since data related to the educational reality of five teachers of Colombian high schools were collected, which are mentioned as: A, B, C, D and E, are collected and described. The selection of these teachers is not probabilistic, the only criterion considered was the acceptance of the invitation made by the researchers through e-mail.

In order to obtain the information, a two-hour meeting was held where the analysis of the didactic-mathematical activity was implemented based on DMK. In this type of analysis, teachers are provided learning situations so that they can analyze and reflect on each of the DMK indicators (Godino *et al.*, 2007). This research presents three situation-problems that include didactic-mathematical knowledge about prime numbers, as they are related to three elements of the historical and epistemic development of these numbers: the search for algorithms for determining prime numbers, the inclusion or exclusion of 1 in this set, and the use of prime numbers in problems related to the greatest common divisor and the least common multiple.

The data were structured through the analysis of categories, in this case, the six aspects of the DMK model. Textual data were taken from the answers given by the teachers in each of the activities, identifying those manifestations that reveal the thoughts, ideas, and knowledge of the participants regarding the categories (Strauss and Corbin, 2002).

3 Results

This section presents in detail the three situations analyzed and their relationship with the aspects of

the DMK and the level of didactic-mathematical knowledge of the participating teachers.

3.1. Didactic-mathematical analysis of situation 1

The first situation was proposed with the objective of analyzing the conception teachers have of what is a prime number, considering its relationship with odd numbers and the inclusion or not of the number 1 as a prime number. This situation is as follows:

Situation 1. Students at an educational institution were asked to write down the prime numbers lower than 10 and the following answers were obtained.

Type 1: 1, 2, 3, 5, 7.

Type 2: 2, 3, 5, 7.

Type 3: 3, 5, 7.

3.1.1. Epistemic aspect in situation 1

The teachers were asked to answer: Which answer or answers should the teacher accept as correct? Which mathematical concepts and/or properties should the students use to give a correct solution? These allow addressing the common and specialized knowledge components. The expected solutions, respectively, were:

- Answers 1 and 2 are correct. This is considering that 1 can be considered prime or not.
- Division, divisibility, divisors, divisibility criteria, prime number, decomposition, multiple, even and odd number, order relation, etc.

All the participant group responded that the Type 2 answer was correct, only E indicated the Type 3 answer as valid. For question 2, A expressed that the concepts to be used were divisors, multiples, and divisibility criteria; B indicated odd and even numbers, divisors, and divisibility criteria; C prime numbers; D chose prime and even numbers; while E answered that



natural numbers, multiples, divisors, factor, division, and decomposition were necessary.

From these answers, it stands out that no teacher considers number 1 as prime, only 2 of them recognize that the concept of prime number is necessary to answer the situation and none of them identifies that the use of the expression “lower than” should be considered. Finally, the teacher who indicated Type 3 as the correct answer reveals that he/she does not differentiate prime numbers from odd numbers.

3.1.2. Cognitive and affective aspects in situation 1

The idea regarding the aspects was to describe the possible difficulties present in the incorrect answers, which led the student to answer incorrectly. The expected solution was: if the teacher indicated that the Type 1 answer was correct, then it was expected to highlight that the student failed to recognize 1 as a prime number and that there was difficulty in differentiating prime numbers from odd numbers. If the teacher had indicated that 2 was correct, it was expected to express the difficulty in recognizing that 1 is not prime. Finally, if he or she indicated that 3 was correct, he or she was expected to identify that the student did not recognize that even numbers are not prime. The responses were:

- A: Perhaps the student is confused as to whether number 1 is a prime number; he/she might be confused that it is a divisor of itself and a divisor of one. For type 3 response, perhaps he/she is confused with the fact that the divisors of two are 1 and 2 and would believe that because it is the first number it should not be included.
- B: Not differentiating odd and even numbers. Not identifying the divisors of a number. Not being clear on the concept or characteristics of prime number.
- C: Confuse odd numbers with prime numbers or are unclear on the definition.
- D: Failure to complete the activity in class and therefore do not have the concepts

defined. They forget the concept of prime number. Multiplication tables and therefore does not determine the divisors.

- E: Handling of concepts, difficulty in dividing, difficulty in decomposing numbers, difficulty in reading comprehension.

The aspect that stands out in the answers is that all the teachers say they know the concept of prime number; however, they had not contemplated it in question 1 of the epistemic aspect as something necessary to solve the situation. In addition, B raises the need to differentiate between odd and even numbers, i.e., again confusing odd and prime numbers. Finally, A in trying to explain errors in solution type 3, did not identify that the numbers being presented are odd.

3.1.3. Mediation and interactional aspects in situation 1

To address these aspects, the question was posed: What strategies would you use as a teacher to guide those students who have not been able to solve the problem? Explain in detail your answer. As a solution, strategies such as problem solving, collaborative work, explanation by the teacher, use of didactic material, development of playful activities, etc., were expected to be mentioned, as well as explanations of how the problematic aspects in student learning would be solved. The responses were as follow:

- A: I would ask the student to divide each number into one, itself and the numbers before it, since the divisions that are exact are the values that work. I would reinforce the concepts of prime and composite number.
- B: Recall the concept or characteristics of a prime number. Recall and identify odd and even numbers. Practice the concept of divisor of a number using simple exercises.
- C: First make them see the difference between odd number and prime number based on the definition and examples.



- D: Extra work with multiplication tables, division process and divisors of a number. Define and clarify the properties of numbers 2 and 1 with the definition of prime number. Create a bulletin board listing the first prime numbers.
- E: Didactic games. Videos. Practical exercises knowing concepts that will help them better understand the topic, by carrying out sequenced activities where they follow instructions.

In these answers, four teachers keep saying that the solution to the difficulties is achieved with teacher strategies such as clarifying the concept, reviewing the algorithms of division and multiplication, and differentiating even, odd, and prime numbers. Only E considers activities of a different nature, proposing playful strategies; however, his response focuses on stating strategies and does not clarify the relationship with the mathematical object, i.e., he considers the strategies as something independent of the object.

3.1.4. Ecological aspect in situation 1

To conclude with situation 1, the following question was asked: For which course do you consider this problem suitable according to the current program? In this aspect, it was expected that this mathematical topic could be worked in any course above the fourth grade of elementary school, since this is what it is indicated in the mathematics program in Colombia. The answers were: A answered third grade of elementary school; B stated that from fourth grade onwards; C expressed that in all grades of elementary and in sixth grade; D chose third, fourth and fifth

grade of elementary school; and E answered that in fifth grade of elementary and sixth grade of High school.

Thus, only B mentions the grades in which prime numbers can be studied according to the national program. Only E indicates some appropriate grades, while A, C and D indicate grades in which, according to the academic program and previous mathematical concepts, it is not coherent to work with prime numbers.

3.2. Didactic-mathematical analysis of situation 2

The second situation was proposed with the objective of analyzing what strategies and procedures teachers use to determine a list of prime numbers lower than a given number and how they present it to their students. This situation was:

Situation 2. Write prime numbers lower than 20

3.2.1. Epistemic aspect in situation 2

- The teachers were asked: Solve the problem posed. What are the limitations of the solution presented by you to the students? The expected answers, respectively, were:
- Solution 1: 2, 3, 5, 7, 11, 13, 17, 19.
- Solution 2: 1, 2, 3, 5, 7, 11, 13, 17, 19.
- The method becomes inefficient as the upper increasing the upper limit.

The responses are presented in Figure 2.



Figure 2

Responses in epistemic aspect of situation 2

Professor	Question 1	Question 2
A	2, 3, 5, 7, 11, 13, 17, 19 are the only ones that can be divided in one and in themselves.	I am not giving enough didactic tools to make it easier for them to solve the problem.
B	The answer to the problem is 2, 3, 5, 7, 11, 13, 17, 19 because these are the numbers lower than 20 that can only be divided by themselves and by the unit.	Does not respond
C	2, 3, 5, 7, 11, 13, 17, 19 only have two dividers	Odd numbers to be confused with primes
D	Does not respond	To solve this problem, I would consider the activities, analysis and strategies of the previous topics. If necessary, the activity developed in class would be performed again, but individually to guarantee that the processes are performed and understood.
E	Does not respond	As I present it, students will find it impossible if they do not know prime numbers.

It is seen that only teachers A, B and C solved the situation posed, evidencing the uncertainty of D and E about the correct way to answer. Regarding the question, only A was able to identify a possible difficulty with what he does by explaining that he lacks strategies to present the solution; while C, D and E responded evasively, focusing on the mistakes that students can make and not on their own actions. Finally, B failed to express the limitations of his response.

3.2.2. Cognitive and affective aspects in situation 2

Teachers were asked to: describe the possible difficulties that may lead students to answer wrongly. From which, they were expected to raise difficulties such as the confusion of prime numbers with other sets such as odd numbers, the inclusion or not of 1 as a prime number, the lack of an efficient algorithm to generate prime numbers, the lack of knowledge of the concept of prime number or the impossibility of calculating the requested numbers due to the lack of mastery

of previous mathematical objects. The answers obtained are:

- A: Are not clear about the concept of prime number, do not know divisors and multiples of a number. Do not know how to divide.
- B: Some of the possible difficulties could be not differentiating odd and even numbers, not identifying the divisors of a number, not being clear on the concept or characteristics of prime number.
- C: They confuse odd numbers with prime numbers or are unclear on the definition.
- D: Do not respond.
- E: Do not have clear concepts.

It is observed that three teachers (A, B and C) agree that the previous concepts such as multiples, divisors, even and odd numbers must be clear. However, only A recognizes the need for understanding the concept of prime number. In addition, D failed to establish possible difficulties in the solution of the situation and E gives a non-specific answer, which does not show an



understanding of what can happen to the students when facing this situation.

3.2.3. Mediatl and interactional aspects in situation 2

The following questions were asked to address these aspects: What type of resource would you use to present the solution to the problem; what questions would you ask your students after presenting the solution; how would you evaluate the learning achieved? The expected answers, respectively, were:

- Sieve of Eratosthenes, virtual applications, ludic activities, the board, multiplication tables, etc.

- Questions such as: what difficulties did they observe in their development? What solution strategies would they use? Was the development of any of them not clear? What can be concluded with the above? What relationships and differences do they find with other numerical sets such as natural numbers, even, odd, multiples of a number, etc.? Among others.
- The explicit enunciation of strategies such as continuous assessment, self-assessment, co-assessment, or the implementation of questionnaires, tasks, problems, and exercises, among others.

The responses are presented in Figure 3

Figure 3

Responses in mediational and interactional aspects in situation 2

Professor	Question 1	Question 2
A	2, 3, 5, 7, 11, 13, 17, 19 are the only ones that can be divided in one and in themselves.	I am not giving enough didactic tools to make it easier for them to solve the problem.
B	The answer to the problem is 2, 3, 5, 7, 11, 13, 17, 19 because these are the numbers lower than 20 that can only be divided by themselves and by the unit.	Does not respond
C	2, 3, 5, 7, 11, 13, 17, 19 only have two dividers	Odd numbers to be confused with primes
D	Does not respond	To solve this problem, I would consider the activities, analysis and strategies of the previous topics. If necessary, the activity developed in class would be performed again, but individually to guarantee that the processes are performed and understood.
E	Does not respond	As I present it, students will find it impossible if they do not know prime numbers.

Regarding question 1, it is observed that B and E mention the Sieve of Eratosthenes (although E does not remember the name), while A would use the multiplication tables and C does not manage to make any instrument. From the above, a limited knowledge of instruments related to the learning and teaching of prime numbers is observed.

Regarding question 2, A and B emphasize the need to address the difficulties faced by the students, while C expresses that he would resort to questioning conceptual aspects of prime numbers without specifying how he would do it. On the other hand, D would emphasize normative aspects such as students' emotions and needs.



Finally, E would delve into the strategies used by the students when they solved the situation.

In the final question, A, C and E fail to mention how they would evaluate, providing generic answers, while B and D state in detail the strategies and instruments they would use, but omit which aspects of the mathematical object they would address..

3.2.4. Ecological aspect in situation 2

To conclude the analysis of situation 2, the following question was asked: For which course do you consider this problem suitable according to the current program? As an answer, it was expected that they would express that the situation can be worked in grades higher than fourth grade, since it is a mathematical problem that is related to topics of higher levels of education.

The answers are: A says it is suitable for third grade of elementary school; B expresses that any grade from fourth grade of elementary school; C chooses all grades of elementary school and sixth grade of high secondary school; D does not answer; and E indicates sixth grade of high school. In these answers it is seen that only B knows the grades in which prime numbers can be worked according to the national program and the nature of the mathematical problem. Only E indicates an appropriate grade but omits that it can be taught in other levels, while A and C indicate grades in which it is not coherent to approach this mathematical topic, according to the program and the previous mathematical concepts necessary when teaching prime numbers.

3.3. Didactic-mathematical analysis of situation 3

Situation number three was proposed with the objective of analyzing whether the teachers recognized the connection of prime numbers with other mathematical objects, specifically the least common objects such as multiple, greatest com-

mon divisor, decomposition into prime factors and divisibility. The situation is:

Situation 3. The students of an institution were asked to solve the following problem: Maria has decided to make bracelets and wants to decorate them with pearls. If she has 24 white pearls and 36 blue pearls, and she wants to make as many bracelets as possible she needs to: use all the pearls, all the bracelets have the same number of pearls, all the handles have pearls of the two colors. How many bracelets will she be able to make?

3.3.1. Epistemic aspect in situation 3

For this aspect, the question asked was: With which previous and more advanced concepts of the school program do you relate the content involved in the solution of this problem? The answers expected were: with the previous concepts, natural numbers, division, multiplication, divisors of a number, divisibility criteria, multiples of a number, even and odd number, simplification of fractions; for the more advanced ones, the greatest common divisor, least common multiple, factoring, rational numbers, among others. The answers were:

- A: Multiples, divisors, prime numbers and composite numbers, decomposition into prime factors and least common multiple.
- B: Odd and even numbers, divisor of a number, prime numbers, composite numbers, decomposition into prime factors, divisibility criteria, greatest common divisor.
- C: prime numbers, decomposition, and mathematical logic.
- D: M.C.D. and divisibility criteria.
- E: Factorial decomposition and least common multiple.

It is possible to observe in the responses several of the expected prior concepts, which is not the case with more advanced knowledge. In fact, teachers were not able to explain which ones they considered as previous and which ones as more advanced. Among the previous concepts,



it stands out that no teacher identified any relationship between prime numbers and fractions, mathematical topics taught from elementary school. In addition, although in the analysis of situation 1 and the answers to the other questions they emphasize difficulties related to the handling of the different numerical sets, only B mentions this aspect.

3.3.2. Cognitive and affective aspects in situation 3

The following instructions were provided for these aspects: point out the possible difficulties present in the incorrect answers. The expected solution was: not considering that prime numbers are used and not composite numbers to perform the decomposition into factors; not knowing the algorithm to calculate the greatest common divisor; lack of knowledge about prime numbers; among others. The answers

- A: Decides to do greatest common divisor separately and does not consider the order of the decomposition. He is not clear on the definition of prime factor decomposition and looks for a divisor that is easy for him.
- B: Some of the possible difficulties could be: Not clear on the concept or characteristics of prime number, not clear that the decomposition process should be done only using prime numbers and from smallest to largest.
- C: Not being clear that decompositions should be done only with prime numbers.
- D: Not clear on the concepts of M.C.D. Not remembering the list of prime numbers or how to find them. Reading comprehension.
- E: Handling elementary operations.

Regarding the answers of teachers, none of them managed to identify that there were different errors and difficulties in each type of solution, so they present a global view of difficulties related to the problem, but not with the particularities of each solution; moreover, none of them found that there was a correct solution. Additionally, except

for E, who gives an answer without specifying, all of them emphasize that there was no clarity in the management of the decomposition into prime factors, but they did not express which aspect of that decomposition is the one that fails in each solution.

3.3.3 Mediatonal and interactional aspects in situation 3

To conclude the analysis of situation 3, the following question was asked: What strategy would you use to correct the errors found? Solutions were expected, such as: take up the situation by developing it step by step; clarify the need to decompose into prime numbers; address the concept of prime number; ask the students the explanation of correct answers, among others. The answers were:

- A: Decides to do greatest common divisor separately and does not consider the order of the decomposition. He is not clear on the definition of prime factor decomposition and looks for a divisor that is easy for him.
- B: Some of the possible difficulties could be: Not clear on the concept or characteristics of prime number, not clear that the decomposition process should be done only using prime numbers and from smallest to largest.
- C: Not being clear that decompositions should be done only with prime numbers.
- D: Not clear on the concepts of M.C.D. Not remembering the list of prime numbers or how to find them. Reading comprehension.
- E: Handling elementary operations. Regarding the answers of teachers, none of them managed to identify that there were different errors and difficulties in each type of solution, so they present a global view of difficulties related to the problem, but not with the particularities of each solution; moreover, none of them found that there was a correct solution. Additionally, except for E, who gives an



answer without specifying, all of them emphasize that there was no clarity in the

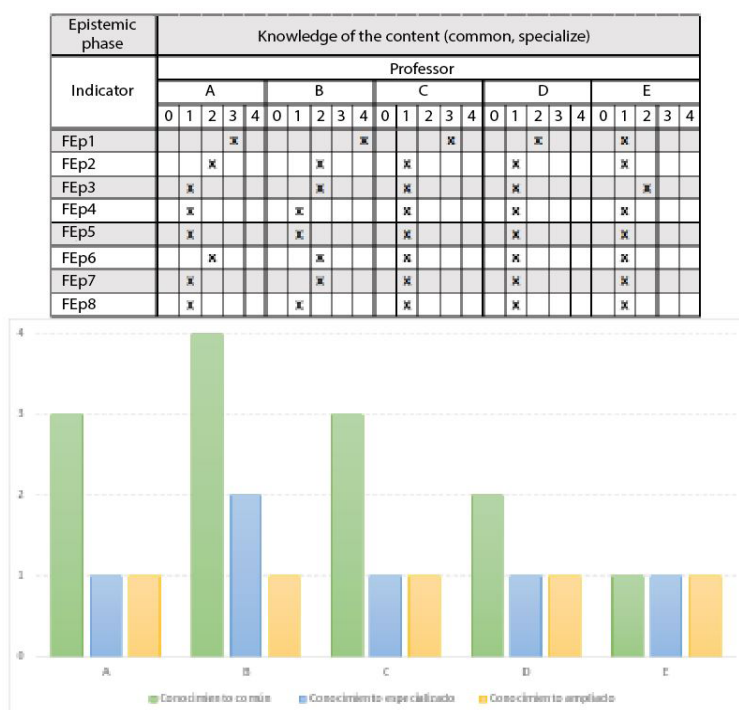
3.4 Classification of knowledge

Based on the indicators presented in Figure 1 and the answers to the questions posed regarding prime numbers, the knowledge of didac-

tic-mathematical of each teacher is classified as zero level (0), low level (1), medium level (2), high level (3) and advanced level (4) in each of the DMK aspects. To begin with, the classification of the level of knowledge in the epistemic aspect is presented in Figure 4.

Figure 4

Classification of knowledge in the epistemic aspect



It is found that A and C have a high common knowledge, B advanced, D medium and E low. This implies that, except for E, the participants have the necessary knowledge to solve situations related to the management of prime numbers in the school context corresponding to the sixth grade of elementary school in Colombia.

However, in the assessment of specialized knowledge, B obtained a medium level and the others a low level, because the type of solutions they can offer to the situations lacks the

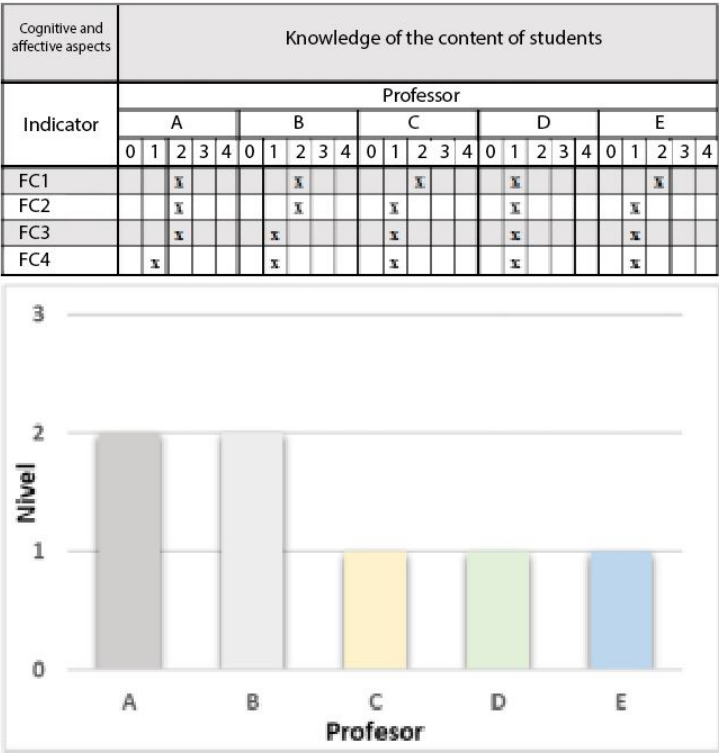
management of a diversity of representations, strategies, and connections with another mathematical topic. This low level is also observed in the extended knowledge, where no teacher was able to relate prime numbers to more advanced mathematical topics in the school program.

Regarding the cognitive and affective aspects, the evaluation presented in Figure 5 shows that A and B reach a medium level of knowledge and the others a low level. The main shortcoming found is the difficulty in presenting the students' personal meanings when



solving situations that deal with prime numbers. This aspect relates to another problematic indicator: not promoting actions that involve students in the solution of such situations

Figure 5
Classification of knowledge into cognitive and affective aspects

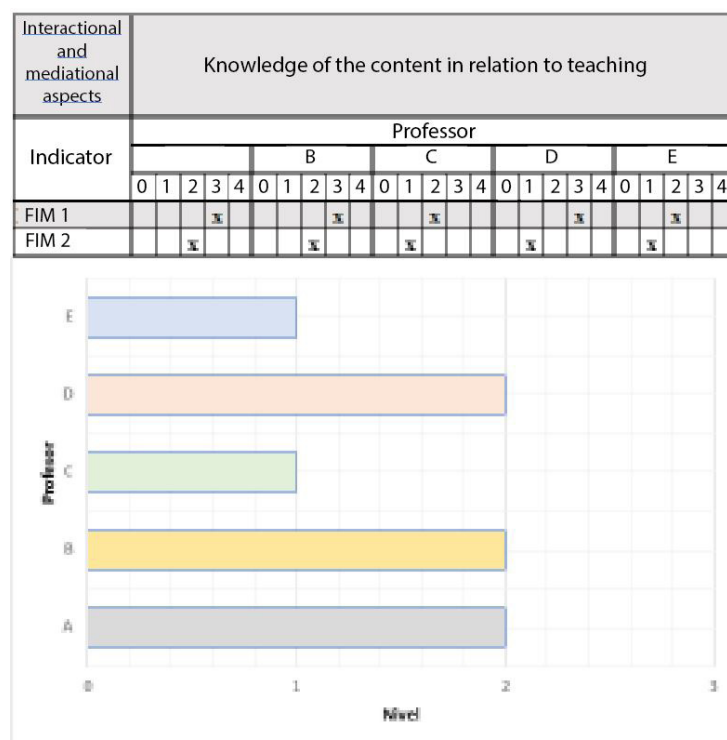


On the other hand, in the interactional and mediational aspects presented in Figure 6, it is explicit that A, B and D have a medium level of knowledge, since they manage to describe the way in which they would conduct their teaching actions to address situations associated with prime numbers. This aspect was not achieved by C and E, who are located at the low level.

The aspect that does not allow participants to reach an advanced level of knowledge is that they were not able to relate their teaching actions with different situations; on the contrary, all of them emphasized the need to solve the difficulties of the students from the same situation that originated.

Figure 6

Classification of knowledge into interactional and mediational aspects



Finally, when analyzing the ecological aspect, it is observed that all participating teachers have a low level of knowledge, since they are unable to identify the academic levels or grades in which prime numbers are present, either as a central object of study or as an object that connects with others. In addition, none of them manages to explain the factors that condition the relationship of prime numbers with their properties, problem solving, nor with the social, cultural, and scientific dynamics around them.

4 Discussion and conclusions

Regarding the epistemic aspect, it was found that the lack of advanced knowledge lies, as explained by D'Amore and Fandiño (2005), in the fact of not knowing the historical and epistemological development of prime num-

bers, which implies not assuming committed and meaningful positions regarding them. In turn, the difficulty in establishing a connection with other mathematical objects may be due to a conception of mathematics in which teachers are unaware that mathematical objects are not isolated entities; on the contrary, they are constantly related (Bagni, 2006; Bagni and D'Amore, 2005). In this case, the lack of knowledge on the connection of prime numbers with other objects generates a weak awareness in teachers of what these numbers offer mathematically and didactically (Grugnetti and Rogers, 2000).

In the cognitive and affective aspects, the main obstacle identified was the inability to establish mechanisms to involve students in the solution of situations related to prime numbers. Radford (2020, 2021) argues that this is because the beliefs and conceptions of teachers regarding the role of students and the teacher place them as



opposites, in which learning is an exclusive attribute of the student and, therefore, teacher does not have to get involved in arousing the student's interest in learning.

Regarding the mediational and interactional aspects, it was observed in teachers the difficulty to manage different didactic trajectories according to the needs, difficulties, mistakes, and obstacles that the students may face. This aspect emerges from the strong need to describe what they observe using qualifiers such as good or bad, right, or wrong, among others, but they do not consider necessary to give space for the recognition of descriptions and deeper reasoning that allow establishing possibilities for improvement (Breda *et al.*, 2018; Font, 2011; Oyarzún and Soto, 2020).

In the ecological aspect, it is seen that the main difficulty for teachers is recognizing the connections between prime numbers and the students' environment. De Gamboa *et al.* (2015) explain that this is due to two types of beliefs; the first one, that the mathematics teacher's role obeys exclusively to teaching the discipline, thus omitting the need to offer the student relations with other disciplines; and the second one, that mathematical relations are complex for students and therefore it is not recommended to present them (D'Amore and Radford, 2017; Llinares, 2016).

The results of each aspect enable to conclude that the DMK model allows identifying the mathematical didactic knowledge of teachers regarding prime numbers, which is essential to establish criteria to improve the processes of teacher training. Furthermore, the possibility of establishing knowledge enables educators and researchers to develop processes of reflection and improvement of didactic practices.

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The learning of logarithmic functions by 12th-grade students based on modeling tasks

El aprendizaje de las funciones logarítmicas por parte de estudiantes de 12.º grado basado en tareas de modelización

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Abstract

The relevance that the application of what is learned in Mathematics to everyday situations has in student learning has led us to develop an experiment on teaching the logarithmic function using modelling tasks. Based on this experiment, we intend to characterize the activities of 12th-grade students in the performance of modelling tasks concerning topics of logarithmic functions and identify the difficulties they may present while solving these tasks using a graphing calculator. When adopting a qualitative and interpretative approach, data were collected through students' written records while solving the proposed tasks using the graphing calculator. The results indicate that the students show that the modelling tasks promoted group work and their interest and participation in class. During the exploration of the tasks, students performed activities that result from the accomplishment of modelling phases such as understanding the statement of the task, organizing and analyzing data, building and validating the model that best fits the data, and exploring the model, either in the introduction of the logarithmic function and its derivative or in the consolidation of the acquired knowledge. In such activities, some students presented difficulties regarding the properties and characteristics of the logarithmic function and its graphic and symbolic representation. The phases of the modelling cycle also made it difficult for students to use the graphing calculator, namely in performing statistical regressions and setting the visualization window.

Keywords: Mathematics, logarithmic functions, learning, mathematical modelling.

Resumen

La relevancia de aplicar lo aprendido en matemáticas a situaciones cotidianas en el aprendizaje de alumnos nos ha llevado a desarrollar un experimento sobre la enseñanza de la función logarítmica mediante tareas de modelización. A partir de este experimento, pretendemos caracterizar las actividades de alumnos de 12º curso, en la realización de tareas de modelización relativas a temas de funciones logarítmicas utilizando una calculadora gráfica, a su vez, identificar las dificultades que pueden presentar al resolverlas. Al adoptar un enfoque cualitativo e interpretativo, se recogieron datos a través de los registros escritos del alumnado mientras resolvían las tareas propuestas utilizando la calculadora gráfica. Los resultados indican que las tareas de modelización promovieron el trabajo en grupo y su interés y participación en clase. Durante la exploración de las tareas, el alumnado realizó las actividades que se derivan de la realización de las fases de modelización como son la comprensión del enunciado de la tarea, la organización y el análisis de los datos, la construcción y validación del modelo que mejor se ajusta a los datos y la exploración del modelo, ya sea en la introducción de la función logarítmica y su derivada o en la consolidación de los conocimientos adquiridos. En estas actividades, algunos alumnos presentaron dificultades en cuanto a las propiedades y características de la función logarítmica y su representación gráfica y simbólica. Las fases del ciclo de modelización también dificultaron el uso de la calculadora gráfica por parte de alumnos, concretamente en la realización de regresiones estadísticas y en la configuración de la ventana de visualización.

Descriptor: Matemáticas, funciones logarítmicas, aprendizaje, modelización matemática.

1 Introduction

When thinking about the learning activities of mathematical topics, the characteristics of the tasks are related to challenge learners to engage and reflect on what they do in these activities (Tekkumru-Kisa, 2020). These characteristics distinguish tasks in terms of the degree of challenge, high or low, and the degree of structuring, open or closed (Ponte, 2005). Among the tasks commonly worked in mathematics classes, we can mention exercises (low level of challenge and closed structure), problems (high level of challenge and closed structure) and research tasks (high level of challenge and open structure). Modeling tasks are part of these types, as problems or investigations, depending on the degree of structure of their statement. The activity resulting from solving this type of task is called mathematical modeling, which is currently part of the mathematics academic program in several countries (Chong *et al.*, 2019; Hoe and Dawn, 2015). In Portugal, it is part of the profile of students who graduate from compulsory education (Ministério da Educação, 2017). Dawn (2018) and Stillman *et al.* (2017) argue that modeling should be part of the activities performed at school in science learning.

Regarding mathematics, the Program for International Student Assessment (PISA) contemplates the definition of mathematical literacy activities related to modeling:

Mathematical literacy is an individual's ability to formulate, apply and interpret mathematics in different contexts. It includes mathematical reasoning and the use of mathematical concepts, processes, facts, and tools to describe, explain, and predict phenomena. (OECD, 2019, p. 75)

There are several authors (e.g., Shahbari and Tabach, 2020; Viseu and Rocha, 2020) and reference works (e.g., NCTM, 2000) that defend the contribution of mathematical modeling in the development of critical skills for students' engagement in their future activities (Reys-Cabrera, 2022). The NCTM (2000) recommends that students experiment with mathematical modeling of real-world problems, both social and physical, throughout their schooling, and this practice should be more present in later grades.

A favorite topic in mathematics for modeling phenomena is functions (Viseu and Rocha, 2020). From elementary school, students work with relationships between quantities, such as proportionality between quantities, for example, through a problem involving the speed of cars, or quadratic functions, through optimization problems of areas of geometric figures (Rocha, 2019). In high school, far-reaching mathematical concepts are introduced, such as derivation, which allows describing and understanding more complex functions, to illustrate, the logarithmic functions introduced in the 12^o grade¹ of high school.

Many everyday phenomena can be modeled using these functions, such as the growth of a bacterial population or the magnitude of an earthquake. Facing a mathematical modeling problem is essential to understand and structure the mathematics involved in the problem. In solving some of these tasks, functions are essential to represent and solve real-life problems (Sawalha, 2018), as is the case with logarithmic functions that translate into numerous everyday situations. According to Kastberg (2002) and Weber (2002), students have difficulty understanding and representing logarithmic functions, because the exploration of logarithmic functions is mainly limited to theoretical aspects. Moreover, the symbolic writing of logarithms and, consequent-

¹ The Portuguese education system includes 12 years of schooling up to higher education. The first nine years correspond to Elementary School (BE) and the last three to High School (SE). The BE consists of three teaching cycles: the first four years (with a single teacher), the second two years and the third three years. In the three years of SE, in which students begin to be oriented towards a group of higher education courses, the mathematics academic program varies depending on whether they follow courses in science, humanities, arts or technology.



ly, of logarithmic functions tends to disrupt their understanding (Mulqueen, 2012). Teaching these functions through real-life problems and situations can improve students' understanding of their topics. Therefore, it is essential to conduct studies that deepen on knowledge by enhancing this understanding (Viseu and Rocha, 2020).

For this, technology is essential to identify and visualize the model that best fits the context of the problem under study. The graphing calculator is essential to conceptualize and understand the different representations of a function (Viseu and Menezes, 2014; Viseu *et al.*, 2020). The relevance of this didactic device increases by allowing to connect analytical and graphical representations of logarithmic function concepts and to compare results. In the performance of modeling tasks in learning logarithmic functions, the graphing calculator is a didactic material that provides students tables, the editing of graphs and the exploration of regression curves that best fit the data resulting from the tasks proposed in class (Viseu and Menezes, 2014).

The combination of solving mathematical modeling tasks using the graphing calculator shows the valuation of what the student learns in solving problematic situations, his/her involvement in the activities proposed in the mathematics class and gives meaning to what he/she learns (Viseu and Rocha, 2020).

Based on these assumptions, this paper aims to characterize the activities that 12th grade students perform with mathematical modeling tasks in learning logarithmic functions and to identify the difficulties they have using the graphing calculator.

1.1 Mathematical modeling

The use of mathematical models in the teaching and learning of science subjects in recent years has been the subject of several studies (Anhalt *et al.*, 2018; Stillmann *et al.*, 2017). The connections between mathematics and the nat-

ural world condition the teaching and learning of mathematics (Barbosa, 2006, 2009; Blum, 2002). In this research, the impact of mathematical modeling in different grades has drawn attention (Stillman *et al.*, 2007). It is critical to analyze how mathematical modeling supports skill development in students' education (Kaiser and Sriraman, 2006; Kaiser and Maaß, 2007; Karawitz and Schukajlow, 2018). The first thing is to define mathematical modeling, distinguishing it from applied mathematics. According to Blum (2002), Blum *et al.* (2007) and Stillman *et al.* (2007), applied mathematics focuses on the use of mathematical tools in a real-world situation. Mathematical modeling focuses on the identification and development of mathematical tools to help solve that problem (Verschaffel, 2000). Mathematical modeling is a process of solving real-world problems (Carreira and Blum, 2021).

Mathematical modeling can be interpreted in different ways. Some authors interpret it as a motivational factor to introduce, develop and consolidate mathematical knowledge and learning (Barbosa, 2009). Others see it as a purely educational approach to develop students' ability to solve concrete problems (Galbraith and Stillman, 2006). Although these are different interpretations, they are not dissociated. By approaching a mathematical modeling problem focused on the solution processes, the learning objectives can be achieved (Barbosa, 2009; Galbraith and Stillman, 2006).

Solving a mathematical modeling problem requires understanding and interpreting the context of the situation under study and identifying the main aspects of the problem. To do this, data can be collected and the relationships and patterns between quantities can be described and understood in order to translate this information into mathematical language (Blum, 2002). This mathematical structure is called a mathematical model (Blum, 2002; Lesh and Fennewald, 2010). The development of a model can be done through various representations to encompass



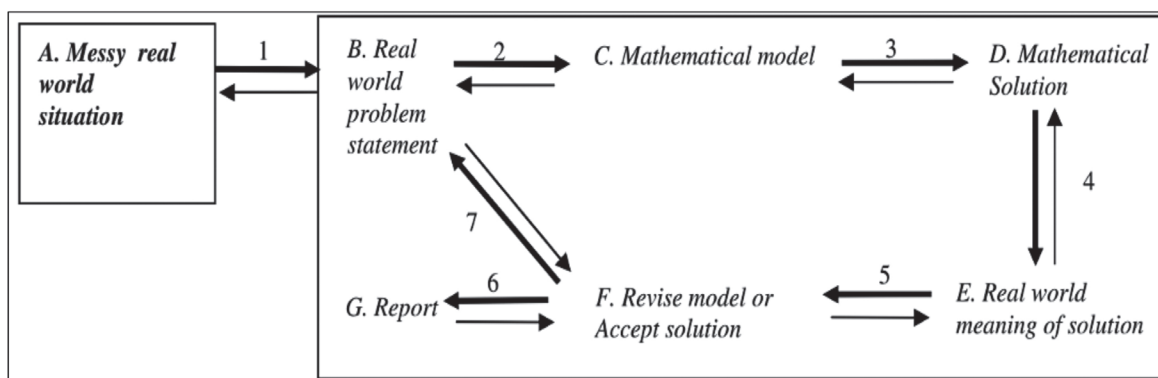
the possible potential of responses to a given situation (Lesh and Fennewald, 2010).

Thus, models run along different dimensions, from the concrete to the abstract, from the specific to the general, from the global to the analytical, from the simple to the complex, from the situational to the decontextualized, and from the intuitive to the formal (Lesh and Fennewald, 2010). The application of a mathematical model to study a problem requires the use of known mathematical tools and methods to extract mathematical results from that model. In turn, these results must be interpreted and validated by analyzing their relevance to the context

and purpose of the real problem. This process can be repeated, from scratch, in the definition of the model, depending on its effectiveness as a solution to the problem (Blum, 2002; Greefrath, 2019; Stillman *et al.*, 2007). This process is called mathematical modeling, which translates into the identification and definition of its phases and transitions in various diagrams or schemes. Stillman *et al.* (2007) adapted a diagram from Galbraith and Stillman (2006) to represent the mathematical modeling cycle, incorporating the reasoning that occurs between the different phases (Figure 1).

Figure 1

The modeling process (Stillman et al., 2007)



In this diagram, letters A-G represent the phases of the modeling process, and the arrows represent the transitions between the phases: (1) Understand, structure, simplify, interpret the context; (2) Assume, formulate, mathematize; (3) Develop and explore mathematically; (4) Interpret mathematical results; (5) Compare, criticize, validate; (6) Communicate, justify (if the model is considered satisfactory); (7) Go through the modeling process again (if the model is considered unsatisfactory).

For Borromeo Ferri (2006), the student is not always motivated to go through all the phases of the modeling cycle when learning mathematical content, and it is necessary to try to adapt this cycle to the classroom con-

text. The authors identify, based on studies and empirical data, the critical aspects that can block students in a given phase. These data were collected in the context of learning mathematical content by modeling problems and using technological tools (Galbraith and Stillman, 2006; Stillman *et al.*, 2007). From the analysis of these data, it is clear the main difficulties of students are the understanding of the statement and its context (Galbraith and Stillman, 2006; Stillman *et al.*, 2007), the identification of dependent and independent variables, the elaboration of the model, as well as the mathematical structure of the real problem (Dede, 2016; Galbraith and Stillman, 2006; Shahbari and Tabach, 2020).



1.2 Logarithmic functions

Understanding the concept of logarithmic function is essential to prepare students to face concrete problems (Kenney and Kastberg, 2013). Formally, the logarithmic function is defined as the inverse function of the exponential function. More precisely, a logarithmic function of base a is defined by $y = \log_a(x)$, $x > 0$, $a > 0$, $a \neq 1$, with $y = \log_a(x)$, if and only if $x = a^y$. Similarly, understanding logarithmic functions involves the ability to interpret the notation used (Kenney and Kastberg, 2013; Weber, 2017). In the expression $y = \log_a(x)$, perceiving the duality between the object x as an element of the domain of the function is a y , and the number obtained in the form $x = a^y$ is an obstacle for most students (Weber, 2002). Since the $\log_a x$ notation is relatively atypical compared to algebraic functions, it is confusing for most students to associate specific properties and characteristics (Kenney and Kastberg, 2013). Lack of understanding the notation of logarithmic functions, specifically the bases and the different nomenclatures $\log_a x$, $\log x$ e $\ln x$, can make it difficult to learn their algebraic properties. Students may be able to interpret linear, quadratic and even exponential functions because their algebraic expressions indicate to some extent what the underlying process is. However, the symbolic notation of logarithmic functions is more complex and ambiguous (Mulqueeney, 2012). Faced with a logarithmic equation, students tend to cancel logarithms of the same base as if they were polynomial functions (Kenney and Kastberg, 2013).

This shows that students have difficulty perceiving the role of logarithmic functions as inverse functions of exponential functions. This difficulty is also perceived in the graphical representation, as they match the graphical representation of logarithmic functions with that of exponential functions (Weber, 2017). A reinforcement of the duality between a bijective function and its inverse through, for example, the square root and the quadratic function, can improve the learning

of logarithmic functions (Kenney and Kastberg, 2013). According to Sawalha (2018), providing students with modeling tasks in learning exponential and logarithmic functions promotes the structuring and understanding of these concepts.

Current recommendations for mathematics teaching point to the development of competencies that students should acquire in their schooling (e.g., NCTM, 2007). According to Niss and Højgaard (2019), mathematical competence is the ability to respond promptly and insightfully to any mathematical challenge. The authors emphasize the difference between mathematical competence and a mathematical competence, the latter being the ability to respond promptly and insightfully to a specific mathematical challenge. On the other hand, mathematical competence is the combination of mathematical skills (Niss and Højgaard, 2019). In defining these subsets of mathematical competencies, the authors emphasize that these must satisfy the fulfillment of mathematical activities. A mathematical activity is the action aimed at asking and answering questions in or through mathematics (Niss and Højgaard, 2019, p. 14). Based on these assumptions, Niss and Højgaard (2019) define four competencies that are essential to participate efficiently in a mathematical activity: fundamental mathematical thinking; mathematical problem development and solving; the use of models and mathematical modeling; and mathematical reasoning. The mastery of mathematical language, constructs and tools allows in the conduction of mathematical activities, mentioned by the authors as other competencies, skills: the management of mathematical representations; the management of mathematical symbols and formalisms; mathematical communication; the management of material resources and mathematical tools. The authors emphasize that all these competencies are different, but not dissociated.

For this reason, we have chosen to give special relevance to three competencies that we consider fundamental in mathematical modeling: the treatment of models and mathematical modeling; the treatment of mathematical repre-



sentations; the treatment of material resources and mathematical tools. The first focuses on the ability to construct mathematical models and to analyze and evaluate models already defined, considering external factors such as data, facts, properties, and the context of the situation. The ability to articulate the different phases of the modeling process is included in this competence (Stillman *et al.*, 2007).

Competence in handling different mathematical representations consists on the ability to interpret and translate mathematical objects, phenomena, correspondences, and processes into a variety of equivalent representations, considering the advantages and limitations in the performance of a mathematical activity. The development, analysis and interpretation of models are directly related to the manipulation of mathematical representations (NCTM, 2000; Shahbari and Tabach, 2020). Finally, technology is essential as a constructive and critical resource in the performance of a mathematical activity. This competence also considers the advantages and limitations of these tools to use them accordingly depending on the mathematical activity (Niss and Højgaard, 2019).

2 Methodology

This study aims to characterize the activities of 12th grade students with mathematical modeling tasks in learning logarithmic functions and to identify the difficulties they have in solving these tasks using the graphing calculator. These difficulties highlight the fact that students do not perform adequately in the solution of these tasks or that result in erroneous solutions (Heyd-Metzuyanim, 2013) and in an inadequate use of the graphing calculator in such solution. Given these objectives, one of the authors carried out a teaching experience based on the use of modeling tasks in the teaching of logarithmic function topics using the graphing calculator as part of his pedagogical practices in the last course of the master's degree in teacher training, which

he integrated into his teaching strategies. This device is part of the didactic material that High School students in Portugal use when doing their activities. The study of logarithmic functions is performed for the first time in the students' school in the 12th grade. This teaching experience occurred during four lessons and included modeling tasks, from which we selected three (Annexes).

To optimize student learning in solving the modeling tasks, the teaching experience developed outlined strategies that assessed student activity through an exploratory teaching format (Ponte, 2005). In the first phase of the lessons, the task was presented to the class to determine whether the students understood the task statement, the data, and what was intended to be determined. In the second phase, students performed the task autonomously, in pairs or in groups. At this point in the lesson, the teacher supported the students, but without interfering with their solving strategies. Once the students' autonomous work was completed, their solutions were collected so that they would not be modified according to the discussion of the class group. In the penultimate phase, the teacher carefully guided this discussion to manage the students' interventions and compare the different solutions. In the last phase, based on what the students did or said, new concepts or procedures were set as a result of the exploration and discussion of the task.

The class selected was a 12th grade Science and Technology class with 31 students, 14 boys and 17 girls between 16 and 18 years old. The class had no repeaters and was composed of students who, for the most part, considered Mathematics and Physics-Chemistry as their favorite subjects. As for performance in Mathematics, the students' grades at the end of the course ranged from 6 to 20, with a mean of about 14.4 (DP=4.21). As for the solution of modeling tasks, the students had already performed tasks of this nature in previous courses



with the class teacher in the study of polynomial functions and rational functions.

Given the nature of the objective, a qualitative and interpretative approach was used to understand the students' activity in solving the proposed tasks in the classroom context (Bogdan and Biklen, 1994). For this purpose, data were collected through the written records of the students in the solution of the proposed tasks, using the graphing calculator to study logarithmic function topics prior to their discussion in the class group. These tasks were developed with the intention that, from their solution, the students would acquire the notion of logarithm of a number in each base, carry out the study of a logarithmic function, determine the derivative of the logarithmic function and systematize the study of the logarithmic function.

The data analysis was based on the content analysis of the students' solutions of the modeling tasks proposed to them, translated according to the topics taught: (i) Introduction to the study of the logarithmic function; (ii) Introduction to the derivative of the logarithmic function; and (iii) Systematization of the knowledge of logarithmic functions. In each of these topics, the modeling phases considered from the theoretical framework are used: (1) understanding of the task statement; (2) data analysis/model construction and validation; (3) model exploration. The analysis of the students' solutions allows to identify the activities they performed in each of these modeling phases and the difficulties they had in their solutions. This analysis focuses on the solutions made in pairs in the modeling tasks related to the introduction of the logarithmic function

and its derivative and in groups in the modeling task related to the systematization knowledge of logarithmic functions.

3 Results

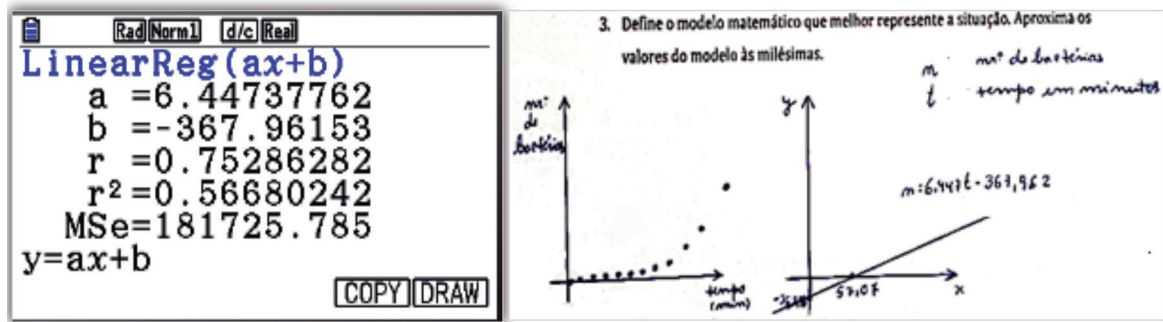
3.1 Introduction to the study of the logarithmic function

In the introduction of logarithmic function, the students began by solving a modeling task in pairs involving knowledge of the exponential function previously studied. The choice of this function as a model arising from the solution of the proposed task was due to the relationship between the exponential function, with a positive real base other than one, and its inverse function, the logarithmic function with the same base (task 1, appendices). When exploring this task, students indicated that they interpreted what the task asked. In the data analysis, model building, and validation phases, students aimed to develop a mathematical model that represented the task situation. Questions 2 and 3 of the task complete and illustrate these phases. Students used diagrams to organize the data on the graphing calculator, identifying the dependent and independent variables. The P3 pair resorted to instrumented action diagrams from the graphing calculator to obtain a linear model and represented a sketch of the point cloud of the number of bacteria as a function of time. Although this sketch shows an exponential curve, it defined a linear model. This shows that they did not perceive well the role of the determination coefficient or that students did not try to find out other models (Figure 2).



Figure 2

P3 Pair's answers to Task 1, items 2 and 3



Likewise, when defining the best model that fits the data, pairs P11 and P13 did not resort to the graphing calculator and defined a mathematical model by means of algebraic processes from the data in the table. These students recognized a geometric progression of ratio 2 and general term 2^n , where n represents the number of binary division processes, which allowed them to define the following functions:

$f(x) = 2^{\frac{x}{20}}$ where x is the time in minutes (P13);

$N(t) = 2^{\frac{t}{20}}$, where t is the time in minutes (P11).

These models correspond perfectly to the data in the table and the binary splitting process if

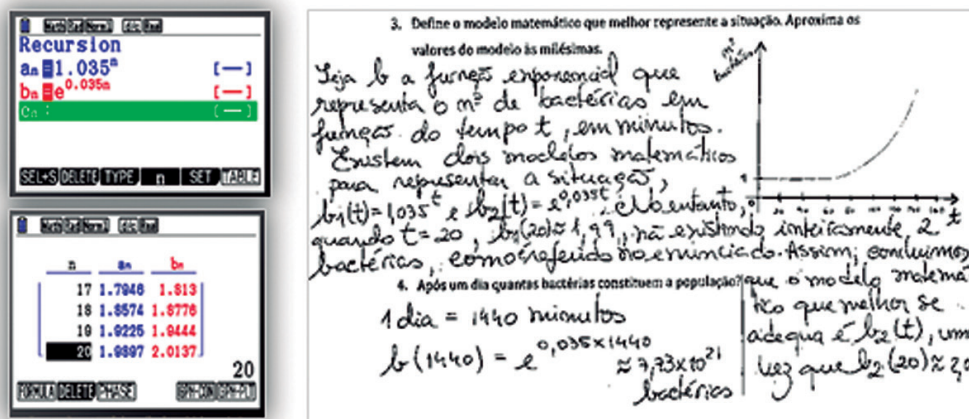
one works with discrete rather than continuous models.

The P7 pair identified the exponential model as the best fit, based on the determination coefficient r^2 , but did not recognize that both models shown on the screen ($y = ae^{bx}$ ou $y = ab^x$) represented the same model. To determine which fit better, these students used the recursion menu of the graphing calculator and plotted the two models as a function of in a table to compare their values.

Some students were critical with the models obtained with the graphing calculator and tried to check the results using another function of $n \in \mathbb{N}$ graphing calculator (Figure 3).

Figure 3

P7 Pair's response to Task 1, items 2 and 3



In exploring the model to answer the remaining items, it is worth noting that the pairs P11 and P13, defined with paper and pencil the same exponential model $y=2^{\frac{x}{20}}$ and tried to solve this question analytically. They decomposed the number 8000 into prime factors to solve the equation $8000=2^x$. However, the obtained decomposition $2^6 \times 5^3$ was a dead end for these students. Pair P13 used the graphing calculator to solve the equation graphically.

Solving questions 5 and 6 of the task led to the inverse process underlying the exponential function to determine the value of x by solving the equation. The application of the properties of inverse functions, i.e., the graphical representation of a function compared to its inverse function, allowed the graphing calculator to represent the symmetric curve of the graph of the exponential function determined by the line of equation $y=x$. Thus, the reciprocal relationship between the exponential function and the logarithmic function with the same base was established $x=a^y \Rightarrow y=\log_a x, a \in \mathbb{R}^+ \setminus \{1\}$. To systematize this knowledge, the students solved tasks on the notion of logarithm of a number in a given base and on the study of logarithmic functions.

3.2 Introduction to the derivative of logarithmic function

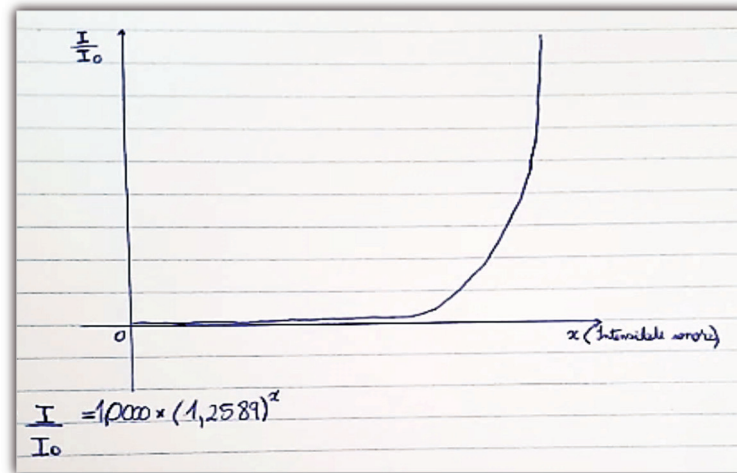
The introduction to the derivative of the logarithmic function resulted from solving a modeling task on sound intensity in decibels (Task 2, Appendix). From the data of the table and the graphing calculator, students had to identify the model that best fit the data presented and answer the remaining questions based on that model. This task was solved in pairs, and the conclusions were discussed and analyzed in class.

The interpretation of the task statement was clear for students; no pair questioned the meaning of the linear scale expression in the question. Most of the pairs found no difficulty in verifying the truth of Bell's statement, understanding that for the data to verify a linear scale, there must be a relationship of direct proportionality between them, as illustrated by P1's statement: the statement is true, since it is not possible to verify a relationship of direct proportionality between the two variables present in the table.

Although the students had already completed a mathematical modeling activity, the notion of a mathematical model was not clear to all, nor was the use of the graphing calculator to determine this model. Eight pairs of students (out of fifteen) correctly identified the role of each variable in the task, correctly organized the data on the graphing calculator, and found the logarithmic model to be the best fit to the data. Some students were unable to find the correct model because they reversed the variables or the lists in the table in the statistics menu. By default, list 1 is assigned to independent variable and list 2 is assigned to dependent variable. In the case of this task, the first column of the data corresponds to the dependent variable and the second to the independent variable. By reversing the role of the variables, these students naturally obtained an exponential model. However, other students did not understand the role of the quantities and plotted the point cloud of the $\frac{I}{I_0}$ relationship as a function of sound intensity in dB, resulting in an exponential model, as exemplified by the answer given by the P13 pair (Figure 4).



Figure 4
P13 pair response to Task 2



Regarding the exploration of the model, the pairs that defined an exponential model found it difficult to answer the remaining items. In item 3, the idea is to calculate the variation in the interval [1010;1030]. However, the graphing calculator does not allow this type of calculation, indicating that these students faced some limitations of the graphing calculator.

In addition to the technical limitations, there are also constraints due to certain mathematical concepts, such as the definition of instantaneous rate of change at a point. As for section 4, since the first derivative of logarithmic functions had not yet been introduced, the students did not have the possibility to solve it analytically. Five pairs of students understood that they could use the graphing calculator to determine the instantaneous rate of change of the model at the abscissa of point 15, as shown by the answer of pair P1: using the derivative of the logarithmic function (on the graph of my calculator) it is possible to determine the instantaneous rate of change of the sound intensity when $\frac{I}{I_0} = 15$. The response of this pair of

students allowed to introduce the derivative of the logarithmic function that modeled the task situation by formally defining the first derivative of a function: $f\left(\frac{I}{I_0}\right) = 1,03 \times 10^{-4} + 4,3429 \ln\left(\frac{I}{I_0}\right)$. Next, the first derivative of any logarithmic basis function was defined by means of the algebraic properties of logarithms and the corresponding derivative rules.

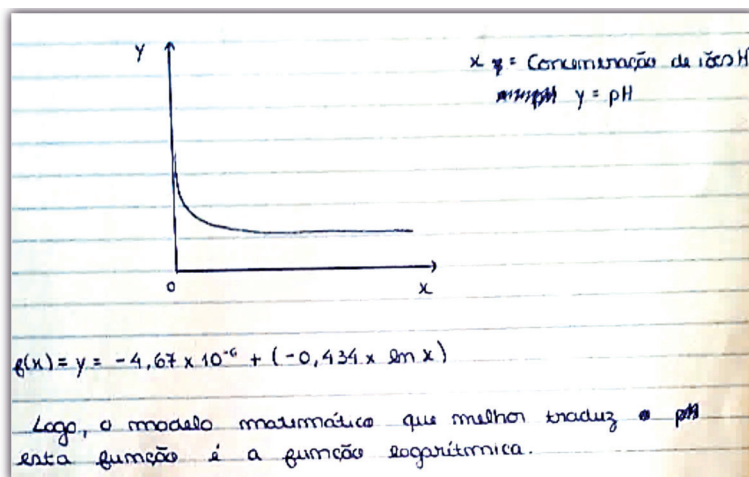
3.3 Knowledge systematization of logarithmic functions

After introducing the logarithm function and its derivative with two modeling tasks in the first two lessons, it was proceeded to apply the knowledge acquired in the study of the logarithm function by solving tasks, as exemplified by task 3 on the pH of a substance, in groups of 3 or 4 students. All students determined a logarithmic model as the best fit to the data in the table. However, only half of the groups determined the best logarithmic model, as illustrated by the response of group G2 (Figure 5).



Figure 5

Group G2 answer to



The groups that answered incorrectly determined a logarithmic model that does not correspond to the best fit to the data in the statement. These groups probably made a mistake in defining and organizing the data on the graphing calculator. Group G6 determined the correct coefficients of the expected logarithmic model but found it difficult to translate these values to the model suggested by the graphing calculator. When reading the coefficients of the obtained logarithmic model, these students did not identify that only coefficient b multiplied the Napierian logarithm of x .

Regarding section 3, it is observed that most of the groups tried to solve it only with analytical methods. However, no group reached the expected result because the students did not determine the domain of the logarithmic model of pH, considering that it could only take values between 0 and 14. Only one group restricted the domain of the logarithmic model, considering the situation of the task. Group G4 previously solved the equation $f(x)=0$, where f represents the model obtained to determine the domain of the model. Based on the data in the table, the values of x for which a solution has an acidic pH were determined. The logarithmic model translates into a strictly decreasing function, which will have an impact in the order of the inequality that translates the state-

ment. Groups that answered incorrectly had difficulty solving the inequality and defining the domain, as shown by the solution of group G8: $-4,7410^{-6} - 0,434 \ln x < 7 \Leftrightarrow -0,434 \ln x < 7 \Leftrightarrow \ln x < -16,13 \Leftrightarrow x < e^{-16,13} \Leftrightarrow x < 9,891 \times 10^{-8}$.

There are several limitations when solving this using the graphing calculator, namely the definition of the display window. When plotting the logarithmic model and the line of equation in a display window adapted to the task situation, it is impossible to observe the intersection of the two curves; since the values are too small, it is impossible to see the intersection even by enlarging the graph. These limitations stem from the limited technological capabilities of the graphing calculator to represent infinitely small or large quantities. The fact that the solution cannot be read graphically may have disturbed the students in their use of the graphing calculator in this section.

Regarding section 5, very few groups had time to explore it. Only two groups responded, identifying that one of the differences between the theoretical pH model and the model obtained through the task was the base of the logarithm, being a decimal logarithm for the theoretical model and a Napierian logarithm for the model, as illustrated by the response of group G3: the standard pH formula is presented through the decimal logarithm. In contrast, the model we present is given as a function of the Napierian logarithm.



When presenting in the class the results of group G3, the procedure for changing the base of the logarithms to any base $a \in \mathbb{R} \setminus \{1\}$ to work with logarithms of equal bases was reviewed. In this way, the accuracy of the obtained model can be compared with the theoretical model of the statement.

3.4 Activities and difficulties experienced by the students

Based on the empirical study by Stillman *et al.* (2007), Table 1 illustrates the performance of students when conducting key cognitive activities to solve the proposed modeling tasks.

Table 1

Success rate of student activities when exploring modeling tasks (%)

Modeling phases	Understanding the task statement			Data analysis and model creation			Exploration of the model		
Cognitive activities	Clarify the context of the problem	Identify magnitudes	Developing hypotheses and assumptions	Identification of variables	Develop relevant hypotheses	Choosing the technology to build the best model	Apply appropriate mathematical knowledge	Use technology to make calculations and graphs	Mathematize the statement and
Task 1 (15 pairs)	100 %	100 %	100 %	100 %	86,7 %	66,7 %	73,3 %	53 %	100 %
Task 2 (15 pairs)	80 %	93,3 %	60 %	93,3 %	53,3 %	53,3 %	20 %	33,3 %	0 %
Task 3 (8 groups)	100 %	100 %	100 %	100 %	100 %	50 %	56,25 %	75 %	25 %

The analysis of the table shows that all students, in some way, performed the expected cognitive activities while exploring the tasks. Some modeling skills, mathematical and technological, are identified, since are essential to task exploration and are in line with the literature (Niss and Højgaard, 2019). One of these is statement interpretation. Most of the students identified, when reading the statement, relevant and non-relevant information, and dependent and independent variables.

In developing the model, it was essential to place and organize the data on the graphing calculator, to know how to work with lists, to represent the point cloud associated with the data, and to define the list that corresponds to each variable. Students tested which model best matched the points representing the data in the

task from the point cloud. It should be noted that some students showed difficulties in organizing the data in a table according to the task situation and in knowing how to use and read the regressions available on the graphing calculator.

For analyzing questions, it was essential to apply mathematical knowledge. From the statement, the students translated and related the information to the model obtained. As the complexity of the tasks increased, students showed difficulties in correctly applying this mathematical knowledge. Recognizing the need to use the graphing calculator to perform calculations, represent graphs, solve equations, and check results is inevitable when exploring these types of tasks. The need for students to know how to graph a function and define a display window appropriate to the situation is also high-



lighted, although this can lead to confusing representations when dealing with disproportionate magnitudes (Campos *et al.*, 2015; Consciência, 2013). However, students cannot identify if what appears on the graphing calculator display is mathematically valid and whether it is consistent with the task situation without a solid theoretical basis (Consciência, 2013).

4 Discussion and conclusions

The students' activity was carried out in small groups to assess the students' actions in an exploratory teaching perspective conducive to the exploration of modeling tasks (Blum and Borromeo Ferri, 2009; Borromeo Ferri *et al.*, 2017; Ponte, 2005). This organization of the students gave them the opportunity to learn to work in groups, develop mathematical communication skills and the attitude of being critical with the results obtained (Rodríguez-García and Arias-Gago, 2020). Gradually, with the students' participation in the activities carried out in their group and in the class, the group improved in the solving of the proposed tasks, corroborating the results obtained by Blum and Borromeo Ferri (2009) and Sawalha (2018). Such participation indicates that it is due to the nature of the modeling tasks, which motivated students to share ideas and strategies and to confront knowledge, processes, and results.

The implementation of the mathematical modeling tasks was based on the modeling cycle defined by Stillman *et al.* (2007), adapting the phases of this cycle to the objectives in the lessons taught in the study of the logarithmic function. Using the graphing calculator, the students carried out the phases of understanding the statement, data analysis, model construction and validation, and model exploration. These phases allow to characterize the sequence of activities they perform when exploring the tasks proposed in the learning of the topics studied.

After reading the proposed tasks, most students can interpret the statements by identifying

the independent and dependent variables, due to the development of their functional reasoning, a skill that is expected to be developed at the end of high school (Ambrus *et al.*, 2018; Ministerio de Educación, 2017). It is a skill that allows students to establish relationships between the data they extract from the interpretation of task statements and the values of variables. The perception of the behavior of values of the variables is evident when these values are organized in tables or scatter plots. The meaning drawn from this analysis highlights the relevance of the connection between the different mathematical representations of function concepts (Viseu *et al.*, 2022). The numerical representation aligns with the symbolic representation (an algebraic expression that translates the model), which in turn is articulated with the graphical representation.

By building the best model that fits the data collected, students make sense to the mathematical model they obtain and the procedures necessary to perform statistical regressions on the graphing calculator. This is part of the instrumented performance schemes that are developed when taking advantage of this technological device (Teixeira *et al.*, 2016), which translates into the identification of the parameter roles that integrate the different models provided by the graphing calculator, the meaning of the value of the determination coefficient and the definition of the display window.

Regarding the exploration of the model, students followed different solution strategies, either by analytical processes or by graphing calculator. When exploring this technological device, students initially presented some limitations to integrate it into their activities. These limitations are due to the lack of familiarity with its use, which influences in the way the teacher guides them: exploring the statistics menu, editing algebraic expressions in the function editing menu and defining the display window. It is observed that the tasks that do not always allow solving problems with paper and pencil make the students feel more familiar with the tool, having



implications in the meaning they give to the topics under study.

The exploration of the statistical menu of the graphing calculator in the solution of the modeling tasks makes it possible to establish the determination coefficient, which informs about the reasonableness or not of the model found to fit the data resulting from the task being solved. The model translates the function that is the object of study, which, in the case of this work, directed the students to learn the notion of logarithm of a number in each base, logarithmic functions and their properties, as well as the derivative of a logarithmic function. These results conclude that the solution of modeling tasks is based on the value of conceptual understanding of the topics under study and of the computational procedures needed to solve a problem (Kenney and Kastberg, 2013). It is about the development of skills that are intrinsic to the solution of modeling tasks, among which are the understanding of different mathematical representations, the sense of mathematical symbology and the ability to communicate mathematically which, in today's world, is mostly done using technological devices (Niss and Højgaard, 2019).

Regarding the difficulties reported by students in solving the modeling tasks, the interpretation of the statement turned out to be a challenge when exploring the model, as mentioned by Stillman *et al.* (2007). Some students found it difficult to connect the context of the task with the mathematical model obtained, which translated into difficulty in understanding the mathematical model they were working with, the functions in general and their properties.

Regarding the definition of the best model that fits the data in the statement, students were not too familiar with the use of the functions of the graphing calculator to organize data, plot point clouds and the preparation of statistical regressions. It is paramount that they know the features of the graphing calculator to be able to organize the data in a table, plot the associated point cloud, build a mathematical model that

fits the task situation and, consequently, solve this type of tasks (Blum, 2002; Campos *et al.*, 2015). These types of activities are reinforced by solving tasks that encourage students to collect and organize data and to model the problem situation under study from a functional perspective. Gradually, students are aware of the usefulness of what they learn at school to understand situations and solve problems they find on a daily basis. This is a teaching perspective that promotes dialogue between two worlds, the real and the mathematical, very different from the perspective that motivates students to reproduce facts and procedures.

In the model exploration phase, some students showed some difficulties when using the graphing calculator to overcome the limitations of specific analytical processes. The definition of the display window was difficult for some students, as it depends on the domain of the model and the context of the task situation. A poorly adjusted display window can lead to misinterpret the graphical representation and become an obstacle when exploring this type of tasks (Arcavi, 2003; Viseu and Menezes, 2014). The proper use of the graphing calculator intrinsically implies having solid mathematical knowledge to relate the concepts and properties of functions to their graphical representation (Consciência, 2013). To fully take advantage of the potential of the calculator, it is essential to know the mathematical concepts and understand the notations displayed on the screen (Galbraith and Stillman, 2006).

In general terms, it is inferred from the results of this study that the solution of modeling tasks promotes in students the acquisition of knowledge (in this case, of facts and procedures of the logarithmic function) and the development of skills (reasoning and problem solving) and attitudes, including autonomy, critical thinking, and the search for scientific, technical, and technological deepening.

There were some limitations when conducting this paper. One of them was due to the use of the graphing calculator. Even though the



students have had a graphing calculator since 10th grade, they were unaware of certain features of the graphing calculator, especially the ability to organize data and perform statistical regressions from these data. Defining a display window to suit the graphical representation or part of it, depending on the objectives of the task, was a challenge for the students. Without a suitable display window, wrong conclusions about the characteristics of the function under consideration could be drawn from reading and interpreting the graphing calculator display. In addition, logarithmic models establish relationships between quantities that are often unreasonable in the graphical representation. Due to the technological limitations of the graphing calculator, particularly the screen resolution (384×216 pixels), it cannot represent specific parts of the graph of the functions underlying these models.

Consequently, the reading and interpretation of the graphical representation obtained with the calculator, as well as the graphical solution of the problem, are limited. Another limitation was related to the design of the tasks involving logarithmic models. It was a great challenge to propose modeling tasks that matched the learning objectives set out in the curriculum and that were associated with significant natural phenomena. The design of real context modeling tasks, focused on the properties of the problem situation and not semi-real and only focused on its mathematical properties (Ponte and Lent, 2012), proved to be a difficult task to perform. Such limitation highlights the use of modeling tasks in teaching strategies in the training of future mathematics teachers, promoting the development of their professional knowledge about the characteristics of the tasks that current mathematics teaching programs recommend integrating in teaching strategies.

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Annex

Homework 01. The growth of bacteria population

Tarea: El crecimiento de una población de bacterias

1. Las bacterias se reproducen asexualmente mediante un proceso llamado división binaria. La división binaria se produce cuando una bacteria duplica su material genético y se divide inmediatamente, dando lugar a dos bacterias idénticas a ella. En condiciones ideales de temperatura y nutrientes, una bacteria tarda aproximadamente veinte minutos en completar el proceso de división.
2. Completa la siguiente tabla.

Tiempo (minutos)	0										
Número de bacterias	1										

3. Utilizando la calculadora gráfica, grafica la nube de puntos del número de bacterias en función del tiempo t .
4. Define el modelo matemático que mejor represente la situación.
5. Después de un día, ¿cuántas bacterias componen la población?
6. ¿Cuánto tiempo tarda la población en alcanzar las 8000 bacterias?
7. Dado que el número de células bacterianas en el cuerpo humano es de aproximadamente 40×10^{12} , ¿cuánto tiempo tardará la población de bacterias en alcanzar este valor?

Homework 2. What is a decibel?

Tarea: ¿Qué es un decibelio? En sus primeros estudios sobre acústica, Bell, inventor del teléfono, se dio cuenta de que la variación del sonido que el oído humano puede sentir no sigue una escala lineal. Si duplicamos la amplitud de la señal, nuestro oído no capta esta transformación. Basándose en sus experimentos, Bell decidió utilizar una escala logarítmica para representar la amplificación o atenuación de un sistema para cuantificar la reducción a nivel acústico en un cable telefónico estándar de 1 milla de longitud. Bell creó la unidad de medida TU (Transmission Unit). Esta unidad de medida pasó a llamarse Bel. Con la práctica, se vio que la unidad era demasiado grande y se decidió dividir la unidad Bel en diez, creando así el Decibelio (dB).

Fuente Sonora	Intensidad Sonora (dB)	Relación entre la intensidad sonora percibida I y el umbral audibilidad I_0
Murmullo	10	10
Conversación normal	20	100
Habitación Tranquila	25	316.2
Biblioteca	30	1000
Vía residencial	40	10000
Lavavajilla	45	31622.8
Oficina	50	100000
Sala de Aula	55	316227.8
Motor de un carro	70	1000000
Aspiradora	65	316227.7
Tránsito congestionado	70	10000000
Cantina escolar	80	100000000

1. A partir de los valores de la tabla, comente la afirmación: "Bell se dio cuenta de que la variación del sonido que puede sentir el oído humano no sigue una escala lineal".
2. Utilizando una calculadora gráfica, represente gráficamente la nube de puntos de intensidad sonora (dB) en función de la relación I/I_0 . ¿Qué modelo matemático representa mejor esta situación?
3. Con respecto al modelo que has definido, compara la variación de la intensidad sonora en los intervalos $[10; 30]$ y $[1010; 1030]$.
4. Determine la tasa de variación instantánea de la intensidad sonora para $I/I_0 = 15$.
5. Determine la tasa de variación instantánea de la intensidad sonora en función de la relación I/I_0 .
6. Describa el comportamiento de la variación de la función que modela la situación en estudio, indicando dónde se acentúa este crecimiento.



Homework 3. The pH of a substance

Tarea. El pH de una sustancia. La acidez de una sustancia se mide por la concentración (en moles por litro) de iones de hidrógeno (H^+) en la sustancia. La forma estándar de describir esta concentración es definir el pH (potencial de hidrógeno) de una sustancia, que indica si una sustancia es ácida, neutra o básica. La escala de pH va de 0 a 14 a una temperatura de 25°C. Si el valor del pH es igual a 7, el medio de la sustancia es neutro, pero si el pH es inferior a 7, es ácido, y si es superior a 7, básico. La siguiente tabla muestra las concentraciones de iones H^+ y los respectivos valores de pH de las sustancias presentes en nuestra vida cotidiana.

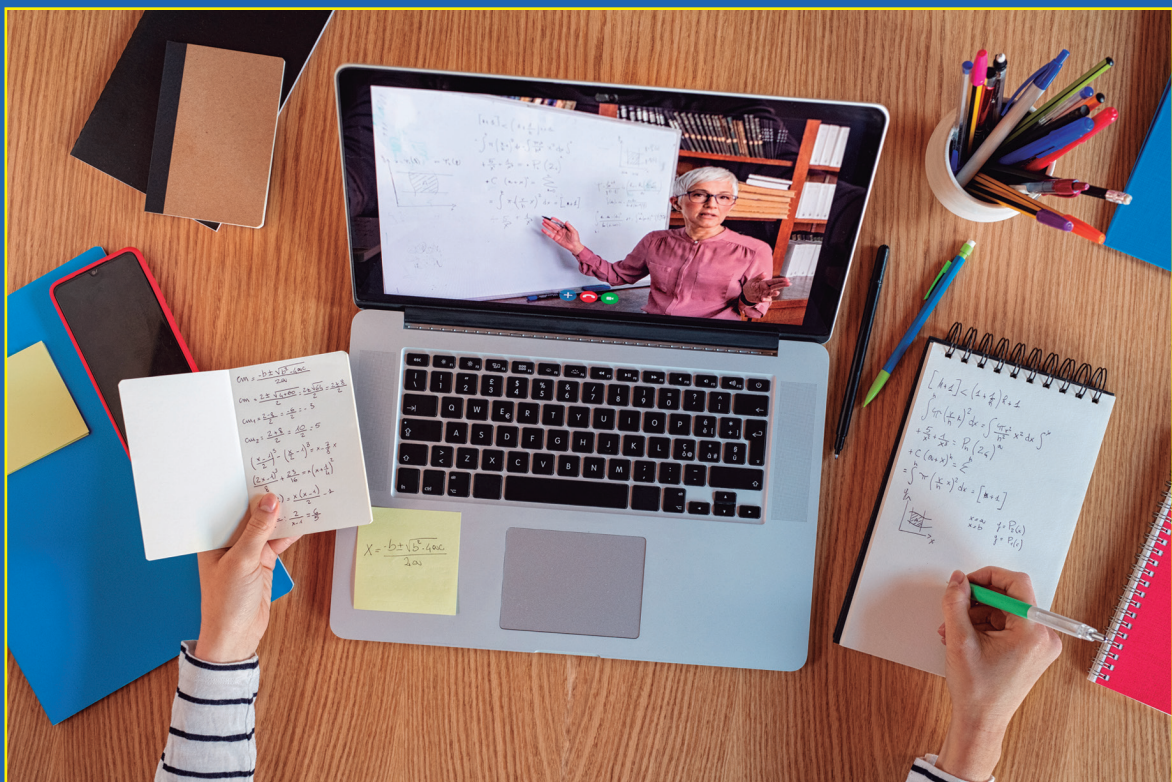
Substancias	Concentração de iões H^+	PH
Zumo de limón	3.981×10^{-3}	2.4
Vinagre	1.2589×10^{-3}	2.9
Zumo de manzana	3.1623×10^{-4}	3.5
Cerveza	3.1623×10^{-5}	4.5
Café	1.0×10^{-5}	5.0
Té	3.1623×10^{-6}	5.5
Leche	3.1623×10^{-7}	6.5
Água pura	1.0×10^{-7}	7.0
Sangre	3.981×10^{-8}	7.4
Agua potable	1.0×10^{-8}	8.0
Jabón de Manos	6.3096×10^{-10}	9.2
Amoniaco casero	3.1623×10^{-12}	11.5
Agua del baño	3.1623×10^{-13}	12.5

1. A partir del análisis de los datos presentados en la tabla, ¿qué modelo crees que traduce mejor el pH de cualquier sustancia en función de su concentración de iones? Utilizando la calculadora gráfica, comprueba si el modelo idealizado traduce esa representación.
2. Tomando como referencia el modelo que has definido, determina la concentración de iones H^+ de un zumo de uva cuyo valor de pH es 3,2.
3. ¿Qué valores expresan las concentraciones de iones H^+ para que el medio de una sustancia sea ácido?
4. A partir de la gráfica de tu modelo, puedes ver que la función representada es estrictamente decreciente en su dominio de validez y que su gráfica tiene una concavidad hacia arriba.
5. Formalmente, el pH viene dado por $pH = -\log[H^+]$. ¿Cuál es la diferencia entre el modelo que has establecido y este modelo formal de pH?



Miscellaneous Section

(Sección Miscelánea)



Source: <https://www.shutterstock.com/es/image-photo/high-angle-view-video-conference-teacher-1676998303>



Creativity and related variables according to educational stage: a systematic review

Creatividad y variables relacionadas según la etapa educativa: revisión sistemática

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Abstract

Creativity is a capacity present in any person that arises spontaneously in order to solve problems or create knowledge. The objective of this study is to identify the variables used in existing research related to creativity and to classify these variables according to the educational stage (infant, primary, secondary and university studies). The methodology has been directed by the PRISMA statement and has been carried out by searching the Dialnet Plus and Web of Science databases. Some of the resources used in the searches have been the use of Boolean operators, a series of filters and the presentation of inclusion and exclusion criteria. The results obtained indicate how intelligence and academic performance are the most studied variables in relation to creativity regardless of the educational stage. It has been extracted how the investigations focused on children and primary school focus on aspects such as personality or control of emotions, while secondary studies analyze problem solving and the works aimed at university students are oriented towards styles of thought. In conclusion, it is necessary to highlight how creativity is present within the educational context and, therefore, it is necessary to know what variables are related to it in order to enhance said capacity through an appropriate intervention.

Keywords: Creativity, correlation, student, evaluation, questionnaire, education.

Resumen

La creatividad es una capacidad presente en cualquier persona que surge de manera espontánea para resolver problemas o crear conocimiento. El objetivo de este estudio es identificar las variables utilizadas en las investigaciones existentes relacionadas con la creatividad y clasificar estas variables según la etapa educativa (infantil, primaria, secundaria y estudios universitarios). La metodología se ha guiado por la declaración PRISMA y se ha llevado a cabo mediante la búsqueda en las bases de datos de Dialnet Plus y Web of Science. Algunos de los recursos utilizados en las búsquedas fueron la utilización de operadores booleanos, una serie de filtros y la exposición de unos criterios de inclusión y exclusión. Los resultados obtenidos indican que la inteligencia y el rendimiento académico son las variables más estudiadas en relación con la creatividad independientemente de la etapa educativa. Las investigaciones enfocadas en infantil y primaria se centran en aspectos como la personalidad o el control de las emociones, mientras que los estudios de secundaria analizan la resolución de problemas y los trabajos dirigidos a universitarios se centran en los estilos de pensamiento. En conclusión, es necesario resaltar cómo la creatividad está presente en el contexto educativo, por ello, es preciso conocer qué variables están relacionadas con ella para potenciar esta capacidad mediante una intervención adecuada.

Descriptores: Creatividad, correlación, estudiante, evaluación, cuestionario, educación.

1 Introduction

Creativity can be understood as innate behavior that involves a personal approach and is not repetitive; just as it is permanently changing and looking for ideas, combining the notions already known, but it must be considered that this is a rather complex term that can cover different fields (Barbachán *et al.*, 2020).

This term has been studied in recent years due to the emergence of certain terms such as resilience, coaching or emotional intelligence, but considering that creativity is not an exclusively modern term, but it is present since the appearance of the human being (Moral-Valiente, 2017).

Considering the origins of this concept, many authors have contributed to studying this phenomenon. The most outstanding is J. P. Guilford, who refers to creativity as the qualities that creative people have, such as originality, fluency, flexibility, and divergent thinking (Guilford, 1980). On the other hand, another classic author is P. Torrance, who relates creativity to a process that aims to test initial hypotheses and interpret the results obtained (Torrance, 1969). The existence of different definitions of creativity is because this term has evolved over the years and, in turn, can take into account different points of view (Corbalán, 2008; Garaigordobil, 2003).

Although there is no unanimous concept of creativity, many authors agree that every person has the possibility and capacity to be creative, since it is conditioned by the motivation, preparation, interest, and willingness to the creation of something original and new (Caeiro-Rodríguez, 2018; Gómez *et al.*, 2017; Hammershøj, 2014; Hernández *et al.*, 2015; Sánchez *et al.*, 2016). In addition, the close relationship existing in the creation of new ideas with different parts of the brain has been studied, allowing people to analyze, associate and interpret the new knowledge that is being acquired (Elisondo and Donolo, 2015; Ramos *et al.*, 2017).

Penagos and Aluni (2000) indicate that it is necessary to have skills and knowledge about

the topic that requires to be creative, as well as it is essential that the individual shows a high intrinsic motivation and a certain capacity to reduce extrinsic pressures. Therefore, there are different types or levels of creativity. Dow and Mayer (2004) rated creativity by referring to the predominant field in which it operates, namely verbal creativity, mathematical creativity and spatial creativity. On the other hand, Fuentes and Torbay (2004) proposed three types of creativity in relation to the amount of imagination the person adds to his/her process or product: objective creativity, imaginative creativity and inventive creativity.

The development of creativity in a person not only promotes the acquisition of skills that enhance problem solving, but also promotes certain social skills such as interaction with others (Cuetos *et al.*, 2020). Therefore, it is necessary to carry out certain strategies or resources that help to develop and/or enhance creativity. Labarthe and Vásquez (2016) held a creative writing workshop that fostered the creative capacity of the participating group. On the other hand, Aqueveque and Romo (2018) carried out an intervention with two groups of infants (one control and one experimental), applying certain multi-sensory activities based on manipulation. The effect of this research was significant, so it can be said that the development of creativity has no age limits. The same is true for sex, where studies have found no significant differences that allow the distinction of creative ability in men or women (González and Molero, 2022a).

Although creativity can be performed in different contexts and at different ages (Marrero *et al.*, 2019; Ortega *et al.*, 2016), there are more studies focus on analyzing this capacity in the educational context (Aldana *et al.*, 2021; González and Molero, 2022b). The importance of this construct at these stages is related to the fact that creativity is related to learning and processes linked to the construction of new knowledge (Elisondo and Donolo, 2016).

Schools seek to transform education through creativity to educate people capable of



developing new ways of learning, thinking, and working, thus empowering active and competent students to make decisions in the processes of change (Canelo *et al.*, 2015). In addition, social changes raise the need to promote in the student skills related to creativity and problem solving (Casado and Checa, 2020), so it is necessary to identify which variables relate to this capacity. Therefore, it is important to highlight the existence of resources such as questionnaires or instruments that have been developed with the purpose of issuing a reasoned judgment on the different lessons learned (González and Molero, 2021; Medina and Verdejo, 2020; Romo *et al.*, 2016).). This idea implies that educational institutions have committed to knowing the creative level of students and teachers (De La Torre *et al.*, 2018). Among some of the most outstanding instruments for the identification of creativity is the Test CREA (Corbalán *et al.*, 2003), which is based on measuring the creativity of students from a graphic material that allows the development of different tasks by discovering and solving problems.

Objective

The objective of this systematic review is to identify which are the most used variables in studies on creativity in students according to the educational level.

2 Methodology

This systematic review has been designed based on guidelines established for the development of quality reviews (Alexander, 2020) and following PRISMA 2020 principles (Page *et al.*, 2021; Yepes- Núñez *et al.*, 2021).

2.1 Search and procedure

A consultation was first carried out on Dialnet Plus and Web of Science databases and the Google Scholar search engine to start this systematic review. For this purpose, the descriptors

“creativity” and “instrument” were considered, as well as other synonyms such as “questionnaire”, “scale”, “assessment” and “inventory”. The Boolean operators used in search formulas have been AND, and the use of quotation marks (“”). Thus, the search has been established through the following search formulas: “Creativity” AND “instrument”; “creativity” AND “questionnaire”; “creativity” AND “scale”; “creativity” AND “assessment”; “creativity” AND “inventory”. An important aspect is that these searches have been conducted in both Spanish and English. In addition, filters such as type of document, language, availability of the text and year of publication have been taken into account. Table 1 shows the different results obtained in the databases consulted according to each search formula.

2.2 Inclusion and exclusion criteria

Once the initial search for studies has been carried out, inclusion and exclusion criteria have been established with the aim of selecting those studies that are most relevant to the subject.

First, in accordance with the inclusion criteria, the following characteristics have been considered: (A) journal article; (b) the language of the publication should be in English or Spanish; (c) direct access to the full text; d) empirical studies; (e) publications between 2000 and November 2020; (f) participants who were students in pre-school, elementary, middle, high school, or university. On the contrary, as exclusion criteria, studies with the following characteristics were not considered: a) documents belonging to book chapter, books, theses and final projects; (b) languages other than Spanish or English such as Portuguese, French, Chinese,...; c) papers without full access to the text; d) qualitative studies; e) publications before 2000; f) participants who were not students, such as adults, older people, workers.



Table 1

Results obtained according to each search formula in the databases

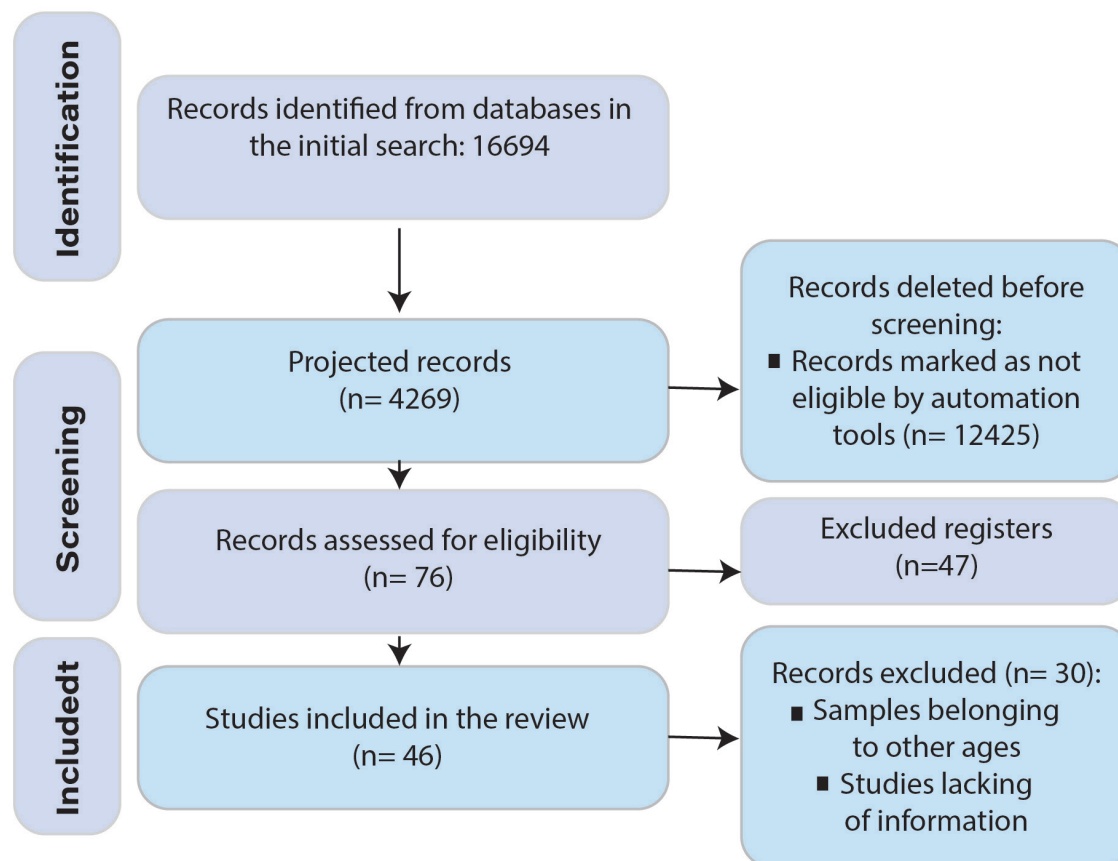
Database	Language	Search formula	Number of articles found after applying filters
Dialnet Plus	Spanish	"creatividad" AND "instrumento"	586
		"creatividad" AND "cuestionario"	271
		"creatividad" AND "escala"	230
		"creatividad" AND "evaluación"	481
		"creatividad" AND "inventario"	42
	English	"creativity" AND "instrument"	205
		"creativity" AND "questionnaire"	142
		"creativity" AND "scale"	109
		"creativity" AND "evaluation"	154
		"creativity" AND "inventory"	21
Web of Science	Spanish	"creatividad" AND "instrumento"	26
		"creatividad" AND "cuestionario"	22
		"creatividad" AND "escala"	25
		"creatividad" AND "evaluación"	67
		"creatividad" AND "inventario"	8
Web of Science	English	"creativity" AND "instrument"	184
		"creativity" AND "questionnaire"	474
		"creativity" AND "scale"	479
		"creativity" AND "evaluation"	605
		"creativity" AND "inventory"	128

Based on these criteria, a total of 46 articles were selected for further analysis in the next section of this systematic review. A total of 16 694 results were obtained by initial search based on the different search formulas listed above. Filters were applied to this first search, obtaining

4269 studies and, later, the inclusion and exclusion criteria were applied, obtaining 76 studies. These studies were reviewed manually, and 46 articles were selected. This whole process can be seen in Figure 1.



Figure 1
Flowchart



3 Results

Once the articles to be included in this systematic review were selected, Figure 2 was elaborated with the intention of visually showing some highlights of these results.

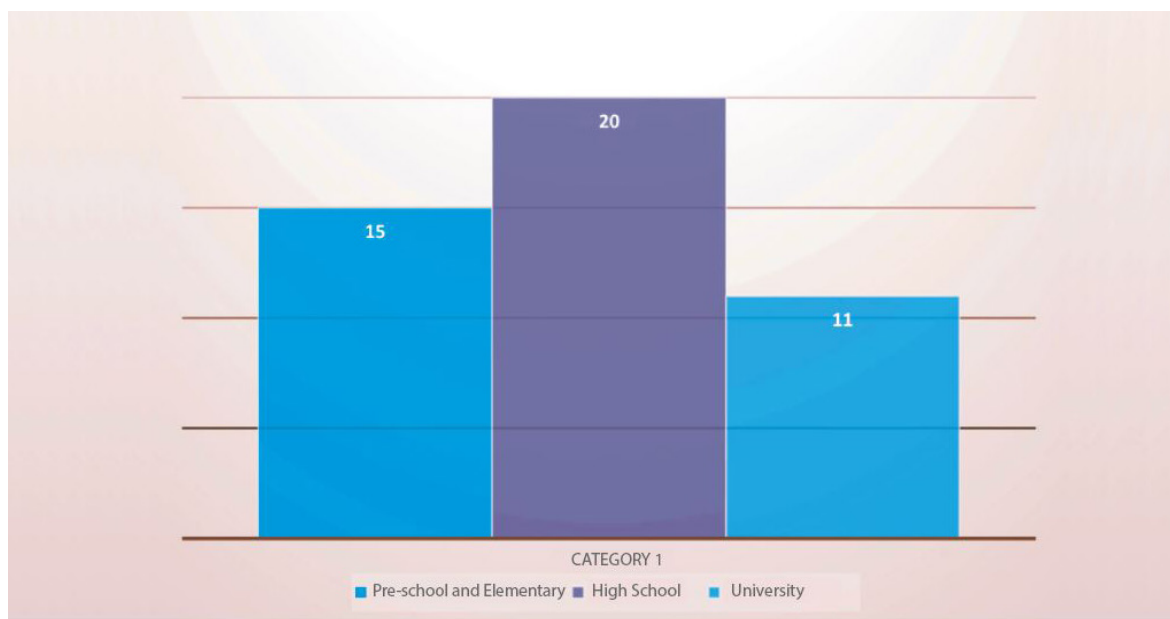
Looking at Figure 2, it can be seen how these studies can be divided into three educa-

tional stages, i.e., pre-school, elementary, High School and university; thus, High school having the most studies. In addition, most of these studies have been published during the second decade of the 2000 and English is the most predominant language. Finally, regarding the age of the participants, most studies focus on individuals between 13 and 18 years old, with the least represented group being 0 to 6 years.



Figure 2

Classification of selected studies by educational stage



To analyze the selected articles, these studies have been divided into three tables according to the educational level to which they belong. Therefore, Table 2 refers to pre-school and elementary students; Table 3 refers to high school students; and Table 4 focuses on university students.

Table 2 shows the main ideas of articles focused on both pre-school and elementary students. The most studied variables in relation to creativity at these ages are intelligence (Gatica and Bizama, 2019; Ortega *et al.*, 2017; Salavera *et al.*, 2019; Vallverdú *et al.*, 2016) and aspects related

to art and music (Fazaie and Ashayeri, 2018; Krumm and Lemos, 2012; Marchena *et al.* 2017). Other variables also studied at these ages are emotions (Hernandez *et al.*, 2020), confidence and learning (Trowsdale *et al.*, 2019), academic performance (Lamana and Peña, 2018), personality (Krumm *et al.*, 2018), aggression control (Jarareh *et al.*, 2016), creative perception and production (Krumm *et al.*, 2015), parental styles (Krumm *et al.*, 2013) and self-concept (Franco, 2006).

Table 2

Main characteristics of the selected studies in pre-school and elementary school students

Author/s and year	Sample	Variables studied	Instrument used to measure creativity
Hernández <i>et al.</i> (2020)	N= 2540	Creativity and Emotions	Emotional Awareness and Creativity Questionnaire
Gatica and Bizama (2019)	6 to 8 years (N= 94)	Fluid Intelligence and Creativity	Creative Intelligence Test (CREA)



Author/s and year	Sample	Variables studied	Instrument used to measure creativity
Trowsdale <i>et al.</i> (2019)	9 to 10 years (N= 135)	Trust, Creativity and Learning	Three measures proposed by the Trowsdale Confidence in Competition, Creativity and Learning Indexes (TICCCCL)
Salavera <i>et al.</i> (2019)	6 to 8 years (N= 957)	Emotional Intelligence and Creativity	Torrance Test of Creative Thinking
Lamana and Peña (2018)	4º Elementary (N= 91)	Academic Performance, Coping and Creativity	CREA Test
Fazaie and Ashayeri (2018)	7 to 9 years (N= 20)	Music Education and Creativity	Torrance Creativity Questionnaire
Krumm <i>et al.</i> (2018)	9 to 13 years (N= 359)	Personality and Creativity	Torrance Test of Creative Thinking and Creative Personality Scale (EPC)
Marchena <i>et al.</i> (2017)	5 years (N= 60)	Musical Intelligence and Motor Creativity	Test of Creative Thinking in Action and Motion
Ortega <i>et al.</i> (2017)	3 to 4 years (N= 60)	Multiple Creativity and Intelligences	Tuttle Thinking Questionnaire
Vallverdu <i>et al.</i> (2016)	6 to 7 years	Multiple Creativity and Intelligences	Torrance Creative Thinking Test
Jarareh <i>et al.</i> (2016)	Pre-school age (N= 30)	Creativity and Aggression Control	Torrance Creativity Questionnaire
Krumm <i>et al.</i> (2015)	9 to 13 years (N= 359)	Creative Perception and Production	Torrance Creative Thinking Test
Krumm <i>et al.</i> (2013)	9 to 12 years (N= 219)	Parental Styles and Creativity	Torrance Creative Thinking Test Creative Personality Scale (EPC)
Krumm and Lemos (2012)	8 to 14 years (N= 301)	Artistic Activities and Creativity	Torrance Creative Personality Scale Test
Franco (2006)	5 years (N= 71)	Self-concept and creativity	Verbal Battery of the Torrance Creative Thinking Test

Table 3 refers to studies on creativity aimed at high school students. This table shows the most common variables related to creativity at these ages.

Table 3

Main characteristics of the selected studies in high school students

Author/s and year	Sample	Variables studied	Instrument used to measure creativity
Piya-amornphan <i>et al.</i> (2020)	14 to 17 years (N= 439)	Physical Activity and Creativity	Creative Thinking and Drawing Production Test (TCT-DP)



Author/s and year	Sample	Variables studied	Instrument used to measure creativity
Pérez-Fuentes <i>et al.</i> (2020)	High school students (N= 742)	Self-esteem, Emotional Intelligence, Personality, and Self-Expressive Creativity	Creative Behavior Questionnaire: Digital (CBQD)
Méndez and Fernández (2019)	11 to 16 years (N= 312)	Motor Creativity	Evaluation of Motor Creativity (ICM)
Zainudin <i>et al.</i> (2019)	High school students (N= 313)	Mathematical Creativity	A Mathematical Creativity Instrument
Pérez-Fuentes <i>et al.</i> (2019)	13 to 19 years (N= 742)	Digital Creativity, Parenting Style, and Academic	Performance Creative Behavior Questionnaire: Digital (CBQD)
Ramírez <i>et al.</i> (2019)	High school students (N= 100)	Runco Ideational Behavior Scale (RIBS)	Runco Ideational Behavior Scale (RIBS)
Caballero and Fernández (2018)	High school students (N= 59)	Creativity and Academic Performance	Creative Intelligence Test (CREA)
Mededovic and Dordevic (2017)	N= 251	Intelligence and Creativity	HEXACO-PI-R Inventory
Belmonte <i>et al.</i> (2017)	12 to 16 years (N= 670)	Intellectual aptitude, Emotional Intelligence, and Creativity	Subtest 3 of the Figurative Version of the Creative Thinking Test (TTCT)
Ramos <i>et al.</i> (2017)	15 to 16 years (N= 51)	Creativity, memory and academic performance	CREA Test
Castañeda <i>et al.</i> (2017)	16 y 17 years (N= 32)	Creativity, attention, academic performance and group interaction	CREA Test
Nakano <i>et al.</i> (2016)	8 to 17 años (N= 987)	Creativity and giftedness	High Ability Assessment Scale (BaAH/S)
Rodríguez <i>et al.</i> (2016)	High school students (N= 51)	Creativity and multiple intelligences	Turtle Creativity Questionnaire
Rico <i>et al.</i> (2016)	12 years (N= 59)	Creativity and Emotional Intelligence	Turtle Creativity Questionnaire
Nakano <i>et al.</i> (2016)	14-18 years (N= 83)	Creativity and Personality	Torrance Creative Thinking Test
Nakano <i>et al.</i> (2015)	8 to 16 years (N= 867)	Intelligence and Creativity	Battery for assessing Intelligence and Creativity
Esparza <i>et al.</i> (2015)	12 to 16 years (N= 78)	Scientific Creativity and Gender Differences and Educational Level	Scientific Creativity Skill Test



Author/s and year	Sample	Variables studied	Instrument used to measure creativity
Alonso <i>et al.</i> (2015)	12 to 18 years (N= 84)	Creativity, Academic Attention and Creative Performance	Intelligence Test (CREA)
Bermejo <i>et al.</i> (2014)	12 to 16 years (N= 98)	Scientific-Creative Thinking and Academic Performance	Scientific-Creative Thinking Test
Gontijo and Fleith (2009)	N= 100	Motivation and Creativity in Mathematics	Torrance Creative Thinking Test and the Creativity Test in Mathematics

Intelligence is the most widely studied term (Belmonte *et al.*, 2017; Mededovic and Dordevic, 2017; Nakano *et al.*, 2016; Nakano *et al.*, 2015; Pérez-Fuentes *et al.*, 2020; Ramírez *et al.*, 2019; Rico *et al.*, 2016; Rodríguez *et al.*, 2016) after academic performance (Alonso *et al.*, 2015; Bermejo *et al.*, 2014; Caballero and Fernández, 2018; Castañeda *et al.*, 2017; Pérez-Fuentes *et al.*, 2019; Ramos *et al.*, 2017). Other studies focus

on other variables related to motor (Méndez and Fernández, 2019; Piya-amornphan *et al.*, 2020), mathematics (Gontijo and Fleith, 2009; Zainudin *et al.*, 2019;), personality (Nakano *et al.*, 2016), and knowledge of the differences between gender and educational level (Esparza *et al.*, 2015).

Finally, Table 4 presents the studies whose sample are university students.

Table 4

Main characteristics of the selected studies in university students

Author/s and year	Participants	Variables studied	Instrument used to measure creativity
Novikova <i>et al.</i> (2020)	N= 128 with an average age of 18.67	Creativity and success in foreign language acquisition	Torrance Abbreviated Test for Adults (ATTA)
Ramankulov <i>et al.</i> (2019)	N= 73 2 Groups made up of 36 and 37 students s	Creativity and foreign language	Torrance Creativity Test
Caballero <i>et al.</i> (2019)	N= 206 with an average age of 21.33	Creativity, gender, age, and selection of the career	CREA test
Oseda <i>et al.</i> (2019)	N= 360 360 students from 5 universities	Emotional intelligence, self-efficacy, and creativity	Armitage and Conner's creative ability measurement tests
Tehranineshat and Rakhshan (2018)	N= 180 180 students: 120 Bachelor's and 60 Master's Degree	Knowledge management and creativity	Randsip Creativity Questionnaire
Elisondo <i>et al.</i> (2018)	N= 132 132 Students from 17 to 40 Creativity, I	Leisure and academic performance	CREA Test Creative Actions Questionnaire (CAC)



Author/s and year	Participants	Variables studied	Instrument used to measure creativity
Kuan-Chen (2018)	N= 139 with an average age of 21.11	Creativity, Creative Products and Cognitive Style	Semantic Creative Product Scale (CPSS)
Núñez-Martínez (2017)	N= 171 171 Students	Creativity and Academic Performance	Torrance Creative Thinking Test
Rodríguez <i>et al.</i> (2015)	N= 360 30 Students	Creativity and Emotional Intelligence	CREA Test
Gutiérrez <i>et al.</i> (2013)	N= 197 with an average age of 24.37	Thinking style, Metacognitive Strategies, and Creativity	Creative Intelligence Test (CREA)
Elisondo <i>et al.</i> (2009)	N= 132 with an average age of 21.60	Intelligence and Creativity	Creative Intelligence Test (CREA)

As in previous ages, intelligence is also the most studied variable in relation to creativity (Elisondo *et al.*, 2009; Oseda *et al.*, 2019; Rodríguez *et al.*, 2015).

The other variables studied are acquisition of the foreign language (Novikova *et al.*, 2020; Ramankulov *et al.*, 2019), academic performance (Elisondo *et al.*, 2018; Núñez-Martínez, 2017), thinking styles (Gutiérrez *et al.*, 2013; Kua-Chen, 2018), knowledge management (Tebranineshat and Rakhshan, 2018), and differences between gender, age, and selection of the career (Caballero *et al.*, 2019).

4 Discussion and conclusions

Creativity is a complex concept that has been present since the very beginning of humanity and that, although there is not yet a widespread concept about this term, many researchers have studied the characteristics of this construct (Barbachán *et al.*, 2020; Caeiro-Rodríguez, 2018; Corbalán, 2008; Garaigordobil, 2003; Gómez *et al.*, 2017; Guilford, 1980; Hammershøj, 2014; Hernández *et al.*, 2015; Morales-Valiente, 2017; Sánchez *et al.*, 2016; Torrance, 1969).

Different types of creativity can be considered (Dow and Mayer, 2004; Fuentes and Torbay, 2004). Thus, certain authors such as Aqueveque and Romo (2018) and Labarthe and Vásques (2016) have carried out certain strategies to pro-

mote creativity and the development of social skills (Cuetos *et al.*, 2020).

When conducted this systematic review, it was possible to verify which variables are the most studied in research on creativity in relation to the educational level. The most studied variable in all academic phases related to creativity is intelligence (Belmonte *et al.*, 2017; Elisondo *et al.*, 2009; Gatica and Bizama, 2019; Mededovic and Dordevic, 2017; Nakano *et al.*, 2016; Nakano *et al.*, 2015; Ortega *et al.*, 2017; Oseda *et al.*, 2019; Pérez-Fuentes *et al.*, 2020; Ramírez *et al.*, 2019; Rico *et al.*, 2016; Rodríguez *et al.*, 2016; Rodríguez *et al.*, 2015; Salavera *et al.*, 2019; Vallverdú *et al.*, 2016). This relationship is linked to how creativity is present in learning and in the processes involved in the creation of new knowledge (Elisondo and Donolo, 2016).

Another variable is academic performance (Alonso *et al.*, 2015; Bermejo *et al.*, 2014; Caballero and Fernández, 2018; Castañeda *et al.*, 2017; Elisondo *et al.*, 2018; Lamana and Peña, 2018; Núñez-Martínez, 2017; Pérez-Fuentes *et al.*, 2019; Ramos *et al.*, 2017).

The other variables studied are not common at different academic levels. Research in pre-school and elementary school focuses on aspects such as plastic arts, emotions, confidence, personality, aggression control, creative production, parental styles, and self-concept (Fazaie and Ashayeri, 2018; Franco, 2006; Hernández



et al., 2020; Jarareh *et al.*, 2016; Krumm *et al.*, 2018; Krumm *et al.*, 2015; Krumm *et al.*, 2013; Krumm and Lemos, 2012; Marchena *et al.*, 2017; Trowsdale *et al.*, 2019),

On the other hand, those related to high school students emphasize motor, mathematical problem solving, personality and gender and educational differences (Esparza *et al.*, 2015; Gontijo and Fleith, 2009; Méndez and Fernández, 2019; Nakano *et al.*, 2016; Piya-amornphan *et al.*, 2020; Zainudin *et al.*, 2019).

Finally, studies of university students relate to foreign language acquisition, thinking styles, knowledge management, and differences between gender, age, and selection of the career (Caballero *et al.*, 2019; Gutiérrez *et al.*, 2013; Kua-Chen, 2018; Novikova *et al.*, 2020; Ramankulov *et al.*, 2019; Tebranineshat y Rakhshan, 2018).

Each paper reveals the importance of creativity in the educational context no matter the age or educational stage of students; certain creativity-related strategies can maximize skills and thus make young people to be active and able to make decisions or solve problems on their own (Canelo *et al.*, 2015; Casado and Checa, 2020). Some of these resources to consider are the existing instruments to identify this capacity, such as CREA Test (Corbalán *et al.*, 2003).

Finally, it is worth noting that intelligence and academic performance are the most studied variables in relation to creativity at all academic levels. However, each educational stage is linked to different elements in the development of creativity, which may be determined by the needs of each educational period. Thus, the early stages of school focus more on creativity with more emotional aspects, and as the courses increase, they focus more on knowledge. Some of the limitations of this systematic review are that some of the studies initially selected did not provide information on the instrument used to measure creativity in the participants, so they had to be excluded later.

Future research can consider the evolution of studies on creativity, which, as mentioned in

the results, have increased significantly in the last decade. Some of the interpretations that can be made on this topic is that researchers now have greater access to studies thanks to new technologies, favoring the search for information. In addition, other practical implications of this work are to analyze the different instruments used to measure creativity. To conclude, it is necessary to highlight the need to know the instruments and the variables involved in the resources to determine creativity; thus, being able to develop appropriate interventions with students to strengthen this capacity and to train skilled young people in the solution of problems in their daily lives.

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Creative education and social justice: a systematic review focused on the Latin American context

Educación creativa y justicia social: una revisión sistemática orientada a conocer el contexto latinoamericano

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Abstract

Teaching institutions and practices must change in order to open education to creativity, which is an effective resource for initiating processes of social transformation. In the present study, we set ourselves the goal of describing how creativity is developed in research in the field of Education for Social Justice around the world, paying special attention to the Latin American context. To achieve this goal, we conducted a systematic review of the international literature produced between 2015 and 2020. Following the model of Sanchez-Meca (2010), we searched the databases of Web of Science and Scopus, the SciELO citation index, and Dialnet's web portal. We identified a potential corpus of 68 publications, which, when applying exclusion criteria, were expressed in a final corpus of 16 publications. We analyzed and characterized the documents in a theoretical and descriptive way, focusing specifically on the concept of creativity they presented. We observed nine empirical investigations, corresponding to experiences or projects in formal (n=8) and non-formal education (n=1), and seven theoretical reflections. We identified two approaches to address creativity: one where it was presented as a diffuse concept, which was not developed theoretically, and another where it appeared as a concrete concept, theoretically developed, and justified. Finally, Latin America shows three examples, which points to a lack of research that associates creativity as a factor for social and educative justice.

Keywords: Social justice, creativity, development education, democracy, Latin America, social inequality.

Resumen

Las instituciones de enseñanza y prácticas educativas deben cambiar para abrir la educación a la creatividad, la cual es un recurso eficaz para iniciar procesos de transformación social. En el presente estudio, nos pusimos el objetivo de describir cómo se desarrolla la creatividad en investigaciones del campo de la educación para la justicia social en el mundo, prestando especial atención al contexto latinoamericano. Para lograr este objetivo, realizamos una revisión sistemática de la literatura internacional producida entre 2015 y 2020. Siguiendo el modelo de Sánchez-Meca (2010), realizamos búsquedas exhaustivas en las bases de datos Web of Science, Scopus, el índice de citas SciELO y en el portal Dialnet. Identificamos un corpus potencial de 68 publicaciones que, tras aplicar criterios de exclusión se expresaron en un corpus final de 16 publicaciones. Analizamos y caracterizamos los documentos de forma teórica y descriptivamente, centrándonos específicamente en el concepto de creatividad que exponían. Observamos nueve investigaciones empíricas, que corresponden a experiencias o proyectos en educación formal (n=8) y no formal (n=1), y siete reflexiones teóricas. Identificamos dos enfoques para abordar la creatividad: uno donde se presentaba un concepto difuso, que no se desarrollaba teóricamente, y otro donde aparecía como un concepto concreto, desarrollado y justificado teóricamente. Finalmente, Latinoamérica muestra tres ejemplos, lo que señala una carencia de investigaciones que asocian la creatividad como factor para la justicia social y educativa.

Descriptores: Justicia social, creatividad, educación para el desarrollo, democracia, Latinoamérica, desigualdad social.

1 Introduction

Education as a social action is essential for the development of societies (Latapí, 1993). Therefore, educational institutions must change radically toward creativity and freedom to fight inequities and inequalities, since education requires teachers to constantly challenge reality (Freire, 1988). Education for social justice encourages students to take an active role in their own education and support teachers in the creation of democratic and critical educational environments (Ayers *et al.*, 2009), therefore, it is one of the main elements of social transformation.

Social justice education issues are diverse, but the equitable distribution of resources and social responsibility of individuals are especially important (Heather, 2006). When educating from justice, students are encouraged to critically examine oppression in the institutional and cultural arena and to generate thoughtful responses and new actions, maintaining hope, and above all, creativity (Heather, 2006, p. 106).

Addressing the concept of creativity as an important factor in education for social justice is complex, because of its polysemic nature (Campos and Palaces, 2018). To understand this relationship, it is necessary to move away from the classical conception of creativity—associated with talent and arts—to define it as an attitude that is developed or inhibited by the context, and that, as a human characteristic, can be taught and learned in all educational contexts (Romero, 2010).

This research presents some of the findings of a systematic literature review that analyzes the concept of creativity in the field of social justice education. The research aims to describe how the international scientific community has conceived the development of creativity in the framework of education for social justice, and to define the specific contributions of Latin American authors.

2 Theoretical framework

2.1 The role of education for a fair society

Different theorists in the modern world have mentioned the power of education for a fair society (Carlisle *et al.*, 2006; Kumashiro, 2013; Hytten, 2015; Sleeter, 2015; Sensoy and DiAngelo, 2017). We speak of education oriented to the approach of rights and that promotes educational institutions that concentrate on the collective demands of societies aware of the difficulties that (re)produce the systems of power (Dubet, 2014).

There is a variety of proposals to generate social justice in education. Bell (1997) points out that it is pivotal to: have full and equal participation of all groups in a society that is seeking to meet its needs; guarantee a democratic, inclusive, and participatory process; and ensure an equitable distribution of resources that guarantees the security of those who need them.

Other authors such as Ayers *et al.* (2009) indicate that education for social justice is not effective without three fundamental elements: “equity, activism, and social literacy” (p. 16). However, despite the theoretical development of education in equality and equity, we are still seeing a reality that is far from the proposals. Diesterweg (1956), points out that educational theory is separated from practice, and therefore, is ineffective and inappropriate to consider the true meanings of social equality.

In this context, what can educators and institutions involved in education for social justice do? Choules (2007), provides an idea: we must strengthen people’s understanding of the world around them, so that they can identify the root of social problems, cultivate their imagination, and collaborate with others to live fuller lives.



2.2 Creativity and social justice

According to a traditional understanding, creativity belongs to a few talented, brilliant, or unique people. This idea focuses on the concept as a synonym for innovation or artistic development (Caerols, 2013). However, according to Vygotsky (2004), creativity is an essential condition in every person's life, and everything that goes beyond routine is because of the creative process.

Kay (1991) defines creative thinking in education as “a process in which the individual encounters, defines or discovers an idea or problem not predetermined by the situation or task” (p. 234). However, seeking solutions to problems in the teaching-learning process is not a new topic: the difficulty of these methodologies arises when the emphasis is placed on immediate or single-answer solutions that hinder lateral thinking (De Bono, 1991). Thus, problem solving has a relationship with human creativity, and the discovery of problems is also an inherent factor in the search for alternatives to social justice.

20th century research findings indicate that people learn more significantly when working on real-world problems (Okuda *et al.*, 1991; Zhang *et al.*, 2011). Therefore, is it a good educational alternative to give creativeness the freedom to challenge what is already established and thus generate choices consistent with social equality? To answer this question Vygotsky (2004) points out that it is imperative to consider creativity in educational processes.

Some leading theorists state that generating creative educational practices for social justice is impossible in education systems that are inflexible, authoritarian, and self-centered (Berg *et al.*, 2009). The authors emphasize that the development of critical and creative perspectives in the training of students is essential. If empathy and the resolution of common problems are added, we are facing a unique alternative for a committed training, so a creative education is fundamental.

2.3 Inequity in Latin America and its impact on Education

In Latin America, the people who make up the poorest 20% live on 4% of each country's total income, and there are currently more than 160 million people who do not have the means to cover their most basic rights (World Bank, 2020). The situation is even worse (CEPAL, 2016) since there has been a widespread stagnation in poverty reduction since 2012.

The impacts of inequity have disrupted the development of education systems (Duarte *et al.*, 2009). Despite the progress made toward the completion of compulsory education, there are still inequalities related to the socio-economic level of households (CEPAL, 2016; 2018). The completion of High School—the minimum level to break the cycle of poverty— corresponds to 82% of young people from 20 to 25 years old from the highest-income quintile, but those in the lowest income quintile only reach 35% (CEPAL, 2018).

According to studies of large populations (Hangartner and Delgado, 2019; Bergsmo *et al.*, 2012), there is a strong negative perception of distributive justice in Latin America: 80% of people perceive injustices. But the biggest concern is that this phenomenon is associated with a high degree of mistrust of political institutions and the state. In 2011, six out of ten Latin Americans trusted little or nothing in the state as guarantor of justice and equality (CEPAL, 2012, p. 97).

Latin America is an example of how social problems directly impact on education. Hence, Tedesco (2017) points out that it is necessary to “introduce a special type of learning experiences that demonstrate effectiveness in moving variables of justice in the educational system” (p. 210). These experiences must be developed with commitment and creativity, considering that education is a means for social change.

Education for social justice does not require awareness of the inequities of social structures and the generation of actions that tend



to reduce discrimination against people regarding their identity and nature (Dubet, 2014). There are different methodological approaches that take on this challenge (Murillo and Hidalgo, 2015; Blasco-Serrano *et al.*, 2019). However, after conducting an exhaustive search, no systematic reviews have been found that address the use of methods or elements of creativity to educate from a perspective of justice and social transformation.

3 Methodology

The study has two objectives: the first is to collect and analyze scientific evidence published in international databases in the last five years, which addresses creativity in the field of education for social justice. The second objective is to recognize and describe the contributions of publications made in Latin America, considering the inequalities and inequities of this territory.

The following research questions were asked to address these objectives:

- What type of studies are these and what characteristics do they have?
- How do they approach the concept of creativity?
- How do they establish relationships between the concepts of creativity and social justice?
- What are their main findings and conclusions?
- What are the contributions of studies coming from Latin America?

3.1 Approach

We conducted a systematic peer review of the literature, following Sánchez-Meca model (2010) and some PRISMA's recommendations (Moher *et al.*, 2009).

3.2 Eligibility criteria

The basic selection criteria established to obtain data required:

Publications should:

- Be the result of empirical research, literature reviews, reflections, or theoretical analysis.
- Refer to the field of research in Education for Social Justice.
- Contain the concepts "Education", "Creativity" and/or "Creative Education".
- Be published in indexed journals.
- Be published from 2015 to 2020.
- Be written in English, Spanish and/or Portuguese.

3.3 Sources of information and search

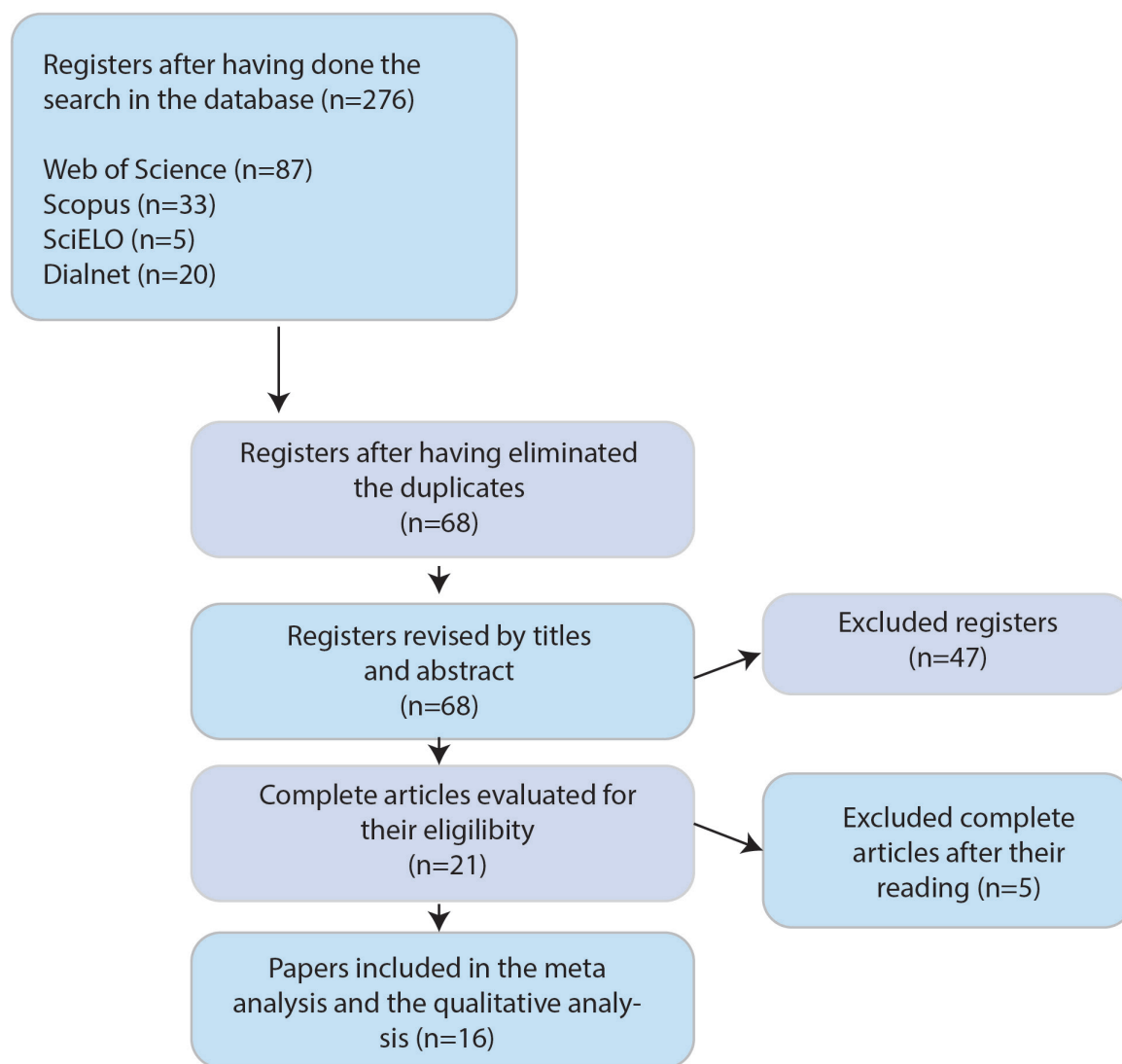
The search was done from 2015 to 2020 in the main Web of Science collection, Scopus bibliographic database and SciELO citation index in the Social Science research domain. The keywords "social justice", "education", "creativity", "creative education" and "education for social justice" were used. Additionally, a review of publications in Spanish was done on the dialnet portal using the keywords "social justice", "education" and "creativity".

3.4 Study selection

The overall keyword search generated a total of 276 research across all databases. After the review of duplicates, a corpus of 68 results was established. Each document was reviewed independently by title and abstract, and finally the full text. At the end of the revisions, 16 documents met the inclusion criteria and were analyzed:



Figure 1
Flowchart of study selection (PRISMA)



3.5 Analysis procedure

The data extraction method for the analysis consisted of a detailed revision of the texts based on the research questions.

Once the information was obtained, it was systematized using Atlas Ti. software. V8. During this process, an independent coding was developed for each researcher, following the indications of Sánchez-Meca "Coding Manual" (2010). From the reports generated by each



researcher, a synthesis of the information was made and divided into two main dimensions: 1) Publications with a concept of creativity defined theoretically and 2) Publications with a concept of creativity not defined theoretically.

4. Results

4.1 Selection of studies

The studies selected for the review (16) are summarized in Table 1:

Table 1

Articles included in the systematic review

N	Authors	Year	Title	Type of research	Country
1	LaDuca, B., Carroll, C., Ausdenmoore, A., Keen, J.	2020	Pursuing Social Justice through Place-Based Community Engagement: Cultivating Applied Creativity, Transdisciplinarity, and Reciprocity in Catholic Higher Education	Empirical qualitative	EEUU
2	Silva, A. N. D., Senna, M. A. A. D., Teixeira, M. C. B., Lucietto, D. A., & Andrade, I. M. D.	2020	O uso de metodologia ativa no campo das ciências sociais em saúde: Relato de experiência de produção audiovisual por estudantes	Empirical qualitative	Brazil
3	Nganga, L.	2019	Preservice teachers' perceptions and preparedness to teach for global mindedness and social justice using collaboration, critical thinking, creativity and communication (4cs)	Empirical qualitative	EEUU
4	García, O.M., & Ciges, A.S.	2019	¡Con mucho arte! Intervención psicopedagógica para la justicia social desde la transformación socioeducativa.	Empirical qualitative	Spain
5	O'Shea, J., & McGinnis, E.	2019	'Do you really want me to tell ya!' critical learning in engaging young people in contact with the justice system as peer educators with social work students	Empirical qualitative	Ireland
6	Paone, T.R., Malott, K.M., Pulliam, N., & Gao, J.	2018	Use of Photovoice in Processing Race-Based Topics in a Multicultural Counseling Course	Empirical qualitative	EEUU
7	Hatton, K.	2017	A critical examination of the knowledge contribution service user and carer involvement brings to social work education	Theoretical review	England
8	Wehbi, S.	2015	Arts-Informed Teaching Practice: Examples from a Graduate Anti-Oppression Classroom	Empirical qualitative	EEUU
9	D'Ambrosio, B. S., & Lopes, C. E.	2015	Insubordinação Criativa: um convite à reinvenção do educador matemático	Theoretical review	Brasil
10	Sava, M., & Marin, V.	2017	Making a Difference in Education through Built Environment Education	Empirical qualitative	Rumania
11	Tilley, E.	2016	Creative activism in the university: A case study of curricular design and implementation	Empirical qualitative	New Zeland
12	Brown, S.	2015	Creativity, Social Justice and Human Rights within Adult Education	Theoretical review	EEUU



N	Authors	Year	Title	Type of research	Country
13	Morales, J.	2018	Aportes de Paulo Freire a la Investigación y a la Lectura Crítica	Theoretical review	Colombia
14	Mesa, M.	2019	La Educación para la Ciudadanía Global: Una apuesta por la Democracia	Theoretical review	Spain
15	Bruce-Davis, M. N., Gilson, C. M., & Matthews, M. S.	2017	Fostering authentic problem seeking: A step toward social justice engagement	Theoretical review	EEUU
16	Albert, S. P.	2019	Educación para la Paz, Creatividad Atenta y Desarrollo Sostenible	Theoretical review	Spain

4.2 Characteristics of studies

Studies were characterized by presenting a mix of theoretical experiences and perspectives. Nine of the publications describe strategies or methodologies applied in the classroom (empirical research) and seven conduct theoretical reflections (theoretical research).

4.2.1 Types of research and context

Empirical research corresponds to educational methodological experiences (n=5) and analysis of institutional projects (n=4) that are developed from a qualitative approach and use different research methods: García and Ciges (2019) and LaDuca *et al.* (2020) present action research; Wehbi (2015); Tilley (2016); Paone *et al.* (2018); García and Ciges (2019) and O'Shea and McGinnis (2019) present the case study; and Nganga (2019) present the phenomenological study.

Regarding the geographical location of empirical studies, specific educational situations in formal education (Silva *et al.*, 2020; LaDuca *et al.*, 2020; Nganga, 2019; O'Shea and McGinnis, 2019; García and Ciges, 2019; Paone *et al.*, 2018; Tilley, 2016; Wehbi, 2015) and non-formal education are analyzed (Sava and Marín, 2017).

A clear tendency to analyze urban environments is identified (94% of studies).

4.2.2 Participants and context

In relation to the characteristics of the participants, only the study by Sava and Marín (2017) conduct activities with children outside the school, the rest of the proposals are in university classrooms with students of different careers (social work, pedagogy, teaching, dentistry, theology).

Some of the studies (n=7) do not develop an in-depth description of the participants and only define their profile:

- In the study conducted by Silva *et al.* (2020), 40 people participate in a methodological problem-solving experience during an academic year.
- The experiences presented by Tilley (2016) include different strategies of creative activism with groups from 20 to 40 people, all identified by diverse profiles (gender, age, ethnicity, origin, etc.).
- O'Shea and McGinnis (2019) narrate the experience of ten students (women) of social work with 13 young offenders (one woman and 12 men). In this work young students and their co-workers reflect on the value of people through creative multimedia productions.
- 47 education students (46 women and one man) participate in Nganga study (2019). The objective of the researcher was to know the impact of the group's 4C (colla-



boration, critical thinking, creativity, and communication) training on their perceptions of social justice.

- García and Ciges (2019) describe the experiences of 60 students and four teachers of a master's degree in Psychopedagogy in different projects of inclusive artistic education.
- Wehbi (2015) shows some examples of integrating arts-based teaching in their social work classes. The 30 participants (not defined by sex or gender) have diverse profiles and experiences of marginalization.
- Paone *et al.* (2018) presents the experiences of a group of 20 Master's students in the educational area with the *Photovoice* learning strategy. In this example, 90% of young women identified their gender as female and 10% as male.

Theoretical research conducted thematic reviews focused on:

- Describe the importance of a theory of creativity, inclusion, and power in the training of social workers (Hatton, 2017).
- Analyze the contributions of creative insubordination to the attitudes and actions of mathematical professors (D'Ambrosio and Lopes, 2015).
- Describe the power of creative and artistic education in adults to generate personal changes that create social justice (Brown, 2015).
- Consult the assessment developed by Paulo Freire on critical reading and research as necessary activities in any educational process (Morales, 2015).
- Propose approaches to know the reality, based on the use of tools that combine rational, visual, emotional, and numerical aspects and that stimulate creativity for a global education for citizenship (Mesa, 2019).
- Examine the nuances around creative problem solving and how professors can generate and strengthen the skills of stu-

dents to engage in social justice (Bruce-Davis *et al.*, 2017).

- Describe how and explain why education for peace is an essential tool for achieving the Sustainable Development Goals set out in Agenda 2030 (Paris, 2019).

4.3 Concept of Creativity in publications

We identify two broad approaches that encompass creativity as the foundation for active education with the objectives of social justice.

4.3.1 Articles presenting a concept of creativity that is not defined in education for social justice

In two of the articles of this review (Silva *et al.*, 2020 and Paone *et al.*, 2018) the concept of creativity is mentioned in the keywords, in the title and/or the abstract, but with terminological or theoretical absences in the rest. In this type of document, creativity is presented as an adjective—among many others—of an educational strategy.

In four of the documents (Morales, 2015; Sava and Marín, 2017; García and Ciges, 2019; Mesa, 2019; LaDuca *et al.*, 2020) creativity is a product or by-product of a larger methodology, perspective, or theory, and it is defined theoretically and conceptually. Morales (2015) describes how the teaching reflection processes of Freire's critical pedagogy are promoters of creative and innovative actions. Mesa (2019) points out how citizenship education promotes creative actions necessary for the achievement of certain future objectives. Sava and Marín (2017) argue how collaborative architecture contributes to the development of competencies, such as critical thinking, teamwork, and the use of creativity. García and Ciges (2019) describe how the link between psychopedagogy and art is unclear and forces people to look equally and creatively at sociocultural relationships. And finally, LaDuca *et al.* (2020) present the advances of a commod-



ification creativity project (Garcés, 2015) that seeks to promote Catholic values among students at a private American university.

4.3.2 *Articles presenting a concept of creativity defined in education for social justice*

There are two types of proposals that express the value of creativity for social transformation in systematic review. Some present theories of creativity and relate them to educational theory, others mention methodologies or actions that promote creativity and define it conceptually by placing it in a paradigm consistent with social transformation.

The first group of texts consists of four proposals:

D'Ambrosio and Lopes (2015) conduct a theoretical review that shows how creative insubordination (Morris *et al.*, 1981) applied in the subject of mathematics helps to eliminate the negative burdens that have historically affected it. They also encourage teachers to generate practices other than those required with responsibility and creativity.

Tilley (2016) narrates methodological experiences based on creative activism. The author describes how the students in her class combined theater, multimedia resources, and creative writing to address the culture of raping at the university. Through a critical Freirean pedagogy, the author generates a concept of creativity directed to the expression of unfair realities and creates a formal denomination to her experiences: "socially engaged critical arts".

Bruce-Davis *et al.* (2017) examine concepts around creative problem solving (Kay, 1991) and how professors can create opportunities to develop and strengthen the skills students need to engage with. In this document, the authors explain how students are motivated to engage with the community and promote positive change, through the combination of social justice issues with the creative problem approach.

Paris (2019) generates a profound theoretical discussion on how education for peace looks

for *attentive creativity* committed to human affairs in the search for solutions to personal and social problems, considered essential to achieve the objectives of sustainable development.

The second group consists of the following publications:

Wehbi (2015) develops a series of examples of integrating practical arts-based teaching into social work classes. Students, through activities based on drawing and action, generated criticism and reflections on the knowledge, education, and practice of social work. The author bases her theory on the creative principles of social change (Helguera, 2011) and Kester's theory of creative vision and committed art (2004).

Brown (2015) conducts a theoretical analysis of how arts-based learning promotes creative learning attitudes in adults, while promoting social justice and human rights. For the author, creativity based on arts plays a liberating role: adults, castrated by a society, show personal expression, intuition, and integrated forms of knowledge (Lawrence, 2018).

Hatton (2017) provides a theoretical justification for demonstrating the importance of the participation of outsiders in the educational processes of future social workers. She begins by examining participation models, in which students are involved with people's needs. This participation, along with key methods of Freire's critical pedagogy, motivates students to activate creative processes to generate new alternatives. The author outlines a theory of creativity based on Vygotsky proposal (2004) where the participation of civil society in the formation of social workers is part of democratic struggles.

O'Shea and McGinni (2019) describe a small-scale project on the use of creative media by social work students to listen claims of vulnerable populations. The project seeks to enable young people to identify dominant paradigmatic approaches to assess the needs of people in prison and thereby generate critical creative dialogue (Fisher, 2013).



Nganga (2019) in her theoretical study justifies the importance of using 4C (Collaboration, Critical Thinking, Creativity and Communication) in the programs to teach knowledge and skills essential to global mentality and social justice. The author points to the need for an education that examines, disrupts, and replaces unfair and oppressive social structures (Sleeter and Grant, 2009) where creativity is essential. This author defines and characterizes creativity as a skill that facilitates the process of managing the complexities of globalization (Beghetto, 2006).

4.4 Research in Latin America

There are few published academic publications on creativity, education, and social justice in Latin America (18% of the corpus). Considering the last objective of the systematic review, a reflective and descriptive summary of these contributions is presented:

The concept of creative insubordination, main topic of the paper conducted by Brazilian researchers D'Ambrosio and Lopes (2015), was first presented by Morris *et al.* in 1981 (USA) in a document that showed the disobedience to bureaucratic activities of some school principals in Chicago. The study of the Latin American authors raises this theory and opens it up to the general educational system (especially the university), explaining how, by ethical choice, professors choose not to follow the norms, because they perceive that many of them do not guarantee the improvement and well-being of the educational community.

Creative insubordination is intended to guarantee that guidelines — for example, a rigid program — does not negatively influence educational actors and thereby limit access to quality education. Creative insubordination has not only been studied in school organization but has also been developed in methodologies (Haynes and Licata, 1995), when teachers identify injustices, they create activities that involve students in

their own conceptions of injustice. Here the creative process for problem solving arises in teachers, who observe that common practices do not respond to the effective right to quality education for all.

Jesús Morales, Colombian researcher and teacher, in his text *Paulo Freire's Contributions to Research and Critical Reading* (2015), dedicates a theoretical analysis to the proposals developed by Paulo Freire on critical reading and research as essential activities in any educational process.

While this text focuses more on those pedagogical actions that protect concepts of social justice, we observe an intrinsic integration on the importance of creativity in Paulo Freire's discourse on critical education.

Freire, analyzed by Morales, is the one who says that approaching knowledge does not respond to an automatic, reckless, solitary, or naive process. Freire appeals for the collective, because it is the interaction of people that leads to the discovery of new ways of building the world and reality. The Freireana theory (1991) promotes the use of thought in its critical, reflective, and analytical way, as fundamental instruments to autonomously transform knowledge, as well as to seek reasons and explanations for what happens in the context.

Silva *et al.* (2020), narrate a pedagogical experience that shows the use of Freire's problematization methodology. The students who participated in the research created documentary projects that were organized in two phases (audiovisual production and post-production) and aimed at presenting social problems in the dental area. The document focuses on arco's Maguerez strategy (Berbel, 2012) with the direct mediation of teachers. The final step in this project is to generate a reflection of reality. The teachers saw how the social, political, and ethical potential of their students changed to a creative process of structuring their own knowledge during the weeks of the activity.



5 Discussion and conclusions

Education that uses creative thinking with the aim of discovering alternatives for a fairer world is practiced from different approaches. In the articles that present experiences, professors who encourage students' creativity are distant from "traditional teaching" and defend non-mass methodologies that involve a renewal of the teaching-learning process (Silva *et al.*, 2020; Nganga, 2019; O'Shea and McGinnis, 2019; Paone *et al.*, 2018; Wehbi, 2015; Sava and Marín, 2017).

Like the previous topic, art, as the backbone of creative thinking (Pinard and Allio, 2005), was one of the alternatives most exposed by the authors: Brown (2015); Wehbi (2015); Tilley (2016); García and Ciges (2019) and Paris (2019), and it takes the process of human creation with aesthetic purpose as a channel of social transformation. In this context, Moreno (2016) states that art-based projects promote change, connecting the needs of the community with its resources and capacity for sustainability by respecting the environment. This proposal allows to see the social problems arising from social injustice and to reason critically about possible collaborative and creative solutions.

At this point, it is good to emphasize that fostering creativity does not necessarily have to involve art, since creativity has a transversal component to the activity, discipline or problem that is addressed. However, art can be an important vehicle for initiating and maintaining a creative attitude over time (Saura-Pérez, 2015).

Other factors analyzed—expressed in the proposals of Brown (2015); Wehbi (2015); Bruce-Davis *et al.* (2017); Paone *et al.* (2018); García and Ciges (2019); O'Shea and McGinnis (2019) and Paris (2019)—is the need of the teaching-learning process to provide students the space to generate ideas, the time to test them and to the moment to reflect and be aware of their environment.

Additionally, Karpova *et al.* (2011), state that curiosity, personal desires, and motivation

to seek answers to a problem involving the learners, are important for deep learning, otherwise people can think creatively but do not act to achieve the goal: in this case, become activists.

As suggested by some authors (Paris, 2019; Mesa, 2019), self-confidence is an important feature, influencing risk-taking and creativity behavior. The presence of a possible failure is a threat that is always present, and to increase the confidence of students Gómez (2007) advises that a welcoming and open classroom environment is needed.

This systematized review seeks —by identifying and characterizing the recently published literature on creativity and social justice— to contribute to the reflection on the importance of generating creative processes in education that aim at social transformation in the world. This need is necessary in Latin America, where inequality has become a problem that impacts the lives of millions of people on a daily basis.

However, this study has limitations, since it does not include proposals that are not published, published in non-academic media, or published in databases other than those selected. On the other hand, it does not present the articles in detail, since it develops a synthesis of their main characteristics in order to respond to the basic methodological criteria of a systematization of the information.

Therefore, researchers, professors, employees, and educational community, especially from Latin America, are invited to reflect on the findings, observe their own reality and contribute with examples, projects and/or experiences that can develop theories on the uses of creative thinking in education for social transformation. Currently, many people are making great changes from localized approaches, so making them visible is a necessity for socio-educational research.

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Self-efficacy, procrastination and academic performance in university students in Ecuador

Autoeficacia, procrastinación y rendimiento académico en estudiantes universitarios de Ecuador

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Abstract

The study of non-cognitive factors that influence academic success has become a topic of growing interest in educational research. This study aims to examine the relationship between university academic performance and two specific non-cognitive factors: self-efficacy and academic procrastination. Similarly, the mediating role that academic procrastination plays in the relationship between self-efficacy and academic performance is explored. The quantitative study was conducted in the Ecuadorian context and was based on a sample of 788 students enrolled in public and private universities in the Metropolitan District of Quito. The analytical strategy consisted on the formulation of path models, based on the methodology of structural equations. The main results of empirical analysis are: a) both self-efficacy and academic procrastination directly affect the academic performance of university students; b) self-efficacy is indirectly related to academic performance through academic procrastination; c) these non-cognitive factors are more determinant for student performance in the early career stage, especially in the case of private universities. Finally, the implications of the findings are discussed, considering the potential development of strategies to promote academic success through interventions that favor the students' sense of self-efficacy and self-regulation.

Keywords: Academic performance, academic procrastination, self-efficacy, academic self-regulation, university, Ecuador.

Resumen

El estudio de los factores no cognitivos que influyen en el éxito académico se ha posicionado como un tópico de creciente interés para la investigación educativa. En este marco, este estudio se propone el objetivo de examinar la relación entre el rendimiento académico universitario y dos factores no cognitivos específicos: la autoeficacia y la procrastinación académica. De igual manera, se explora el rol mediador que la procrastinación académica desempeña en la relación entre la autoeficacia y el rendimiento académico. El estudio cuantitativo se realizó en el contexto ecuatoriano y se basó en una muestra conformada por 788 estudiantes matriculados en universidades públicas y privadas del Distrito Metropolitano de Quito. La estrategia analítica consistió en la formulación de modelos de ruta, fundamentados en la metodología de ecuaciones estructurales. Los principales resultados del análisis empírico son los siguientes: a) tanto la autoeficacia como la procrastinación académica afectan directamente el desempeño académico de los estudiantes universitarios; b) la autoeficacia se relaciona indirectamente en el rendimiento académico a través de la procrastinación académica; c) estos factores no cognitivos son más determinantes para el desempeño estudiantil en la etapa inicial de carrera, especialmente en el caso de universidades privadas. Finalmente se discuten las implicaciones de los hallazgos efectuados, considerando el desarrollo potencial de estrategias de promoción del éxito académico mediante intervenciones que favorezcan el sentido de autoeficacia y la autorregulación del estudiantado.

Descriptor: Rendimiento académico, procrastinación académica, autoeficacia, autorregulación académica, universidad, Ecuador.

1. Introduction and state-of-the-art

The higher education system is a very important indicator for a country, since it represents the development and evolution in all areas of a society. With university education, governments face great challenges in meeting minimum quality standards (González-Zabala *et al.*, 2017). Significant changes have occurred in Latin America during the last decades: a growing demand of the population for accessing higher education, an increase in the number of universities, the opening of new careers and an increasing interest in the development of scientific research (Jara-López and Vargas-Jiménez, 2016). These trends have enabled countries to monitor Higher Education Institutions (HEIs) by establishing control mechanisms through an evaluation and accreditation process to measure the quality of the services they offer (Rojas, 2011; Véliz Briones, 2018).

This reality is not different in Ecuador, since the end of the 2000 a public policy was designed to guarantee free education in higher public education to standardize the process to access to it, and avoid a decrease in the quality of third-level curricula in general (Quinatoa-Andrango, 2019; Rojas, 2011). With the establishment of the Constitution of 2008, the National Secretariat for Higher Education, Science, Technology and Innovation (SENESCYT) was created, which is the agency in charge of implementing the National Leveling and Admission System (SNNA), which would be responsible for the evaluation and allocation of access quotas to undergraduate courses in public universities, applying the National Examination for Higher Education (ENES). Since then, and although it has undergone a series of modifications, the test has remained until now as a mandatory requirement for accessing public university (Zambrano, 2016).

Since its origin, this type of exam aimed to promote meritocracy through an admission process that enables students to reach a quota

in the career and public university of their choice, guaranteeing SNNA actors the right to access and stay in higher education under conditions of equal opportunity (Jara-López and Vargas-Jiménez, 2016). However, the objective has not been entirely achieved, because inequalities prevail in terms of preparation for the exam which normally favor aspiring graduates in private schools, who also tend to have more access to preparation courses (Tobar-Pesántez and Solano-Gallegos, 2018). As a result, a significant number of applicants, mainly from public high school, tend to experience difficulties in achieving a score that allows them to access their preferred career, having to choose an alternative career at public universities or to pay for a private university (Zambrano, 2016). Thus, private HEIs are places for students whose first career was not accepted or for those who did not reach the minimum score for any career (Ruiz *et al.*, 2018).

Since enrollment in private universities has increased, it is common to observe in these universities more acute problems related with vocational incompatibility when the student chooses the career. According to Zumárraga *et al.* (2017) in an investigation carried out in an Ecuadorian private university with a national scope, about 24% of students choose a career that does not fit their professional preferences. This vocational problem is one of the main factors affecting student drop-out in higher education (Castro *et al.*, 2016). Using figures from the World Bank, during the last decade university drop-out among people from 25 to 29 years old in Latin America and the Caribbean has been around 21%, rising to 30% in Ecuador (Ferreya *et al.*, 2017). In this sense, the specialized literature agrees to identify academic achievement as a fundamental determinant of student failure and drop-out of third-level programs (Díaz-Peralta, 2008).

While university academic performance has traditionally been related to the knowledge of academic content and skills of students, aspects that are largely conditioned by the quality



of the education taught at the middle level, an important part of this academic performance is explained by non-cognitive factors, which are also known as non-academic factors, soft skills, or psychosocial factors (Farruggia *et al.*, 2018). In this sense, the meta-analysis developed by Robbins *et al.* (2004), based on the accumulation of empirical evidence from 109 relevant research, reveals that non-cognitive factors explain the variability of academic performance in university students, compared with more traditional academic predictors such as socioeconomic status, average grade at school or standardized tests of knowledge. Also, non-cognitive factors show the advantage of being potentially malleable by concrete interventions, enabling educational institutions and professors to perform these individual attributes through different strategies, applied especially in the classroom context, thus improving the student's learning capacity (Allen *et al.*, 2009; Farrington *et al.*, 2012).

Although educational research has shown a growing interest in studying the relationship between non-cognitive factors and academic performance, as well as their practical implications, there are still gaps in knowledge about how these non-academic variables interact and intertwine to influence the academic performance of university students (Farrington *et al.*, 2012). Among the non-cognitive factors addressed by university-centered educational literature, self-efficacy and procrastination have not been deeply studied. Accordingly, this research aims to study the effect of self-efficacy and academic procrastination on academic performance in university students, in addition to exploring the mediating role that academic procrastination plays in the relationship between self-efficacy and academic performance. Likewise, setting Ecuador as a context, and considering the vocational problems of students when selecting the careers in private universities mostly, this paper proposes to investigate whether the relationship between self-efficacy, procrastination and academic performance

is modified by comparing early career students from public and private HEIs.

1.1 Self-efficacy, procrastination, and academic performance

The available literature has placed self-efficacy and procrastination as constructs that can significantly predict academic performance (Kim and Seo, 2015; Rodríguez-Durán and Barraza-Macías, 2017). According to the studies conducted, it is observed that a good academic performance cannot be guaranteed only by the intellectual capacity of individuals, since the action of other non-cognitive variables can determine a different performance in two students with the same degree of knowledge and academic ability (Ruiz-Dodobara, 2017).

Self-efficacy is defined as the beliefs that the individual has about his or her own abilities to act in the way required to achieve expected results (Bandura, 1997). At the educational level, self-efficacy is a strong predictor of university academic performance and consists of a student's personal belief in his or her ability to successfully carry out the academic activities required (Rodríguez-Durán and Barraza-Macías, 2017; Alegre, 2014). This would be reflected in students with a high level of self-efficacy expectations that would have high academic achievement, and vice versa (Contreras *et al.*, 2005). Similarly, students with a high level of self-efficacy will tend to evaluate their abilities in a positive way in the long term, leading to the completion of studies (Navarro-Charris *et al.*, 2017).

On the other hand, academic procrastination refers to the behavioral pattern characterized by postponing tasks, with the prior knowledge that such behavior will have negative effects or consequences (Angarita-Becerra, 2012). Similarly, the person who procrastinates experiences subjective discomfort (anxiety, guilt, etc.) when is aware of the consequences of having postponed (Przepiorka *et al.*, 2016).



In the university setting, procrastination has been considered as a type of irrational, deliberate and negative behavior for academic performance, mainly caused because the student does not like the task to be performed (Álvarez-Blas, 2010), reason for which the student will tend to postpone it until the end, and sometimes even failing to present it on time (Chan-Bazalar, 2011). Among the negative consequences of academic procrastination, repetition of subjects, low performance and academic dropout are common (Garzón-Umerenkova and Gil-Flores, 2017). In relation to the accumulation of empirical evidence, the meta-analysis carried out by Kim and Seo (2015), from 33 relevant studies on the subject, supports the existence of a negative and consistent relationship, especially in America and Europe, between academic procrastination and academic performance.

1.2 Self-efficacy and academic performance: The mediating role of academic procrastination

While empirical research has provided ample evidence of the effects that academic self-efficacy and procrastination have on academic performance, it still unknown how these non-cognitive factors articulate to affect the performance of university students (Farrington *et al.*, 2012). In this sense, the hypothesis is proposed regarding the fact that academic procrastination works as a mediator of the relationship between self-efficacy and academic performance. As discussed in the previous section, academic procrastination is a clear negative predictor of academic performance of university students, but in turn, this recurrent procrastination of academic tasks and responsibilities can be explained as the result of low levels of self-efficacy. There is a broad theoretical consensus on procrastinate behavior as a self-regulatory failure (Chan-Bazalar, 2011; Ferrari, 2001; Garzón-Umerenkova and Gil-Flores, 2017); however, this perspective is incomplete, since it does not consider the motivational

element that enables a person to self-regulate effectively and avoid incurring patterns of postponing activities.

In this point self-efficacy takes on a fundamental role because it is not enough to know about the cognitive and metacognitive strategies that can be used for implementing self-regulatory learning processes, but it is also necessary to be confident to execute them and sustain them in time (Bandura, 1997; Klassen *et al.*, 2008). In other words, for an individual to select appropriate learning strategies, assess his/her knowledge, self-monitoring, and understand the importance of using concrete strategies, he/she requires to believe that will be able to manage these elements to drive learning (Klassen *et al.*, 2008). In this sense, self-efficacy in self-regulation, understood as individual beliefs in one's own ability to employ self-regulatory strategies (Usher & Pajares, 2008), is key to enabling the student to meet the demands of academic training, favoring the systematic implementation of the skills and strategies necessary to control and organize the learning process more autonomously.

Thus, the lack of confidence in one's own ability to self-regulate leads to failures in the application of self-regulating learning strategies, which in turn leads to academic procrastination behaviors. Self-efficacy in self-regulation is a specific form of self-efficacy (Zuffianò *et al.*, 2013), and a positive association has been found between general self-efficacy and self-efficacy in self-regulation (Usher and Pajares, 2008).

Based on the theoretical argumentation, it is feasible to say that academic procrastination acts as a mediator of the relationship between self-efficacy and academic performance. The sense of self-efficacy allows the student to be able to carry out different activities and promotes motivation with respect to the self-regulated accomplishment of the tasks and action necessary to achieve their learning goals (Ruiz-Dodobara, 2017), which will lead to a less procrastinating behavior and therefore, a greater



probability of successful academic performance (Álvarez-Blas, 2010; Güngör, 2020).

Finally, when seeing the relationship between self-efficacy, procrastination, and academic performance in the university context of Ecuador, it is necessary to consider the differentiated reality that private and public universities experience regarding their enrollment and admission processes for new students, due to the current higher education access exam. As noted above, since private universities in many cases end up playing an absorptive role for those students who do not achieve a quota at the public university (Ferreira, 2017), these types of institutions face major vocational problems when choosing the career. The latter along with the difficulties related to the transition from High school to the university (Páramo-Fernández *et al.*, 2017), would allow us to assume that problems of academic performance, linked to issues of self-efficacy and academic procrastination intensify at the initial levels of university education, mainly in those students who belong to private institutions. The latter because a student who lacks vocational affinity when choosing the career is more likely to feel unself-effective in pursuing university studies (Wessel *et al.*, 2008). From the above, this work is considered to explore whether the relationship between self-efficacy, procrastination, and academic performance in the initial stage of career experiences changes when comparing public and private universities.

1.3 Hypothesis and research question

Hypothesis 1 (H1). Self-efficacy is positively related to academic performance.

Hypothesis 2 (H2). Self-efficacy is negatively related to academic procrastination.

Hypothesis 3 (H3). Academic procrastination is negatively related to academic performance.

Hypothesis 4 (H4). Academic procrastination acts as a mediator of the relationship between self-efficacy and academic achievement.

Question 1 (Q1): To what extent does the relationship between self-efficacy, academic procrastination and academic performance change when comparing public and private universities, considering the first levels of professional training?

2 Methodology

2.1 Participants and procedure

The empirical study involved the participation of 788 university students from the Metropolitan District of Quito (DMQ), Ecuador. The sampling procedure used was for convenience, with quotas for sex and type of university (public/private). Women accounted for 50.6 % of the participants and men for 49.4 %. The average age of the sample was 21.1 years ($SD = 2.77$). 56.3% of students belong to public universities and 43.7% to private universities. In addition, 72.7% of participants reported family incomes less than or equal to 1576 USD (4 unified basic wages). Regarding the level of studies completed, 56.0 % of the participants enrolled in the first career levels (1-3), 27.9 % in the middle levels (4-6) and the remaining 16.1 % were in the final stage of their respective programs (7° level onwards).

The data collection was carried out by applying a general survey on academic behavior developed by the Educational Innovation Group on Vocational and Professional Guidance of the Salesian Polytechnic University, Quito (GIEOVP-UPS). The information was collected on a face-to-face basis during June and July 2019. In relation to ethical considerations, this research considered some steps to protect the rights of the participants, in accordance with the ethical principles established by the American Psychological Association (2017). These actions consisted of ensuring the anonymity of the participants by avoiding requesting names or any other identifying information,



establishing and communicating the appropriate confidentiality protocols regarding the management of the obtained data, telling the objectives of the study, and requesting prior voluntary consent.

2.2 Instruments and measurements

Self-efficacy: It was measured using the General Self-Efficacy Scale (GSS) formulated by Alegre (2013), which consists of 20 items related to self-perception that, in general terms, a university student has about his or her abilities to develop appropriate actions for the achievement of goals or the resolution of problems. The GSS has a Likert-type rating scale with response options ranging from 1 [strongly disagree] to 5 [strongly agree]. By confirmatory factorial analysis (CFA), it was verified that the GSS has a one-dimensional structure, obtaining an adequate adjustment to the data for a unifactorial model: $\chi^2 = 942.81$ [$p < .001$]; comparative fit index [CFI] = .91; incremental fit index [IFI] = .91; adjusted goodness-of-fit index [AFI] = .86; standardized root of mean quadratic residue [MRR] = .043.¹ The self-efficacy index was obtained by calculating the sum of the GSS item scores (theoretical range [20-100]; $M = 77.63$; $DT = 11.97$; $\alpha = .94$).

Academic procrastination: The Academic Procrastination Scale (APS) developed by Busko (1998) was used to measure it, considering the version validated for the Ecuadorian university context by Zumárraga-Espinosa and Cevallos-Pozo (2021). In this case, the APS consists of 12 reagents: three for the postponement of activities dimension and nine for the academic self-regulation factor. APS items use a 5-point rating scale, ranging from 1 [never] to 5 [always]. According to the CFA conducted, the APS has factorial validity for the two-dimensional model, with satisfactory goodness-of-fit indicators reported: $\chi^2 = 234.61$ [$p < .001$]; CFI = .93; IFI = .93; AGFI = .93; SRMR = .066. Considering the recommendations of Zumárraga-Espinosa and Cevallos-Pozo (2021) for the use of an

overall academic procrastination index, this index was obtained combining the two dimensions of the APS, i.e., adding the score of all the items that make up the APS (theoretical range [12-60]; $M = 29.78$; $SD = 6.94$; $\alpha = .81$).

Academic performance: Students' self-perceived academic performance was consulted through a reagent. To this end, the participating group was asked to rate its own performance during the last two academic periods, looking at a rating scale ranging from 1 [very bad] to 10 [outstanding] ($M = 7.49$; $SD = 1.13$).

Control variables: Sex, age, and family income reported by participants are included as control variables when empirically evaluating the hypothesis and research question of the study.

2.3 Data analysis

The analytical strategy used consisted of the formulation of path models, based on the methodology of structural equations, and an estimation of parameters for maximum likelihood. In line with this statistical technique, a hypothetical-theoretical model of simple and partial mediation is proposed to examine the hypotheses and research question. In all analyzes of structural equations, the variables of interest were residualized with respect to the control variables (sex, age, family income) to neutralize potential confusing effects that may distort the results. The bootstrap method was used to estimate indirect effects, using 5000 subsamples and 95 % confidence intervals. The statistical software used was AMOS 23.

¹ The recommended benchmarks for concluding that a factor model has an acceptable fit to the data are CFI, IFI $\geq .90$; AGFI $\geq .85$; SRMR $< .10$ (Byrne, 2010; Schermelleh-Engel et al., 2003).



3 Results

3.1 Self-efficacy, academic procrastination, and academic performance

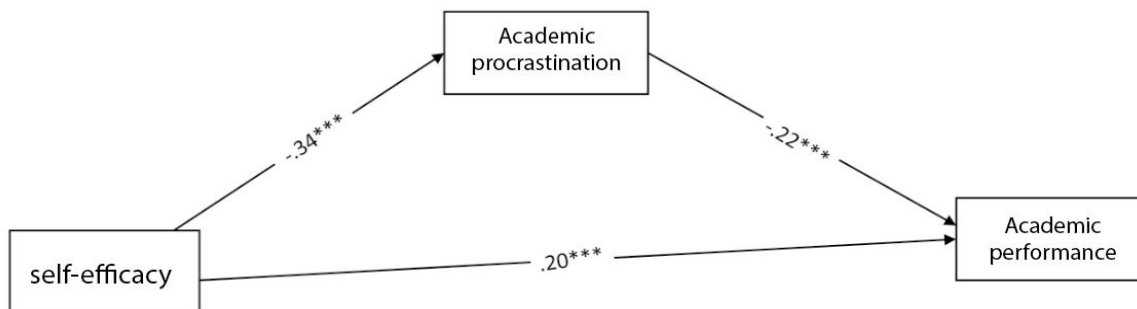
The route model results, corresponding to the hypothetical mediation model, are shown in Figure 1. The goodness-of-fit indicators show that the model has an acceptable fit to the data: $\chi^2 = 21.25$ [$p < .01$]; CFI = .93; IFI = .94; AGFI = .97; SRMR = .035. First, self-efficacy is positively and significantly related to academic performance ($\beta = .20$; $p < .001$), which provides empirical support for H1. As expected, college students who feel more confident in their abilities to solve

tasks of any kind tend to report higher levels of academic achievement. In contrast, self-efficacy is negatively related to academic procrastination ($\beta = -.34$; $p < .001$). In this way, it was observed that the most self-efficacious participants are less likely to procrastinate intensely, compared to those with low levels of self-efficacy (H2 verified).

On the other hand, H3 also has empirical support since academic procrastination produces a negative and significant effect on academic performance ($\beta = -.22$; $p < .001$), which implies that students who procrastinate more frequently in relation to their academic tasks and activities, tend to report poorer academic performance

Figure 1

Partial simple mediation model: Self-efficacy, academic procrastination, and academic performance



Note: *** $P < .001$. Standardized regression coefficients are reported. Model goodness-of-fit indicators: $\chi^2 = 21.25$ [$p < .01$]; CFI = .93; IFI = .94; AGFI = .97; SRMR = .035. R^2 Academic procrastination = 12.5%; R^2 Academic performance = 13.5%. Data collected by GIIQVP-UPS, Campus-Quito, during 2019.

Regarding H4, the analysis of the indirect effect of self-efficacy on academic performance, mediated by academic procrastination, yielded a positive and significant result ($\beta = .075$; $p < .001$). Since this is a standardized indirect effect, if a student's self-efficacy increases by 1 standard deviation, this will result in an approximate increase of .075 standard deviations in academic performance, through a reduction in academic procrastination.

3.2 The case of early learners: a comparative reading by type of university

To explore whether relationships of interest change in the initial stage of university education, the analysis concentrated only on those students who were completing the first three levels of their university careers ($N = 441$). Considering this subsample, the route model evaluated in the



previous section was again executed, differentiating between students from public and private universities. Thus, the results of the analysis conducted are presented in Figure 2.

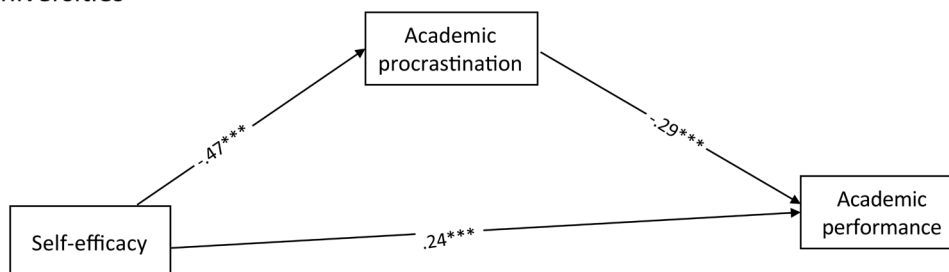
Regarding PI, it can be observed that, on average, the relationships among the variables of interest tend to intensify in the early career levels, especially in the case of the direct effect of self-efficacy on academic performance. In addition, considering the initial stage of the career, the indirect effect of self-efficacy on academic

performance, mediated by academic procrastination, is higher in university students belonging to private institutions ($\beta = .136$; $p < .001$), compared to those studying in public universities ($\beta = .045$; $p < .05$). This suggests that the direct and indirect impact (through academic procrastination) of self-efficacy on academic performance is more important in the initial stage of university education, mainly in those who study in private universities ($R^2 = 20.9\%$).

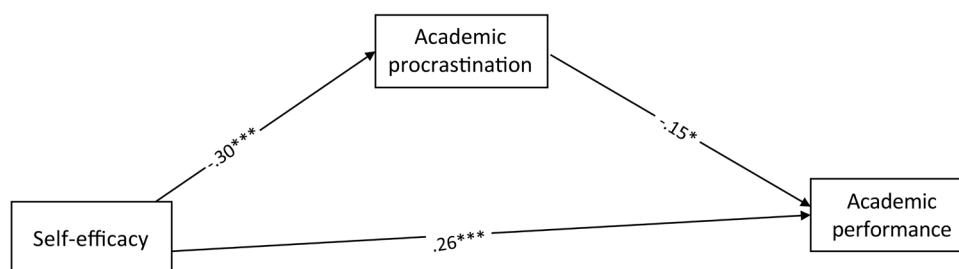
Figure 2

Route models: private universities vs. public universities: initial phase of the career

Private universities



Public universities



Note: $***p < .001$; $*p < .05$. Standardized regression coefficients are reported. Private universities [N = 198]: $\chi^2 = 14.42$ [$p < .05$]; CFI = .92; IFI = .92; AGFI = .92; SRMR = .059; R^2 Academic procrastination = 24.0%; R^2 Academic performance = 20.9%. Public universities [N = 243]: $\chi^2 = 16.26$ [$p < .05$]; CFI = .84; IFI = .86; AGFI = .93; SRMR = .056; R^2 Academic procrastination = 11.4%.

Data collected by the GIEOV-UPS, Campus-Quito, during 2019.



4 Discussion and conclusions

This research aimed to study the relationship between self-efficacy, procrastination, and academic performance among university students in Ecuador. In this respect, the findings derived from the analyzed partial simple mediation model in Ecuador, provide confirmatory evidence for the existence of direct relationships between self-efficacy and academic procrastination (negative), self-efficacy and academic performance (positive), as well as between academic procrastination and academic performance (negative), as expected according to the theoretical approaches initially addressed. The reported positive relationship between self-efficacy and academic performance is consistent with H1 and agrees with recent empirical studies at the university level, such as those of González-Cantero *et al.* (2020) and Sadoughi (2018). Similarly, H2 could be empirically supported from the negative and significant result for the effect of self-efficacy on academic procrastination, which is compatible with evidence recently provided by educational literature (Dike and Emmanuel, 2019; Güngör, 2020). Regarding the negative relationship between academic procrastination and academic performance stated in H3, the favorable statistical results presented, which made it possible to verify this hypothesis, are equal and are added to the results reported by recent empirical research (Hidalgo-Fuentes *et al.*, 2021; Pekpazar *et al.*, 2021).

However, in addition to the direct effects found, this paper was intended to contribute to the understanding of how self-efficacy and procrastination relate to each other to affect university academic performance. In this sense, the results of the mediation analysis carried out are consistent with the thesis that places academic procrastination as a mediating variable of the relationship between self-efficacy and academic performance, empirically supporting hypothesis 4 (H4). Thus, in addition to directly influencing the academic performance of university students, the sense of self-efficacy also produces an indirect effect through academic procrastination. In other

words, part of the negative impact of low levels of self-efficacy in university academic performance is explained by the fact that a weak sense of self-efficacy leads to intensifying procrastinatory behaviors that lead to worsening academic scores and outcomes.

On the other hand, although this study provides evidence in favor of the mediating role of academic procrastination in the relationship between self-efficacy and academic performance, the empirical exploration of this mediating relationship is still quite insidious in the current literature. However, the approach to this mediation hypothesis was based on theoretical (developed in the initial section) and empirical bases.

On the empirical side, the basic criteria established by the specialized literature were taken as reference, which indicate that for a mediation relationship to happen, it must be verified that the independent variable influences the mediator, and that, in turn, the mediator affects the dependent variable. In this respect, the relationship between self-efficacy and academic procrastination, assuming a causal direction that positions self-efficacy as a determinant of procrastination, is supported by strong empirical longitudinal evidence (Ziegler and Opdenakker, 2018). The same is true of the causal relationship between academic procrastination and academic performance, also supported by cumulative evidence of meta-analytical (Kim and Seo, 2015) and longitudinal cut (Gareau *et al.*, 2019).

Additionally, the data suggest that self-efficacy and academic procrastination have a greater impact on the academic performance of those students in the early career levels. Similarly, the results of the comparative analysis indicate that the academic performance of students in private HEIs tends to be more affected by individual attributes such as self-efficacy and academic procrastination in the specific case of the initial stage of vocational training. The latter agrees with the approach about the vocational mismatches that adversely affect career choices made by students from private universities, due to the selectivity of the process of access to public higher education (via acceptance exam) and the dynamics of



absorption that of private universities for those who do not get the desired quota in public university.

Implications, limitations, and future studies

The results presented give rise to a series of practical implications for Ecuadorian universities. Since self-efficacy evaluations carried out by students are a critical factor for academic success in the initial career stage, it is important that HEIs, especially private, dedicate efforts to strengthen their vocational guidance programs and their respective induction processes to increase the number of students who start their studies with adequate confidence levels in their own ability to respond to the academic demands of their vocational training.

On the other hand, initial diagnostic systems on procrastination habits could be implemented using properly validated psychometric tests. In this way, it would be feasible to detect students with this problem and to apply actions oriented to exercise their ability to self-regulate, as well as their intrinsic motivation on concrete and measurable goals. Such interventions must be followed up by teachers throughout the career, and therefore they must also be trained in these types of strategies, so that they can assist the students in the optimal planning of their academic activities and the strengthening of the confidence that they will be able to carry it out satisfactorily.

Finally, HEIs could implement, from the beginning of the university life, courses based on self-regulation strategies that allow students to develop the ability to persist in the completion of their academic tasks, avoiding procrastination. Strategies can be implemented such as splitting tasks into subparts to make it easier to do it, estimating the time each task can take, creating lists, or other resources that can track complete and uncompleted tasks, preparing the material and environment for doing the task, among others.

Regarding the limitations of the study, since non-probabilistic sampling was used for

collecting the data, the findings presented cannot be generalized to the entire Ecuadorian university population. It is therefore recommended that future work be based on probabilistic evidence of national representativeness. However, the value of exploratory research lies in detecting relevant relationships among the variables studied, so that the statistical results presented function as initial empirical evidence to guide future studies toward a deeper understanding of the subject, through more refined and rigorous research designs, including the use of probabilistic samples, longitudinal information, and even more experimental cutting approaches (Sarstedt *et al.*, 2018). Consequently, further investigations are expected to continue reviewing the proposed mediation model to more rigorously contrast the results presented.

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Positive parenting and self-regulation of learning in adolescents

Parentalidad positiva y autorregulación de aprendizaje en adolescentes

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Abstract

The Self-Determination Theory study the motivation for student learning in relation to parenting, facilitating the factor of autonomy for self-regulated learning. In this conceptual framework, the objective of this work is to analyze the influence of parental achievement goals, support for autonomy and parental control on self-regulation of learning in adolescents in the city of Cuenca (Ecuador). A quantitative, descriptive and correlational cross-sectional focus is assumed. 1056 adolescents (47 % female and 53 % male) from 18 educational institutions, aged 14-19 years, and 1692 parents (56 % mothers and 44 % fathers) participated. The scales of: Self-regulation of Learning (Chávez y Merino, 2016), Achievement-Oriented Goals (Inda-Caro *et al.*, 2020) and Autonomy Support Scale (Mageau *et al.*, 2015) were applied. The data reveal a positive relationship between support for autonomy provided by parents and the autonomous regulation of adolescent learning. Consequently, it is confirmed that positive parenting is a promoter of this type of learning, of an active and self-directed nature, which could be associated with satisfactory academic results. From this derives the need for close family-school collaboration so that parents strengthen the type of parental behavior that facilitates the involvement of the student as protagonist of their learning process.

Keywords: Positive parenting, self-regulated learning, active learning, parental achievement goals, motivation, parental control.

Resumen

La Teoría de la Autodeterminación estudia la motivación para el aprendizaje de los y las adolescentes en relación con la parentalidad, siendo esta última un factor facilitador de la autonomía para el aprendizaje autorregulado. En este marco conceptual, el objetivo de esta investigación es conocer el papel de las metas de logro parental, el apoyo a la autonomía y el control parental en la autorregulación del aprendizaje en adolescentes de la ciudad de Cuenca (Ecuador). Se asume un enfoque cuantitativo, correlacional de corte transversal. Participaron 1056 adolescentes (47 % mujeres y 53 % varones) de 18 instituciones educativas, de 14 a 19 años, y 1692 progenitores (56 % madres y 44 % padres). Se aplicaron las siguientes escalas: Autorregulación del Aprendizaje (Chávez y Merino, 2016), Metas Orientadas al Logro (Inda-Caro *et al.*, 2020) y Autonomy Support Scale (Mageau *et al.*, 2015). Los datos revelan una relación positiva entre el apoyo a la autonomía proporcionado por los progenitores y la regulación autónoma del aprendizaje de adolescentes. En consecuencia, se confirma que la parentalidad positiva es un factor promotor de este tipo de aprendizaje, de carácter activo y autodirigido, que podría estar asociado con resultados académicos satisfactorios. De ello se deriva la necesidad de una estrecha colaboración familia-escuela para que los padres y madres robustezcan el tipo de comportamiento parental que facilite la implicación del estudiante como protagonista de su proceso de aprendizaje.

Descriptores: Parentalidad positiva, aprendizaje autorregulado, aprendizaje activo, metas de logro parental, motivación, control parental.

1 Introduction

The Self-Determination Theory, SDT, is a theoretical model of motivation that highlights the natural tendency of students to explore their environments, to grow, to learn, and to develop (Deci and Ryan, 2000; Ryan and Deci, 2017). It assumes that all students have motivational resources that enable them to engage in one's own learning and achieve a positive school trajectory (Deci and Ryan, 2002; Reeve *et al.*, 2004; Vansteenkiste *et al.*, 2010). However, it also admits that some social contexts provide adequate support while others impair human development. In this direction, SDT focuses on the dimensions of parents—affection, support for autonomy and structure—that are valuable as they facilitate the three innate needs of people: competence, autonomy, and relationship. On the contrary, the dimensions of rejection, psychological control and disorganization hinder sons and daughters from feeling competent, autonomous, and connected with other (Grolnick, 2009; Grolnick *et al.*, 2015; Soenens *et al.* 2019).

The above approach leads us to point out that the 'family' ecosystem is a social area with great contrasts and contradictions (Torrubia *et al.*, 2017). The influence of the family on its members is seen by being the subsystem closest to the subject, the immediate environment that contains the person, and the one that most intervenes in its formation (Bronfenbrenner and Morris, 2006). From this perspective, "the family, understood as a warm, intimate and supportive environment, offers possibilities for personal development and a source of well-being; and is the basic socializing agent and learning scenario" (Torío López, 2018, p.17). The optimal development of a child is the result of positive parental practices that favor the affective bond in the parental relationships. For this reason, according to Ryan and Deci (2000), an adequate exercise of parenthood meets the basic needs of competence, autonomy, and bonding, to foster both intrinsic motivation and internalized forms of extrinsic motivation in sons and

daughters. Additionally, many teachers struggle daily to motivate students who lack of enthusiasm, refuse to cooperate, or even show aggressive behavior in the classroom. They are behaviors far from the type of self-regulated learning that is required for the adolescent's school success. The main role of parents in the socialization of their sons and daughters can contribute to overcoming family-school partnership.

The aim of this research are the dimensions of parental goals, parental support for autonomy and psychological control, variables that in positive parenthood can boost regulated motivation for good adolescent learning performance. In short, it is intended to know whether positive parenthood is a factor that promotes autonomy and the self-regulation of learning in the student. Finding answers to this concern in a specific context is the purpose of this work.

2 Theoretical framework

2.1 Self-regulation of learning

Self-regulation of learning "is the process through which students activate and maintain cognitions, behaviors, and affections that are systematically oriented to achieving their goals" (Schunk and Zimmerman, 1994, p. 309). In this way, self-regulation of learning has positive correlations with academic performance, as Rosario *et al.* demonstrated (2014) in a study in which students with more school failure had less self-regulation of learning.

In the theoretical framework of STD, the importance of developing the internal potential of human beings as elements that model personality and self-regulatory ability is emphasized, demonstrating the inherent character of individuals toward growth and assimilation (Ryan *et al.*, 1997). In this perspective, in the internalization process present in the socialization of individuals, behavior is not always intrinsically motivated; and an important distinction is made between autonomous and controlled regulation (Ryan and Deci, 2008). In both cases, it is an ener-



gized/managed process by the same individuals who will guide the behaviors to their learning to monitor, regulate and control their thoughts, motivations, and behaviors until achieving them (Stover *et al.*, 2017). However, in the first case –autonomous regulation – autonomous learning is encouraged, i.e., self-regulated learning that is inherently motivated. In contrast, there are different forms of extrinsic motivation in controlled learning; social factors encourage or threaten internalization and integration of the regulation of these behaviors (Deci and Ryan, 1985, 2000; Ryan and Deci, 2017; Stover *et al.*, 2017). Consequently, STD believes that the environment, including the relationships among socializing agents, plays an essential role in making people's behavior autonomous, or be well controlled. In this way, the student experiences a notorious learning through cooperative strategies of teaching, services and socio-educational learning that bring education institutions and society closer to the academic improvement of the student (González-Alonso *et al.*, 2022).

2.2 The goals of parental achievement

The sub-theory of the content of goals advocates that people orient their behavior toward goals or lifelong goals that can be articulated into two broad categories: intrinsic goals and extrinsic goals (Ryan and Deci, 2008). The first, associated with intrinsic motivation, includes four aspects: personal growth, affiliation, health, and community contribution; it relates to the satisfaction of basic psychological needs and contributes to psychological well-being. Extrinsic goals, associated with extrinsic motivation, are articulated into three categories: fame, physical appearance, and financial success; they relate to external manifestations, reactions of others, interpersonal comparison, need for approval, and are associated with poor people's well-being (Deci and Ryan, 2017).

In the academic world, goals have two orientations; on the one hand, the approxima-

tion to success and avoidance to failure; and, on the other, the task. In the latter, students are interested in improving their skills and building new knowledge. However, in the orientation toward success in homework, students aim to protect their own image before themselves and others (Pintrich, 2003). From this theoretical approach, Dahling and Ruppel (2016) show that students with low orientation to the task show less academic self-efficacy, while those students with a high orientation to the task show better school performance. Mageau *et al.* (2016) formulate three types of orientations of achievement goals when applying this conceptual model to the goals of parental achievement in the area of family socialization: a) goals oriented to the mastery of the task: focused on the learning and the goal cognition of the activity that the children develop; b) goals oriented to success in the outcome of the task: aimed at demonstrating that children are better than others in the execution of a given activity; and c) goals aimed at avoiding task failure: explain the behavior of those parents who prevent their sons and daughters from making mistakes so that they do not feel inferior or disqualified by others. For Gonida and Cortina (2014), the goals of parents—goals of parental achievement—can direct parental practices, particularly toward support for autonomy and psychological control. The task-oriented parental goal (intrinsic motivation) supports for autonomy and facilitates autonomous regulation of learning; in contrast, parental achievement goals directed at task success and avoidance of failure (extrinsic motivation) are linked to psychological control and explain controlled regulation of student learning.

2.3 Support for autonomy and psychological control

The parental dimension for STD, *support for autonomy*, refers to parental practice that guides children to think their own way and make decisions according to their attitudes, interests, and



values (Grolnick *et al.*, 2014; Ryan *et al.*, 2006). It is important to emphasize that support for autonomy is the base of intrinsic motivation and internalized forms of extrinsic motivation. In addition, it is positively associated with the perception of general competence in children and adolescents, which facilitates a sense of success in academic tasks (Grolnick, 2009; Grolnick *et al.*, 2015). In this perspective, studies indicate that sons and daughters who show better academic and psychosocial skills have parents who promote their initiative, listen to their opinions, and enable them to make decisions (Joussemet *et al.*, 2005; Joussemet *et al.*, 2014). In the study of parenthood, psychological control is contradictory with the promotion of the autonomy in children (Rodríguez Meirinhos *et al.*, 2019). In the STD framework, “psychological control refers to those behaviors that interfere with the thoughts and feelings of sons and daughters” (Barber, 1996, p. 3297). Rodríguez Menéndez *et al.* (2018) say that psychological control integrates three essential components: a) manipulation and coercion practices that dominate and pressure the child in the demands or expectations of parents; b) intrusion into the emotions of the children; and c) lack of respect to children.

Assuming the above, psychological control is a parental practice with negative consequences for children’s development because it frustrates their autonomy. In this direction, studies indicate that psychological control increases the risk of children to have problematic behaviors at home, at school, or with their peers, and facilitates the development of anxiety symptoms, which could have a negative impact on adolescent academic performance and interpersonal relationships (Barber and Xia, 2013; Kuppens *et al.*, 2013; Pinquart and Kauser, 2018; Scharf and Goldner, 2018). Additional studies corroborate that, when family upbringing favors support for autonomy, this parental practice explains a better internalization of learning and an autonomous motivation in children and adolescents, as well as improved psychological well-being; while psy-

chological control, a coercive environment, and external motivation inhibit or reduce the interest of intrinsically motivated activities (Bernier *et al.*, 2010; Brenning *et al.*, 2015; Grolnick, 2009; Grolnick *et al.*, 2014).

Because of the latter, the objectives of the study seek to understand the situation in this regard in a local context. As a general objective, this research aims to know how parental goals operate in self-regulation of learning in the case of adolescents and families in the city of Cuenca (Ecuador). The specific objectives are a) to identify self-regulation of learning in adolescents, its characteristics, and the type of predominant regulation; b) to analyze this variable in relation to the goals of parental achievement (mastery of the task, success in the task and avoidance of failure) and c) to check for possible relationships of this variable with the support for autonomy versus psychological control. Additionally, to look for differences according to the sex of adolescents and parents.

3 Methodology

The research is quantitative, with a cross-sectional correlational design that allows the relationship between goals of parental achievement, support for autonomy and parental control in self-regulation of learning among adolescents in the city of Cuenca.

3.1 Population and sample

The mean age of fathers was 51.95 ($SD=6.04$), and the mean age of mothers was 49.95 ($SD=5.29$).

With the information provided by the Coordination of Education Zone 6, the population of 25 870 adolescents enrolled in first, second and third year of high school in the educational institutions of the city of Cuenca (Ecuador), located in the urban area was determined. With the subsequent authorization of local education authorities, educational institutions were selected. For this purpose, a multi-



stage sampling was used, determining 18 institutions; subsequently, the random selection of each level of high school was made by conglomerates. The sample size calculation was performed with a 95 % confidence level and 3 % margin of error. Once the sample was calculated, the choice of institutions was done at random with a total of 10 public institutions and eight private from the city of Cuenca (Ecuador). This study is based on a sample of 1056 adolescents, of whom 47% are women, 53% are men; the mean age was 16.10 years ($SD= 1.10$). 1692 parents participated, 56% are mothers and 44% are fathers. The mean age of mothers was 42.73 ($SD= 6.65$), and the mean age of fathers was 45.36 ($SD= 7.89$). Regarding the level of education of mothers, it is reported that 39.1% completed elementary school, 31% completed high school and 29.9% completed higher education level. Regarding fathers, 35.3% reported they completed elementary school, 31.6% completed high school and 33.1% completed higher education.

3.2 Instruments

The instruments used to measure the study variables were three scales, which have been adapted from scales of international use.

1. Learning Self-Regulating Scale (Chávez and Merino, 2016) adaptation of the Self-Regulated Learning Questionnaire, SRQ-L (Williams and Merino, 2016). The questionnaire filled out by adolescents includes two subscales a) *Autonomous Regulation* (item: 1,3,6,9,11 and 12) and b) *Controlled Regulation* (item: 2,4,5,7,8,10,13 and 14). There are 14 items for a Likert-type response scale, with five possible degrees of agreement, from (1) “Not true for me” to (5) “Totally true for me”—as in the original scale. The internal consistency of the instrument yields a coefficient: $\alpha .75$, for autonomous learning, and $\alpha .72$, for controlled learning.
2. Parental scale of achievement-oriented goals (Inda-Caro *et al.*, 2020), adaptation of *Parental Achievement Goals* AGQ (Mageau *et al.*, 2016). This instrument consists of 11 items, according to a Likert scale, with seven possible degrees of agreement, ranging from (1) “I do not agree” to (7) “I strongly agree”. It was filled in by the parents who reflected in three types of goals: a) task-oriented, which measure the interest of parents in the effectiveness of learning (item: 3, 6 and 10); b) goals aimed at success in the outcome of the task, which reflect the interest of parents in achieving recognition for their sons and daughters (item 2, 4, 7 and 8); and, c) goals aimed at avoiding task failure, which include parental behaviors to prevent children from being perceived as less competent than their peers (item: 1, 5, 9, and 11). The internal consistency of the instrument yields the following coefficients: $\alpha 0.75$ (mothers), and $\alpha 0.72$ (fathers).
3. Permanent Parental Autonomy Support Scale, P-PASS (Mageau *et al.*, 2015). This scale addresses two dimensions: a) support for autonomy, with three indicators: election of the offer in certain limits; explain reasons behind demands, rules and limits; and be aware of accepting and recognizing the child's feelings; and b) psychological control, with three indicators: threatening to punish the child; inducing guilt; and promoting achievement goals. It is an instrument with 24 items, with a Likert-type response scale with 7 response alternatives (1-7). The scale has a high reliability index, high internal consistency: support ($\alpha_{\text{mother}} = 0.88$ $\alpha_{\text{father}} = 0.85$); control ($\alpha_{\text{mother}} = 0.86$; $\alpha_{\text{father}} = 0.84$)

3.3 Data Collection Procedure

In the first day in the institutions, the students were given a closed envelope addressed to their parents containing the following documents:



information of the research; questionnaire for parents (mother and/or father), with the corresponding consent to use the data; consent to authorize the participation of their sons and daughters in the research. In the second visit, the envelopes with the completed documents were collected and, once the authorization of the parents was verified, the questionnaires were applied to the students. The questionnaire was applied in the classroom during the academic year, in October, November and December (school year 2019-2020).

3.4 Data analysis

Measures of central tendency and dispersion were obtained. Based on the sample size, the Kolmogorov Smirnov test was applied, which allowed the data normality hypothesis to be rejected (p value $< .05$). Therefore, non-parametric methods such as Mann Whitney, Wilcoxon, and Friedman's U-test were used for cross-group comparison, as well as Spearman's correlation coefficient to test the existence or absence of correlation between "parental goals" and "support for autonomy versus psychological control" with

"self-regulation of learning". Data processing was performed using the SPSS 25 statistical package.

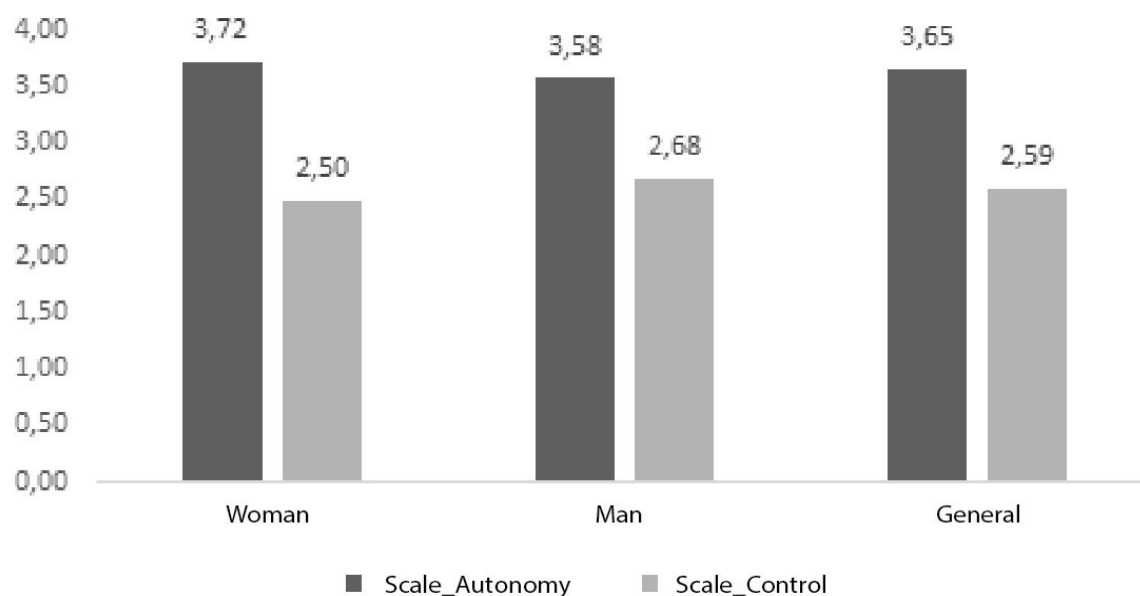
4 Results

4.1 Self-regulation of learning in adolescents

As can be seen in Figure 1, considering the sample as a whole, adolescents show a higher average on the autonomy scale than on the control scale ($M_{aut} = 3.65$ and $SD_{aut} = 0.7$; $M_{cont} = 2.59$ and $DT_{cont} = .71$). This data shows a clear positive orientation toward the autonomous self-regulation scale, followed by controlled regulation. A more in-depth analysis, considering both sexes separately, indicates that the data collected using the scales of autonomous regulation and controlled regulation showed significant differences between men and women ($Z = -26.31$, p value $= .0001$, $d = 1.26$). On the autonomy scale, the latter obtained a higher score ($Z = -3.246$, p value $= .049$), while on the control scale, a higher score was observed for males ($Z = 4.06$, p value $= .001$). These results show differences in women regarding autonomous regulation that could favor a type of learning and self-regulation. Consequently, the sex of the participants makes a difference with respect to the way in which student learning is self-regulated.



Figure 1

Self-regulation of learning in adolescents according to their sex

Source: Our elaboration.

4.2 Self-regulation of learning in adolescents: How it relates to parental achievement goals

Parental achievement goals (mastery of homework, success in homework, and avoidance of failure) were analyzed considering the gender of parents. As shown in Table 1, the data first place the goals oriented to the “mastery of the task (effectiveness of learning), with the highest mean ($m_{d.parents} = 6.42$ and $SD_{d.parents} = .80$; $m_{d.mothers} = 6.40$ and $SD_{d.mothers} = .81$); followed by the goal “task success” ($m_{d.parents} = 5.93$ and $SD_{d.parents} = 1.06$; $m_{d.mothers} = 5.78$ and $SD_{d.mothers} = 1.16$); and, thirdly, “failure avoidance goal with scores below aver-

age ($m_{e.parents} = 2.65$ and $SD_{e.parents} = 1.58$; ($m_{e.mothers} = 2.56$ and $SD_{e.mothers} = 1.58$).

Self-regulation of learning in adolescents and parental achievement goals correlate positively, data are provided in Table 2. In the case of the mother, the goal “failure avoidance” has a significant positive correlation with “controlled regulation” ($\rho = .205$ pvalue = .0001). In the case of fathers, there is a significant positive relationship between the goal “success in the task” and the “autonomous regulation” of adolescents ($\rho = 0.089$ pvalue = .02781); and also between the goal “avoidance of failure” and “controlled regulation” ($\rho = 0.189$ pvalue = .0001).



Table 1

Parental Achievement Goals

Parental Goals	Mother		Father	
	Mean	SD	Mean	SD
Task Proficiency	6.40	.81	6.42	.80
Task Success	5.78	1.16	5.93	1.06
Task Avoidance*	2.56	1.58	2.65	1.58

Note: * $p < .05$

It should be noted that the goal “avoidance of task failure”, associated with extrinsic motivation, correlates positively in both parents with the controlled regulation of learning, this being the most significant statistical relationship of the study. It is also noted that the goal “success of the task” in fathers correlates positively with the “autonomous regulation” of the adolescent, while the goal “avoidance of failure” in mothers is the one that shows a positive correlation with this type of regulation.

An apparent contradiction is observed in the case of mothers. The goal of “failure avoidance” correlates positively with both controlled regulation and autonomous regulation, probably because this goal can have an external orientation, linked to psychological control, explaining controlled learning. STD believes that behaviors that have been extrinsic motivated can be internalized when socializing with people, especially those behaviors aimed at academic achievement in students.

Table 2

Correlation Between Self-Regulating Learning Scales and Parental Achievement Goals

Self-regulation of learning	Parental Achievement Goals					
	Mother			Father		
	Task Mastery	Task Success	Avoidance of failure	Task Mastery	Task Success	Avoidance of failure
Autonomous Regulation	.067	.067	.090*	.038	.089*	.036
Controlled Regulation	-.001	.058	.205**	-.064	.052	.189**

Note: ** Significant correlation at .01 level; *Significant correlation at .05 level

Source: Our elaboration.

On the other hand, following STD, whose autonomous regulation model enables students to be involved and persist in their academic tasks while controlled regulation is the least favorable for good academic performance, the results could indicate that the parental goals that guide the autonomous regulation of sons and daughters are the most favorable to their academic goals.

4.3 Self-regulation of learning in adolescents related to autonomy support versus psychological control

Table 3 shows Spearman correlations that indicate the relationship between the two dimensions of the PPASS scale, autonomy support and psychological control, and the two self-regulating programming scales: “autonomous regula-



tion and controlled regulation”. There are statistically significant positive correlations in parents of both sexes between autonomous regulation of learning and support for autonomy, higher in the case of mothers (rho.270 vs. rho.254); and, on the other hand, between controlled regulation and

psychological control, higher in fathers (rho.319 vs. rho.295). This is an indication that the sex of parents is related to support autonomy, in the case of mothers, and psychological control, in fathers.

Table 3

Parental support for psychological autonomy-control, self-regulation

Sex	Instrument	Dimension	Autonomous regulation	Controlled regulation
Mother	PPASS	Support for autonomy	.270**	.037
		Psychological control	.044	.295**
Father	PPASS	Support for autonomy	.254**	-.019
		Psychological control	.047	.319**

Note: *** Significant at level .001, ** Significant at level .01, * Significant at level .05.

5 Discussion

Adolescents in Cuenca have a clear positive orientation toward autonomous regulation of learning, as Peruvian and Portuguese adolescents (Chávez and Merino, 2016; Rothes *et al.*, 2017). As mentioned by STD, the intention is that “students will have the capacity to intervene intentionally in their learning environment, guiding and transforming the events in accordance with their academic objectives” (Rosario *et al.*, 2014, p. 786). Also, although autonomous regulation predominates in the group, there is a significant statistical difference between girls and boys in the two subscales; while self-learning predominates in girls, controlled learning is more common in boys. This gender difference is consistent with other research (Arias *et al.*, 2018; Parra *et al.*, 2014; Suárez-Valenzuela and Suárez-Riveiro, 2019) that have explained the higher adaptation level to school environments, learning strategies, and efficiency that characterize girls. It is observed that the sex of adolescents explains the differences between the two self-regulation scales of learning.

On the other hand, parents of adolescents show that their goals are primarily oriented to

the mastery of the task and the success of the task, followed by the goal of avoiding task failure. This result coincides with an investigation that reported that Canadian mothers prefer the mastery of the task, which takes to autonomy; followed by the success of the task that explains psychological control and parental interference in the task of sons and daughters (Gonida and Cortina, 2014). It is also similar to the findings of Inda-Caro *et al.* (2020) where parents prioritized the same goals, identified from the career of their children. In the STD context, the goal of mastery the task is intrinsically motivated and the two remaining goals, task outcome, and task failure avoidance, are linked to extrinsic orientation (Deci and Ryan, 2017).

Findings also show that the parental goal of avoidance task failure is statistically significant in mothers with the scale of controlled regulation. In this case, following Gonida and Cortina (2014), it may be associated with controlling practices to prevent sons and daughters from making mistakes. As for parents, the highest scoring goal is oriented to success in the task, which is linked to autonomous regulation. This result is also related to the study just mentioned,



whose authors observed practices of support for the autonomy of minors in the parents, which consist of listening to their point of view, encouraging them to carry out activities and feeding back their tasks positively.

The goals of parental achievement: task success and avoidance of failure guide parental psychological control practices. In addition, it is noted that a differentiating element of parental goals is the sex of parents; according to Schvaneveldt (2014), a possible explanation for this difference might be culture. In Ecuador, parental practices in mothers, influenced by physical and emotional closeness, include high levels of support along with high levels of control and monitoring of the academic activities of their sons and daughters. Finally, the results show that parental support for autonomy is related to self-regulated learning. The data obtained agree with studies that indicate that support for autonomy is associated with better competencies and academic tasks in adolescents (Grolnick, 2009; Grolnick *et al.*, 2015; Joussemet *et al.*, 2005; Joussemet *et al.*, 2014). This result corroborates that intrinsic motivation and internalized forms of extrinsic motivation are explained by the support for autonomy; moreover, these motivations are positively related to self-regulation (Grolnick, 2014; Ryan and Deci, 2017). In addition, the results show that psychological control of both parents are related to controlled regulation of learning. Previous research has shown that disrespectful behavior by parents has a negative impact on academic performance and that interest in intrinsically motivated academic tasks is diminished (Barber and Xia, 2013; Kuppens *et al.*, 2013; Piquart and Kauser, 2018; Scharf and Goldner, 2018).

6 Conclusions

The results of the study showed that autonomous regulation of learning, facilitating active and self-directed learning, predominates among students. It is a result that must be reinforced

by strategies of self-regulation of learning and evaluation toward the achievement of objectives and the improvement of academic performance in adolescents, as well as the conditions in which the student can be involved and participate in his or her learning, including family-school intervention. It should be stated that in the case of adolescents in Cuenca, family variables—parental achievement goals and parental practices—promote or threaten both intrinsic motivation and internalized forms of extrinsic motivation. Therefore, it is necessary to support the formation of a positive parenthood that will encourage parents to promote healthy psychological growth and academic development of children and adolescents by supporting autonomy and avoiding psychological control practices.

This research has deepened on the contextualized knowledge of the topic; however, there is little research that explicitly studies the relationship between parental achievement goals and self-regulation of learning. Further research must be done on autonomous regulation of learning as a predictor variable of optimal academic performance of students. In this regard, the Theory of Self-Determination provides a solid theoretical framework for advancing possible hypotheses that will illuminate the way forward.

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Multigrade teachers in Esmeraldas (Ecuador): a silenced reality

Maestras de multigrado en Esmeraldas (Ecuador): una realidad silenciada

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Abstract

The existence of single-teacher or multi-grade teachers in rural areas of Ecuador is a reality of education. The lack of recognition by the administration leads to political and institutional discrimination that calls intercultural education into question. The multigrade activity in rural Ecuador faces complex and differentiated challenges compared to traditional teaching, as in these cases, multiple levels (1-7) of elementary school are taught by a single teacher. This task is largely carried out by women who are unfamiliar with the cultural conditions of these places, and in many cases are not qualified. This paper aims to be the voice of 7 single-teacher women teachers from the coastal area of Ecuador according to Brumat's (2011) proposal, who work in the rural parish of La Unión, belonging to the Quinindé canton, in the province of Esmeraldas. Especially with regard to their training and insertion in the rural context. A qualitative and exploratory study was carried out. The data collection instrument consisted of a semi-structured interview, and its analysis was carried out using the MAXQDA programme. The daily life of these teachers can be characterised by feelings of loneliness and isolation, and a difficult coexistence in disadvantaged sectors where the cultural complex, gender and racial discrimination and discrimination based on place of origin are deeply rooted.

Keywords: Rural education, gender discrimination, educational administration, multigraded classes, Ecuador, intercultural dialogue.

Resumen

La existencia de profesoras unidocentes o de multigrado en las zonas rurales del Ecuador forma parte de la realidad educativa cotidiana. La ausencia de reconocimiento por parte de la administración provoca una discriminación política e institucional que cuestiona la educación intercultural. La actividad unidocente que se desempeña en la zona rural del Ecuador se enfrenta a desafíos complejos y diferenciados respecto a la enseñanza tradicional, al ocuparse en estos casos de múltiples niveles (1-7) de educación básica por una única maestra. Esta tarea es desempeñada en gran parte por mujeres que desconocen las condiciones culturales de estos lugares, y en numerosas ocasiones no poseen la cualificación necesaria. Este trabajo pretende dar voz a siete profesoras unidocentes de la zona costa del Ecuador según la propuesta de Brumat (2011), que trabajan en la parroquia rural La Unión, perteneciente al cantón Quinindé, en la provincia de Esmeraldas. Especialmente respecto a su formación e inserción en el contexto rural. Se siguió un estudio de corte cualitativo y exploratorio. El instrumento de recolección de datos consistió en la entrevista semiestructurada, y su análisis se realizó a través del programa MAXQDA. La vida cotidiana de estas profesoras puede caracterizarse por sentimientos de soledad y aislamiento, y una difícil convivencia en sectores desfavorecidos donde el complejo cultural, la discriminación de género, racial y por el lugar de origen están arraigados.

Descriptores: Educación rural, discriminación de género, administración educativa, clases multigrado, Ecuador, diálogo intercultural.

1 Introduction

The existence of multigrade teachers in rural areas in Latin America is an unknown reality (Arias-Ortega *et al.*, 2021); Carvajal-Jiménez *et al.*, 2020). The dichotomy between the rural and the urban world enables to visualize the characteristics, circumstances and special needs that require specific attention and analysis (Brumat, 2011; Calderón, 2015; Waissbluth, 2019). In Ecuador, there is little research that seems to address the rural educational reality and the daily life of multigrade (Vaca *et al.*, 2020).

This area is characterized by high levels of poverty, risks of social exclusion, discrimination and deficiency in public services (Arévalo-Avecillas *et al.*, 2018; Padilla, 2018). These problems are faced by teachers in multigrade schools (Mora, 2020; Waissbluth, 2019). In 2019, the Ecuadorian rural area presented 41.8% of poverty and 18.7% of extreme poverty, compared to 17.2% and 4.3% in the urban area, respectively (INEC, 2020).

According to UNESCO (Calderón, 2015), these educational organizations appear in the 50 and are characterized by the concentration of the responsibility of a single teacher for all grades, from first to seventh of elementary school. Likewise, the teacher may also be responsible for the performance of administrative tasks, student drivers and community members, or for the distribution of teaching materials, among other functions (Waissbluth, 2019).

For Cruz and Juárez (2018), since there is not extensive information on multigrade schools, it is recommended to address this reality (Vaca *et al.*, 2020). Those authors, in their work on single schools, analyze the experiences of four single schools with low academic scores compared to urban schools.

Among the challenges faced by the rural educational reality in Ecuador are high student dropouts, the digital gap, bullying and illiteracy (Calderón, 2015; Waissbluth, 2019). As regards dropout, the most frequent causes are the lack of financial resources (46.75%) and the need to work

(17.56%) since boys and girls play an essential role in the family economy; illiteracy in the rural area is 12.9% compared to 3.9% in the urban area (Defensoría del Pueblo de Ecuador, 2018).

On the other hand, the digital gap is significant in a sector where only 24.5% have access to a computer nationwide. 16.1% have internet access in rural areas, compared to 46.6% in urban areas (INEC, 2018).

Coexistence in this rural context is often characterized by racist attitudes or gender discrimination (Jiménez, 2020), isolation and cultural incomprehension with the community. Thus, for example, 58.7% of rural women have suffered some form of violence, especially those women belonging to indigenous groups (67.8%) and Afro-Ecuadorian groups (66.75%) (UNICEF, 2018). Students are most affected by psychological and verbal harassment (Flores and Sigcha, 2017; Jiménez, 2020).

In Ecuadorian culture, there are different meanings in language with racist and social exclusion connotations. In addition, in the mestizo discourse there is permanent ambiguity regarding the identity of the Ecuadorian, which is translated into stereotyped visions of the existing cultural diversity. Thus, it is common in public spaces to use expressions like “longo” a negative word to refer to other mestizos and indigenous people; “cholo” to refer to mestizos who seem more concerned about having fun than working; “chaso” or “pastuso” to emphasize the rurality of the mestizo, or “mantubio” to refer to the working and independent mestizo. Another group that historically suffers expressions and discrimination is Afro-descendant, identifying them with a group related with leisure or crime (Ayala, 2002; Tutiven *et al.*, 2018).

Therefore, teacher training in these educational contexts requires specific training in this complex reality. Brumat (2011) mentions the disconnection between the training programs in degree and postgraduate courses directed to teachers with the needs and characteristics of rural life, predominating the urban.



According to Rojas-Durango *et al.* (2013) this gap makes it impossible to incorporate the needs and wealth of communities, leading to political-institutional discrimination. In view of this situation, Cragolino and Lorenzatti (2016) defend the need for teachers to update strategies that allow them to work in different degrees and to face the diversity of these students, as well as the problems that they deal with as a result of poverty and discrimination. For example, the rural areas have the greatest presence of historically excluded and discriminated groups, such as indigenous and Afro-Ecuadorians with 78.5% and 25.6%, respectively at the country level (Defensoría del Pueblo de Ecuador, 2018).

The Organic Law on Intercultural Education (LOEI) (Asamblea Legislativa, 2011) aims to position interculturality as an essential axis in education for the good life or *sumak kawsay* (Lara and Herrán, 2016; Lara, 2019; Vernimmen-Aguirre, 2019; Valdez-Castro, 2021), necessarily including the rural world as advocated in article 31 of the Constitution of the Republic of Ecuador (Asamblea Constituyente, 2008). The education of the good living has been an interesting and new proposal in Ecuador, because it integrates ecology, dialog of knowledge, rationality, pedagogies such as death or prenatal, universality or egocentrism as essential variables for its understanding (Lara and Herrán, 2016; Lara, 2019; Suárez *et al.*, 2019).

Thus, the LOEI (Legislative Assembly, 2011) in article 1 mentions that “it guarantees the right to education, determines the general principles and purposes that guide Ecuadorian education in the context of good living, interculturality and plurinationality”; in paragraph (v) concerning equity and inclusion, it states that “equality of opportunity is guaranteed to communities, peoples, nationalities and groups with special educational needs and develops an ethic of inclusion (...)”.

This work aims to approach the daily life of multigrade Ecuadorian teachers, experienced in seven women who perform their activity in the Esmeraldas coast area. The concept of the

conduction of daily life is used as a tool to capture human subjectivity from the social context in which the person lives and relates (Brumart, 2011; Kristensen and Schraube, 2014). It focuses especially on three major challenges:

- Identify the training needs of multigrade teachers.
- Know the meaning of multigrade.
- Listen to the experience of multigrade teachers in the rural community.

2 Methodology

A methodology based on the qualitative approach was used, since the research focused on obtaining the perspectives and points of view of multigrade teachers as to their emotions, priorities, experiences, and meanings they give to their daily life, in relation to the rural contexts in which they work.

It is exploratory research, located in the rural parish of La Unión, belonging to the canton of Quinindé, in the province of Esmeraldas, Ecuador, zone coast. A sample made up of the seven rural teachers of this parish was used. The selection sought to maximize the usefulness of information from small samples (Flyvbjerg, 2006). In this case, information was collected from a sample of seven multigrade teachers belonging to rural contexts that are remote from each other and difficult to access. Sampling was performed for its convenience due to its accessibility and willingness. In all cases, there was no paved road allowing access by motor vehicles. Its composition was homogeneous according to the following similar parameters among the individuals that make the sample: A) the seven individuals are multigrade teachers; b) sex (the entire sample are women, characteristic that was at random); c) age: when making a distribution of absolute frequencies of age by classes as follows: 25-30, 30-35, 35-40 and 40-45 —continuous variable concentrated in the second class, 30-35); d) professional experience and qualification:



ranges from nine to 11 years and 71.4% have a third-level education degree (see Table 1).

Table 1

Characterization of participating individuals

Doc.	Sex	Experience	Age	Level of study	Specialization
E1	Femenine	10	33	Third Level	Bachelor's Degree in Child Education
E2	Femenine	11	34	Third Level	Bachelor's Degree in Child Education
E3	Femenine	9	35	Third Level	Bachelor's Degree in Child Education
E4	Femenine	9	33	Fourth Level	Master's Degree in Education
E5	Femenine	11	32	Third Level	Bachelor's Degree in Child Education
E6	Femenine	10	28	Third Level	Engineer in Business Administration
E7	Femenine	9	45	Bachelor's	Degree in Science

Individual and semi-structured qualitative interviews were used as a data collection technique, due to its openness, flexibility, and opportunity to be carried out in different time-space moments, in a dialog based on the categories defined *a priori* (see Table 2) (Savin-Baden and Major, 2013). A fluent conversation was achieved, in which perceptions and knowledge about the meaning of multigrade, professional updating, and experiences of discrimination lived were revealed, constituting as categories of study. The instrument was developed by the authors and submitted to consultation workshops with specialists in which it was corrected, re-elaborated and the validity of the contents determined according to their correspondence with the categories determined (see Table 2) (Mattos and Cruz, 2011). Recording the interviews prevented data loss for oblivion or other reasons. Each time the instrument was applied to the interviewees, similar results were obtained, observing moments of theoretical saturation.

The design used was phenomenological (Duque and Aristizabal 2019) in obtaining the perspectives of the participants to explore,

describe and understand their common experiences regarding multigrade, professional updating and discrimination. There were four phases: literature review, data collection planning, data collection and analysis, and discussion and conclusions.

In the initial phase, a bibliographic review was carried out on the subject of study, the identification of the categories and subcategories, and the preparation of the interview. In the second phase, data collection planning was carried out: individual interviews were scheduled during May 2019, once the informed consent of the participants and the necessary authorizations had been obtained. Before starting the interview, the objective of the research, the anonymity and confidentiality of the responses were recalled. The interviews were conducted by a single researcher, the dialogs were recorded and then transcribed. The average interview time was about one hour.

Content analysis was used for data processing. In addition, a mixed analysis was applied (Rodríguez *et al.*, 2005). To this end, first, three categories were deductively determined *a priori*, from which seven subcategories were defined (see Table 2).



Table 2

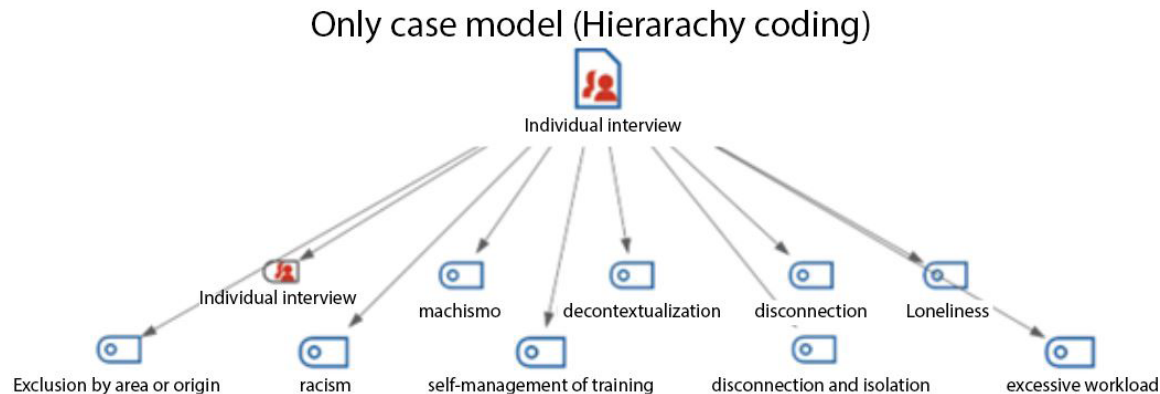
A priori construction of categories and subcategories

Categories	Subcategories
Meaning of multigrade	Self-perception of multigrade Professional satisfaction of multigrade
Professional training	Relevance of updating courses received Attitude for professional updating
Experiences of discrimination during multigrade	Racism Machismo Community exclusion

Once the data were collected, an open and inductive coding was performed. In this phase, the dialogs were transcribed, and the answers were then grouped by questions creating blocks of information according to the categories and subcategories. This first document was ana-

lyzed through MAXQDA Software (Kuckartz and Rädiker, 2019), which provided a code hierarchy map through the software's MAXMaps visual tool and allowed a code reduction from twenty-three to nine (see Figure 1).

Figure 1

Code Hierarchy Model

Note. MAXMapas, MAXQDA, 2020, multigrade project.

Subsequently, the document was divided into three main sections (Document 1, 2 and 3), which encompass the questions by category, generating a case model (see Figures 2, 3 and 4). The various paragraphs of the transcripts were enumerated for their codification. The codes were induced by the answers obtained and by the concepts found in the literary review developed in the initial phase. These codes made it possible

to point out the matches and group the results through the MAXMaps tool for models and code hierarchy, which facilitated the interpretation of the information and the preparation of the results.



3 Results

The results are presented according to the groupings made from the categorizations and coding, which allowed to group them into interpretative texts that represent the perceptions and attitudes of the interviewees about multigrade and its meaning, professional updating, and discrimination.

3.1 Meaning of multigrade: quiet isolation

Paragraphs from one (1, 9 and 17 belong to the moderator) to 24 of the first document were used to obtain the first result, which correspond to the first category “meaning of multigrade” and the subcategories that emerge from it: self-perception of multigrade and professional satisfaction (see Figure 2). Although in the code hierarchy model for this category, four codes (disconnection, isolation, excessive workload, and loneliness) were initially delimited; in the analysis of the individual document, other representative codes were observed in the case model that were considered interesting for this result; these were: autonomy, frustration with not achieving goals, tranquility, self-fulfillment, and professional experience.

A single case model was constructed with this information using the MAXMaps tool that shows the general characteristics of the seven teachers interviewed regarding the category meaning of multigrade (see Figure 2).

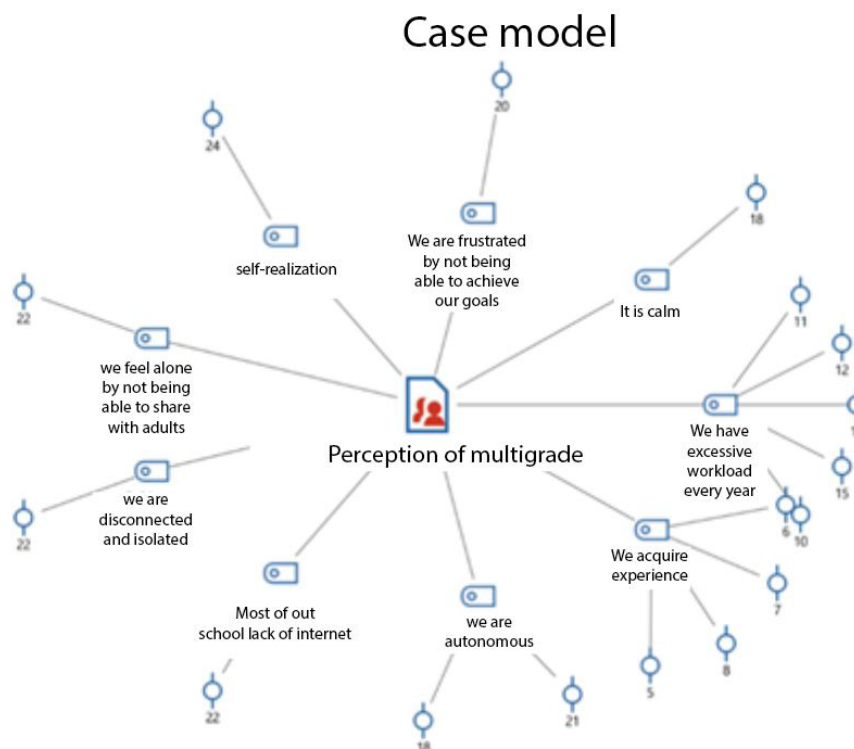
Multigrade, described by the interviewees, is full of different perceptions, which enable a description of how teachers perceive multigrade and their level of satisfaction with their work.

In terms of the level of satisfaction, different positive aspects are mentioned, such as autonomy, tranquility, experience, the nature of the work or personal performance. Thus, while there is a feeling of isolation and loneliness, to some extent it is valued as an opportunity for autonomy and tranquility “although it is difficult at times, the environment is very calm and gives me the possibility to adequate my time” (E1); it is the moment to acquire unique experience to be able to combine teaching at different levels: “I can do many different activities since I teach all the subjects, and I think that being with children of different ages helps with this experience. I can work with any grade” (E4).

Similarly, there is the presence of a sense of personal and vocational accomplishment: “I can proudly see my effort in my students who continue their studies, and above all remind me with affection” (E7), or “I like to work with children and share my knowledge” (E3).



Figure 2
Case model, category meaning of multigrade



Note. The figure shows the codes and number of the paragraph corresponding to the transcript in which they are present. The most recurrent codes were: We have excessive work; we work with all years (paragraphs 6, 10, 11, 12, 14 and 15); and experience is acquired in the different subjects and years (paragraphs 5, 6, 7, 8 and 10, document one, questions and answers corresponding to the first category, perception of multigrade).

MAXMapas, MAXODA, 2020.

On the other hand, among the negative aspects, they express the frustration of not achieving the proposed objectives, feelings of loneliness and isolation, few relational experiences with adults, the lack of opportunities for professional exchange and training. The excessive work, along with the need to meet competencies and learning outcomes, is frustrating to them: “it is difficult to explain all subjects at the same time, and it is also not possible to achieve the proposed goals. As much as I try, I cannot achieve the objective, I would do it if it were a single grade” (E3), or “it is a little more difficult than teaching a single

year of elementary school. In my previous job, I was a single-degree teacher, so it was easier and calmer” (E2).

In this regard, they share their feeling of loneliness and isolation, not only in relation to relationships with other professionals, but also with adults:

No, I don’t like being alone. I have more contact with students, but I cannot share or talk to adults. I have been desperate many times because I cannot share with colleagues or we cannot support at each other because we are alone, and in very distant communities. (E5)



Isolation is not only in terms of physical space, but also in terms of the possibilities of access to educational experiences organized by the district of education. They point out that when they have some questions, they are mainly replaced by self-formation or by the advice of the other multigrade teachers: “with self-training” (E3), and “with the help of the teachers who have the same doubts” (E4).

Regarding the use of information and communication technologies, they mention the existing disconnection of the school with telephone and internet: “without mentioning that in most of our schools we have no signal and are not connected” (E5).

Likewise, the meaning of multigrade for rural teachers is characterized by being a different activity from the traditional one. They are responsible for various tasks, as they are required to attend students of the seven levels of elementary school, as well as to do administrative and maintenance activities:

Being only a teacher for all grades and subjects is really very hard, and keeping in mind that we also have administrative work because we are the principals, ... and many times we even have to clean up and try to keep things clean and running. (E5)

The training to deal with students with various disabilities lies in each one's own initiative:

I would like to train to teach children with different disabilities. I have two cases and I would like to know how I can help; I have searched on the internet some activities that I can do, but I would like an expert to come. (E3)

3.2 Training to be multigrade teacher: “work to live”

Paragraphs 1 to 32 of document two were used for this result, which collected the questions and answers corresponding to the second category, Professional training. Paragraphs 1, 9, 17 and 25 correspond to the questions raised by the interviewer and enable to observe the behavior of the subcategories: relevance of the updating courses received and attitude toward professional training (see Figure 3). Two codes were identified for this purpose: Decontextualization and self-management of training. The model of the case drawn up in the MAXMaps tool can be seen in Figure 3.

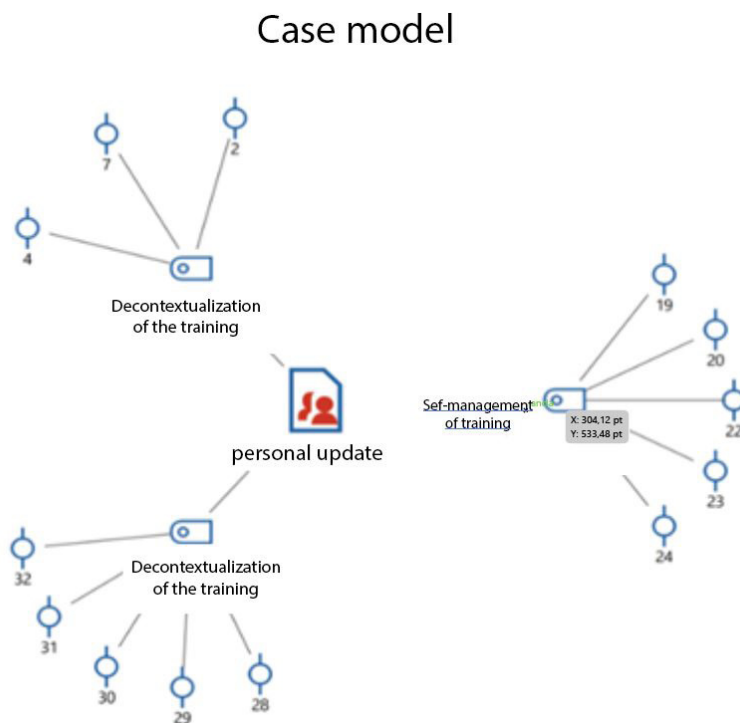
The professional updating of multigrade teachers normally happens by self-management. They agree with the decontextualization of the courses they have attended, and which are offered by the district of education, with respect to the needs of their daily reality.

Hence, when asked about the ways in which they are updated, it mainly depends on their own initiative: “autonomously because the trainings that I have participated have not been well planned and I have not understood” (E5), “looking for information, or something that I need on the Internet” (E1), “with self-training, because if we wait for them to call us it will take a lot of time” (E3), or “reading manuals or asking colleagues what to do in difficult cases” (E7).

However, they participate in district trainings, although they do not like it because “they always say what to do in general, but do not know the specific case of each school, or the needs we have” (E6), or “they make us do activities, but they do not explain well how to apply them according to the different needs that each teacher has in his or her school” (E3).



Figure 3

Model of a case, category professional training

Note. The figure shows the codes and number of the paragraph corresponding to the transcript in which they are present. In this case, even though the code for decontextualization of training is the most frequent (paragraphs 2, 4, 7, 28, 29, 30, 31 and 32) versus self-management of training (paragraphs 19, 20, 22, 23 and 24), equality between both is observed, different from what happened in the previous category, where there was an imbalance between the frequency of the different codes.

MAXMapas, MAXQDA, 2020, multigrade personal project.

The lack of interest of teacher relies on calls of trainings oriented only to a certain specific level, being more theoretical than practical and not lasting enough, “they always do trainings or workshops for teachers who teach one degree, we do not have many options because we are multigrade teachers. We are a little abandoned and they just tell us to do the same. But you cannot do it because all the grades are together” (E5), “I have been trained but I have not been given strategies, it is more about theory or concepts” (E1), “four months ago I was in a training in a study circle with a very good psychologist,

but we discussed few things because of the time” (E7).

3.3 Multigrade: facing discrimination

This last result shows the processing of the paragraphs that make up the third document analyzed, “experiences of discrimination during multigrade”, corresponding to the same category. The subcategories racism, machismo and exclusion from the community were presented. For this purpose, the codes were determined:

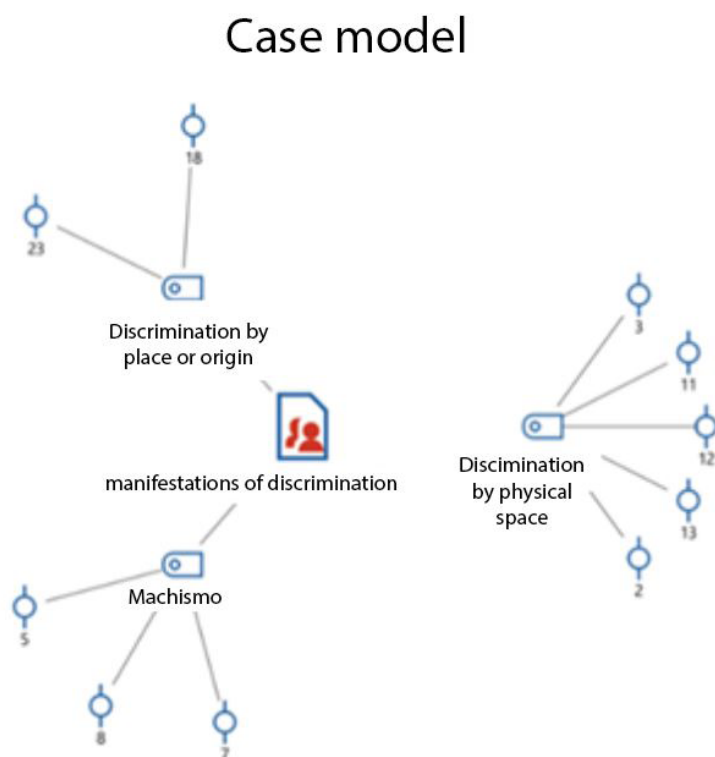


discrimination based on race or physical condition, machismo and discrimination on the basis of place or area of origin. With these codes, the

MAXMaps tool modeled a case shown in Figure 4. la herramienta MAXMapas modeló un caso que se muestra en la figura 4.

Figure 4

Model case, category experiences of discrimination during multigrade



Note. The figure shows the codes and number of the paragraph corresponding to the transcript in which they are present. In this case, the code for decontextualization of training is the most frequent (paragraphs 2, 4, 7, 28, 29, 30, 31 and 32) versus self-management of training (paragraphs 19, 20, 22, 23 and 24).

MAXMaps, MAXQDA, 2020, a multigrade personal project.

There are constant experiences of discrimination suffered by the students and the teachers in the community in the different testimonies of the interviewees. For example, they deal with racist attitudes to black students: “Most discriminate black students” (E1), condemning the attitude of parents “some parents who are racist believe better than others” (E1).

Likewise, teachers also experience discrimination due to their skin color, because they come from the highlands, or even because they

come from the rural area: “I have been told black, and actually I am black and I am very proud to be” (E4), or “I have not been discriminated by being black, but I have been discriminated because I am from the Highland” (E1). The cultural complex and the lack of recognition of its origins is also observed: “When starting to work in the institution that I currently work, they did not want me to work there, because I studied there, and they said that could not be possible” (E6).



Among other discriminatory attitudes are machismo and harassment in school: "I don't know what it is called, but many male students think they are better than girls" (E6), "machismo, for example, they say that wives need to work in the house and men are the ones who provide food" (E7), or "one child thought he could control and give orders to the rest; and another student who discriminates two fat children because of their physical appearance" (E2).

4 Discussion

According to the data obtained, it was possible to observe that the daily life of multigrade teachers represents a complex educational reality, and it is very abandoned in Ecuador. Interculturality, inclusive education, teacher training and updating are essential challenges in rural areas.

The meaning of multigrade by teachers is characterized because it is very different from traditional teaching. Uninterest with the task they perform is due to the feeling of administrative, professional, and cultural isolation. Training and experience opportunities are the most valued aspects.

Regarding teacher training, it is possible to perform this profession without a university degree or without the proper specialization, observing a contradiction with the levels taught in multigrade schools, since they are in charge of all the levels of elementary school.

Thus, the lack of training of teachers and levels of attention mentioned by different authors such as Salazar-Gómez and Tobón (2018), who express the absence of public planning for the selection and updating of teachers, is reported. The low level of teaching competencies necessary for their development in this context is another alarm that leads to a gap by the educational administration and the uninterest and dropout of multigrade teachers.

Additionally, the workload or the feeling of isolation in these situations of rurality and disconnection. In this sense, the attention and

updating of teachers is a relevant issue for the identity of the teacher and his/her aptitude to face all the responsibility, colliding otherwise as a risk factor (Calderón, 2015). Isolation goes beyond the lack of minimum resources because teachers also refer to the lack of attention from the educational administration by not recognizing their work in the type of trainings that are offered and in the schedule in which they are planned, leading to the self-management of their training. The solidarity between partners is interesting in terms of their training needs.

Cultural conditions of machismo and discrimination found in their workplaces are even more worrying. The cultural complex mentioned by Lara and Herrán (2016), along with the lack of recognition of women's teaching work, alert the strong distance even from the legal reality that focuses on interculturality (Lara, 2019). Combining cleaning and maintenance work of the school with their role as teachers could increase the macho and welfare culture in this rural area, as pointed out by Mogollón and Solano (2011). Multigrade teachers are apparently more rejected than supported.

5 Conclusions

Being a multigrade teacher in the coast of Ecuador is a task performed by many women who suffer discrimination from a political-institutional, racial, regional and gender perspective. Discrimination is observed in social and professional isolation, as well as in the disconnection from educational policies. The absence of their situation in the proposals for professional updating confirms this fact. Additionally, there is a culture of rejection on the part of the rural community in which they coexist in their daily activities and the silence of the educational administration.

Beyond the traditional work of a teacher, they must deal with the administrative workload and the physical functioning of the center. This is a topic that promotes machismo in these areas.



The challenge for multigrade not only relies in the complex and demanding teaching responsibility, but also in the rural educational culture. The daily life of a woman who is a teacher in this context is represented from a reality silenced by the educational administration, isolation, difficult coexistence with the community, and discrimination. It would be interesting if future research would include the testimonials of parents in these rural contexts, as well as district authorities as representatives of the administration.

However, the passion, commitment, and bonds of solidarity between them are remarkable. Their professional updating is characterized by the self-management of teachers. The preparation obtained in formal educational processes seems to be insufficient and distant from the real needs that teachers require in these contexts.

The educational culture of the kawsay sumak to which Ecuador aspires as a society is far from the story of teachers whose experiences are collected in this work. Regarding knowledge and the horizontal relationship that an intercultural education and the good living seeks, it is observed an egocentric culture in public policy by discriminating against these teachers, along with egocentric behaviors like machismo and stereotyped behavior in the rural population toward multigrade teachers.

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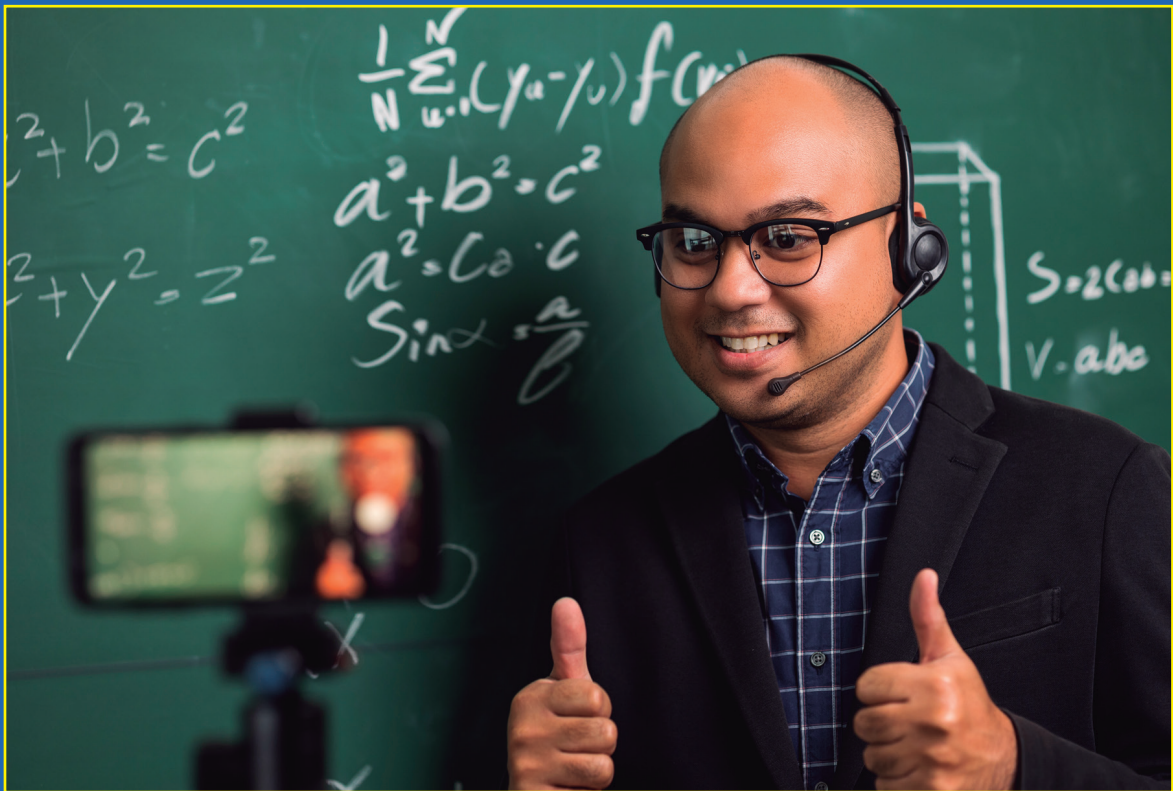


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Publication guidelines

(Normas Editoriales)



Source: <https://www.shutterstock.com/es/image-photo/indian-young-teacher-man-wearing-headset-1928433692>

Publication Guidelines of «Alteridad»

<http://alteridad.ups.edu.ec/>
p-ISSN:1390-325X / e-ISSN:1390-86

1. General information

«Alteridad» is a bilingual scientific publication of the Salesian Polytechnic University of Ecuador (UPS), published since January 2006 uninterruptedly, on a semi-annual basis (January-July).

It is an arbitrated scientific journal, which uses the peer-review system under the double-blind review, in accordance with the publication standards of the American Psychological Association (APA). The compliance with this system ensures authors an objective, impartial and transparent review process, making it easier for authors to be included in reference international databases, repositories and indexes.

«Alteridad» is indexed in the Web of Science's Emerging Sources Citation Index (ESCI), at the Scientific Electronic Library Online (SciELO), in the REDALYC Scientific Information System, in the directory and selective catalog of the Regional Online Information System for Scientific Journals of Latin America, the Caribbean, Spain and Portugal (Latindex), in the Directory of Open Access Journals (DOAJ), in the European Reference Index for the Humanities and Social Sciences (ERIHPLUS), in the Ibero-American Network of Innovation and Scientific Knowledge (REDIB), on the Dialnet Portal; it is evaluated in the Information Matrix for Journal Analysis (MIAR), the Integrated Classification of Scientific Journals (CIRC), and the Qualis review system for CAPES journals. In addition, it is in repositories, libraries and specialized catalogs around the world.

The journal is published in two versions: electronic (e-ISSN: 1390-8642) and printed (ISSN: 1390-325X) in Spanish and English; each manuscript is identified with a Digital Object Identifier System (DOI). All articles published in «Alteridad» have the Creative Commons Attribution-Non-Commercial-Share Equal license (RoMEO blue journal).

2. Scope and policies

2.1. Topics

«Alteridad» is a journal specialized in Education and its transdisciplinary lines such as Didactics, Public Policies, School Management, Edu-communication, ICT, Social Pedagogy, among others; and all those disciplines related to the main topic.

2.2. Sections

The journal has a semi-annual periodicity (20 articles per year), published in January and July and has two sections of five articles each by number; the first referring to a **Monographic** topic



prepared in advance and with thematic topic and the second, a section of **Miscellaneous**, composed of varied contributions within the theme of the publication.

2.3. Contributions

All manuscripts must be original, and must not have been published in any other journal or must not be in the arbitration or publication process in another journal. Empirical research results are published in Spanish, Portuguese or English, and studies and state-of-the-art are also admissible:

- **Researches:** 5000 to 6500 text words, including title, abstracts, descriptors, tables and references. Particular assessment will be made of research results, methodological rigor, the relevance of the subject, the quality of scientific discussion, the variety, timeliness and richness of bibliographic references (preferably publications indexed in JCR and Scopus). At least 35 references are expected.
- **Literature studies and reviews:** 6000 to 7000 text words, including tables and references. The debate generated, the relevance of the subject, the originality, current and selective contributions and references of around 70 works (preferably from publications indexed in JCR and Scopus) will be particularly valued.

3. Editorial process

3.1. Submission of manuscripts

Manuscripts must be submitted only and exclusively through the Open Journal System (OJS), in which all authors must register in advance, although only one will be responsible for the correspondence. No author may submit or review two manuscripts simultaneously, estimating a time of four consecutive numbers (2 years). An article may have a maximum of 3 authors, although if justified depending on the size of the study, there may be up to 5.

«Alteridad» informs the reception of the manuscript submitted by the authors; the information related to the acceptance or rejection of the manuscript is sent by email and the platform; and in the case of acceptance, the author is also informed of the editing process.

In the website of the journal, in the Guidelines section, are presented the Guidelines for the Authors, the format of the structure of the articles, the cover page and cover letter, the pre-submission list, the evaluation forms by the external reviewers and a guide for the submission of the article through OJS. Before the submission, it is strongly recommended that the manuscript be checked with the Pre-Check Protocol. Two documents should be sent simultaneously:

- a) Cover page and cover letter (use the official model), on which will appear
 - Cover page (Title, Abstract and key words provided in the Manuscript).
 - Full name of each of the authors, organized in priority order; followed by the professional category, institution, email of each author and ORCID number. It is mandatory to indicate if the authors have a PhD academic degree (include Dr. before the name).
 - A **Cover letter** will also be included indicating that the manuscript is an original contribution, has not been sent or evaluated in another journal, with the signature of the authors, and acceptance (if applicable) of formal changes to the manuscript compliant with the rules and partial transfer of rights to the publisher.
- b) Fully anonymized **manuscript**, in accordance with the rules referred to in section 4.



3.2. Revision process

Upon having received the document and in a maximum period of 30 days, the correspondence author shall receive a notification, indicating whether the manuscript is estimated or dismissed for the arbitration process by the scientific reviewers. In the case that the article has formal problems, or does not address the educational subject, or has a high similarity percentage to another document(s), the editorial board shall dismiss the work without the option to return it. Conversely, if it has superficial problems, it will be returned to the author for the corrections before starting the evaluation process. The submission date of the article will be considered based on the final submission when the article is presented with the corrections.

The articles will be scientifically evaluated by an average of three experts of the topic. Reports will indicate the following recommendations: Accept the Submission, Publishable with Modifications, Sent the manuscript back for its Review, Not Publishable. The acceptance or rejection of the manuscript for its publication will be decided from the analysis of external reports. In the case of dissenting results, it shall be forwarded to a new opinion, which shall be final. The protocol used by reviewers is public (researches; studies and state-of-the-art).

In general, once the external scientific reviews are taken into view, the criteria justifying the decision on the acceptance/rejection of the manuscript by the Editorial board are:

- Current and novelty.
- Relevance and significance: Advancement of scientific knowledge.
- Originality.
- Reliability and scientific validity: Proven methodological quality.
- Organization (logical coherence and formal presentation).
- External support and public/private funding.
- Co-authoring and internationalization degree of the proposal and the team.
- Presentation: Good writing.

The timeline for the scientific evaluation of manuscripts, after the previous estimation procedures by the Editorial Board is a maximum of 100 days. In relation to the manuscripts sent for Calls for papers, their scientific review dates begin once the call finishes. Manuscripts that are positively evaluated and require modifications must be sent with the changes, within the next 15 days.

3.3. Editing and publishing of the manuscript

The edition and layout processes of the accepted articles is performed by the Technical Board of the journal in coordination with the Abya-Yala Editorial. «Alteridad» reserves the right to make style corrections and editorial changes if necessary to improve the manuscript. A proof of printing in PDF format will be sent to the authors for the correction of typography and spelling in a maximum of three days.

Abya-Yala Editorial will carry out, free of charge for the authors, the professional translation of the final version of the manuscript into the English language (or Spanish, according to the original version), which will guarantee the consultation and international dissemination of the manuscript. The articles will be published on the journal's platform in a timely manner. All articles, in their two language versions (Spanish and English), are published in PDF, HTML, EPUB and XML-Jats format.



3.4. Promotion and dissemination of the published article

The authors are committed to give maximum diffusion to their article published in «Alteridad». In this sense, they are encouraged to promote their published article on academic networks (Academia.edu, ResearchGate, Mendeley, Kudos), social (Twitter, Facebook, LinkedIn, also publishing the DOI), institutional repositories, web or blog staff, among others. Authors are also encouraged to share the published article through email lists, research groups and personal contacts.

«Alteridad» has a Metric Measurement System (PlumX) that allows verifying the compliance with this commitment. For the submission of future articles by authors of «Alteridad», the impact of previous works will be taken into account.

4. Structure of the manuscripts

The manuscripts shall be submitted in typeface Arial 10, simple spacing, fully justified and without tabs or white space between paragraphs. Only large blocks (title, authors, abstracts, key words, credits, and captions) will be separated with white space. The page must be 2 centimeters in all its margins. Manuscripts must be submitted in Microsoft Word document (.doc or .docx), requiring the file to be anonymized in File Properties to avoid the information related to the identification of the author/s.

4.1. Cover page

Title (Spanish and English): Concise but informative, in Spanish on the first line and in English in the second, consisting of as many significant terms as possible. The title is not only the responsibility of the authors, and changes can be proposed by the Editorial Board. A maximum of 80 characters with space are accepted.

Abstract (Spanish and English): It must be presented in a concise way and in this order: Justification, objectives, methodology used (approach and scope), more relevant results, discussion and main conclusions. It must be written impersonally “The present work analyzes...”. In the case of the Abstract, the use of automatic translators will not be accepted because of their poor quality. It will be between 220/230 words.

Key words (Spanish and English): 6 keywords must be presented for each language version directly related to the topic of the manuscript. The use of the keywords presented in UNESCO's Thesaurus will be positively valued (<http://bit.ly/2kIgn8I>) or the controlled vocabulary of IRESIE (<http://bit.ly/2mzg4m8>).

4.2. IMRDC Structure

For those works involving empirical research, the manuscripts will strictly respect the IMRDC structure, with the headings of Economic Supports and Notes being optional. The works involving Literature Studies and Revisions may be more flexible under their headings, especially in Methodology, Results and Discussion. In all types of works, bibliographic references are mandatory.

1. **Introduction and state of the play:** It should include the theoretical foundations and purpose of the study, using bibliographic citations, as well as the review of the most significant literature of the topic at the national and international level. The use of high-impact references (JCR and Scopus) will be positively valued.



2. **Methodology:** It must be written in a way that the reader can easily understand the development of the research. It should contain the explanation on the approach (quantitative, qualitative or mixed) and the scope (exploratory, descriptive, correlational or explanatory). When appropriate, it shall describe the sample and the sampling form, as well as it must refer to the type of statistical analysis applied. If it is an original methodology, it is necessary to set out the reasons that have led to its use and describe the possible limitations.
3. **Results:** Efforts will be made to highlight the most relevant results and observations of the investigation, describing, without making judgments, the material and methods used for the analysis. The results will be presented in figures or/and tables according to the journal's standards (See section 4.4). They will appear in a logical sequence in the text, tables or figures, avoiding data redundancy.
4. **Discussion and conclusions:** Discussion and conclusions: It will summarize the most important findings, relating the observations with interesting studies, pointing to contributions and limitations, without resulting in data already commented in other sections. In addition, the discussion and conclusions section should include deductions and lines for future research.

4.3. Economic support and notes

Economic support (optional): Council Science Editors recommends that authors specify the source of funding for the research. Works on the endorsement of competitive national and international projects will be considered a priority. In any case, for the scientific assessment of the manuscript, it must be anonymized with XXXX only for its initial evaluation, in order not to identify authors and research teams, which must be set out in the Presentation Letter and subsequently in the final manuscript.

Notes (optional) will go, only if necessary, at the end of the article (before references). They should be used to clarify terms or make marginal annotations. Note numbers are placed in superscript, both in the text and in the final note. Notes collecting simple bibliographic citations (without comments) are not allowed, as these should be in the references. If it contains a cite, the reference must also be found in the Bibliography section.

4.4. Bibliography

Bibliographical citations should be reviewed in the form of references to the text. Bibliography that is not cited should not be included in the text. Its number must be sufficient and necessary to contextualize the theoretical framework, methodology used and research results in an international research space: Minimum 35 for empirical research manuscripts, and around 70 for literature studies and reviews.

They will be presented alphabetically by the author's first last name (adding the second one only in case the first one is very commonly used, and joined with a hyphen). The quote should be extracted from the original documents, preferably journals and to a lesser extent books. Given the significance of citation indices and impact factor calculations, the use of references from indexed publications in JCR and/or Scopus and the correct citation following APA 6 norms is valued (<http://bit.ly/2meVQcs>).

It is mandatory that quotes with DOI (Digital Object Identifier System) be reflected in the References (can be obtained on <https://search.crossref.org/>). All journals and books without DOI must contain a link (in its online version, if applicable, and in a shorten version using Bity: <https://bitly.com/>), and the websites must include the consultation date using the format provided.



Journal articles must be presented in English, with the exception of those in Spanish and English, in which case they will be presented in both languages using square brackets.

Norms for the references

a) Periodic publications

- **Journal article (one author):** Ochoa, A. (2019). The type of participation promoted in schools is a constraint factor for inclusive education. [El tipo de participación que promueve la escuela, una limitante para la inclusión]. *Alteridad*, 14(2), 184-194. <https://doi.org/10.17163/alt.v14n2.2019.03>
- **Manuscript from a journal (until twenty authors):** Guarderas, P., Larrea, M., Cuvi, J., Vega, C., Reyes, C., Bichara, T., Ramírez, G., Paula, Ch., Pesantez, L., Íñiguez, A., Ullauri, K., Aguirre, A., Almeida, M., & Arteaga, E. (2018). Sexual harassment in Ecuadorian universities: Content validation for instrument development. [Acoso sexual en las universidades ecuatorianas: Validez de contenido de un instrumento de medición]. *Alteridad*, 13(2), 214-226. <https://doi.org/10.17163/alt.v13n2.2018.05>
- **Manuscript from a journal (without DOI):** López, L., & Ramírez-García, A. (2014). Medidas disciplinarias en los centros educativos: ¿Suficientes contra el acoso escolar? *Perfiles Educativos*, 36(145), 32-50. <https://bit.ly/37Xd5mw>.

b) Books and chapters of books

- **Complete books:** Cuéllar, J.C., & Moncada-Paredes, M.C. (2014). *El peso de la deuda externa ecuatoriana*. Abya-Yala.
- **Chapter of books:** Padilla-Verdugo, J. (2014). La Historia de la Educación desde los enfoques del conocimiento. In E. Loyola (Ed.), *Ciencia, Tecnología y Sociedad (CTS). Miradas desde la Educación Superior en Ecuador* (pp. 107-128). Abya-Yala. <https://bit.ly/3etRnZH>

c) Electronic means

- Aunión, J. (2011, marzo 12). La pérdida de autoridad es un problema de toda la sociedad, no es específico del aula. *t*. <https://bit.ly/2NlM9Dp>

Guidelines for headings, tables and figures

The headings of the article shall be numbered in Arabic. These will be without full case of capital letters, no underscores, no bold ones. The numbering must be at most three levels: 1. / 1.1. / 1.1.1. A carriage return will be established at the end of each numbered heading.

Tables and figures must be presented in the text in Microsoft Word® located on the place where the authors consider they should be. They shall be used only when necessary and suitable, their use should be limited for reasons of spaces (maximum 6 between tables and figures). Both must be listed in Arabic and titled with the description of their content. If the source of the table or figure is not of its own elaboration, the authors must incorporate the source consulted below the table [for example, Source: Romero-Rodríguez (2016, p. 32)].

Tables must be elaborated in Microsoft Word document, thus tables cut and pasted from other documents that cannot be edited in the diagramming process will not be accepted. The figures, in addition to being incorporated in the Microsoft Word document®, must be sent as supplementary



material during the submission in the «Alteridad» OJS, with a quality greater than 600 dpi, in TIFF, JPEG or PNG files.

5. Fees and APC

«Alteridad» is an Open Access journal, included in the Directory of Open Access Journals (DOAJ) that offers all its production online for the entire scientific community. It also does not set any economic fees throughout the editorial process for the publication of the articles, including scientific review, layout and translation thereof. There is no publication fee, no Article Processing Charge (APC) associated with this publication, neither for authors nor for readers. The journal is also licensed by Creative-Commons Attribution-Non-Commercial-Share Equal (RoMEO blue journal), which allows free access, download and archive of published articles. All expenses, inputs and financing of «Alteridad» come from the contributions made by the Salesian Polytechnic University.

6. Ethical responsibilities

Each author shall submit a responsible statement of authorship and originality, as well as their ethical responsibilities.

- **Originality:** The works must be original and should not be evaluated simultaneously in another publication, being the responsibility of the authors to comply with this standard. The opinions expressed in the published articles are the responsibility of the author/s «Alteridad» as CrossRef's international partner, uses the CrossCheck® and iThenticate® anti-plagiarism tool to ensure the originality of the manuscripts.
- **Authorship:** The list of signatory authors should include only those who have contributed intellectually to the development of the work. Collaborating in data collection is not sufficient criteria of authorship. «Alteridad» rejects any responsibility for possible conflicts arising from the authorship of the manuscripts published.
- **Transmission of copyright:** the transfer of rights of the manuscript published in «Alteridad» will be included in the cover letter. The Salesian Polytechnic University (the publisher) has the copyright of published articles; it favors and allows the reuse of these under the license of use indicated above.



Normas de Publicación en «Alteridad»

<http://alteridad.ups.edu.ec/>
p-ISSN:1390-325X / e-ISSN:1390-8642

1. Información general

«Alteridad» es una publicación científica bilingüe de la Universidad Politécnica Salesiana de Ecuador (UPS), editada desde enero de 2006 de forma ininterrumpida, con periodicidad fija semestral (enero-julio).

Es una revista científica arbitrada, que utiliza el sistema de evaluación externa por expertos (*peer-review*), bajo metodología de pares ciegos (*doble-blind review*), conforme a las normas de publicación de la *American Psychological Association* (APA). El cumplimiento de este sistema permite garantizar a los autores un proceso de revisión objetivo, imparcial y transparente, lo que facilita a la publicación su inclusión en bases de datos, repositorios e indexaciones internacionales de referencia.

«Alteridad» se encuentra indexada en el *Emerging Sources Citation Index* (ESCI) de *Web of Science*, en la *Scientific Electronic Library Online* (SciELO), en el Sistema de Información Científica REDALYC, en el directorio y catálogo selectivo del Sistema Regional de Información en Línea para Revistas Científicas de América Latina, el Caribe, España y Portugal (Latindex), en el *Directory of Open Access Journals* (DOAJ), en el *European Reference Index for the Humanities and Social Sciences* (ERIHPLUS), en la Red Iberoamericana de Innovación y Conocimiento Científico (REDIB), en el Portal Dialnet; está evaluada en la Matriz de Información para el Análisis de Revistas (MIAR), en la Clasificación Integrada de Revistas Científicas (CIRC), y en el sistema Qualis de revisión de revistas de CAPES. Además, se encuentra en repositorios, bibliotecas y catálogos especializados de todo el mundo.

La revista se edita en doble versión: electrónica (e-ISSN: 1390-8642) e impresa (ISSN: 1390-325X) en español e inglés; siendo identificado cada trabajo con un *Digital Object Identifier System* (DOI). Todos los artículos publicados en «Alteridad» tienen licencia Creative Commons Reconocimiento-No-Comercial-Compartir igual (RoMEO blue journal).

2. Alcance y política

2.1. Temática

«Alteridad» es una revista especializada en Educación y sus líneas transdisciplinarias como Didáctica, Políticas Públicas, Gerencia de Centros Escolares, Educomunicación, TIC, Pedagogía Social, entre otras; y todas aquellas disciplinas conexas interdisciplinariamente con la línea temática central.

2.2. Secciones

La revista tiene periodicidad semestral (20 artículos por año), publicada en los meses de enero y julio y cuenta por número con dos secciones de cinco artículos cada una, la primera referida a un tema **Monográfico** preparado con antelación y con editores temáticos y la segunda, una sección de **Misceláneas**, compuesta por aportaciones variadas dentro de la temática de la publicación.



2.3. Aportaciones

Todos los trabajos deben ser originales, no haber sido publicados en ningún medio ni estar en proceso de arbitraje o publicación. Se editan preferentemente resultados de investigación empírica, redactados en español, portugués o inglés, siendo también admisibles estudios y selectas revisiones de la literatura (*state-of-the-art*):

- **Investigaciones:** 5000 a 6500 palabras de texto, incluyendo título, resúmenes, descriptores, tablas y referencias. Se valorarán especialmente los resultados de la investigación, el rigor metodológico, la relevancia de la temática, la calidad de la discusión científica, la variedad, actualidad y riqueza de las referencias bibliográficas (preferiblemente de publicaciones indexadas en JCR y Scopus). Se esperan mínimo 35 referencias.
- **Estudios y revisiones de la literatura:** 6000 a 7000 palabras de texto, incluidas tablas y referencias. Se valorará especialmente el debate generado, la relevancia de la temática, la originalidad de las aportaciones y referencias justificadas, actuales y selectivas de alrededor de 70 obras (preferiblemente de publicaciones indexadas en JCR y Scopus).

3. Proceso editorial

3.1. Envío de manuscritos

Los manuscritos deben ser enviados única y exclusivamente a través del *Open Journal System* (OJS), en el cual todos los autores deben darse de alta previamente, si bien uno solo de ellos será el responsable de correspondencia. Ningún autor podrá enviar o tener en revisión dos manuscritos de forma simultánea, estimándose una carencia de cuatro números consecutivos (2 años). Un artículo podrá tener como máximo 3 autores, aunque si se justifica en función del tamaño del estudio, podrán ser hasta 5.

«Alteridad» acusa recepción de los trabajos enviados por los autores, informa por email y la plataforma del proceso de aceptación o rechazo; y en el caso de aceptación, del proceso de edición.

En el Portal oficial de la revista, en la sección Normativas, están las Normas para Autores, el formato de estructura de los artículos, la Portada y Carta de presentación, el chequeo previo al envío, los formularios de evaluación por parte de los revisores externos y una guía para el envío del artículo a través de OJS. Antes de su envío se recomienda encarecidamente que se compruebe el manuscrito con el Protocolo de chequeo previo. Deben remitirse simultáneamente dos documentos:

a. Portada y Carta de presentación (usar el modelo oficial), en la que aparecerán:

- **Portada** (Título, Resumen y Descriptores previstos en el Manuscrito).
- **Nombre y apellidos completos** de cada uno de los autores, organizados por orden de prelación; seguido por la categoría profesional, centro de trabajo, correo electrónico de cada autor y número de ORCID. Es obligatorio indicar si se posee el grado académico de doctor (incluir Dr./Dra. antes del nombre).
- Se incluirá además una **declaración** (Cover letter) de que el manuscrito se trata de una aportación original, no enviada ni en proceso de evaluación en otra revista, confirmación de las autorías firmantes, aceptación (si procede) de cambios formales en el manuscrito conforme a las normas y cesión parcial de derechos a la editorial.

b. Manuscrito totalmente anonimizado, conforme a las normas referidas en el epígrafe 4.



3.2. Proceso de revisión

En un plazo máximo de 30 días, a partir de la recepción del documento, el autor de correspondencia recibirá una notificación, indicando preliminarmente si se estima o desestima para el arbitraje por los revisores científicos. En el caso de que el artículo presente deficiencias formales, no trate el tema educativo, o tenga un elevado porcentaje de similitud con otro(s) documento(s), el Consejo editorial desestimaré el trabajo sin opción de vuelta. Por el contrario, si presenta carencias superficiales de forma, se devolverá al autor para su corrección antes de comenzar del proceso de evaluación. La fecha de recepción del artículo no computará hasta la recepción correcta del mismo.

Los artículos serán evaluados científicamente por una media de tres expertos en el tema. Los informes indicarán las siguientes recomendaciones: Aceptar el envío, Publicable con modificaciones, Reenviar para revisión, No publicable. A partir del análisis de los informes externos, se decidirá la aceptación o rechazo de los artículos para su publicación. En el caso de resultados discrepantes se remitirá a un nuevo dictamen, el cual será definitivo. El protocolo utilizado por los revisores es público (Investigaciones; Estudios y revisiones de la literatura).

En general, una vez vistas las revisiones científicas externas, los criterios que justifican la decisión sobre la aceptación/rechazo de los trabajos por parte del Consejo Editor son los siguientes:

- Actualidad y novedad.
- Relevancia y significación: Avance del conocimiento científico.
- Originalidad.
- Fiabilidad y validez científica: Calidad metodológica contrastada.
- Organización (coherencia lógica y presentación formal).
- Apoyos externos y financiación pública/privada.
- Coautorías y grado de internacionalización de la propuesta y del equipo.
- Presentación: Buena redacción.

El plazo de evaluación científica de manuscritos, superados los trámites previos de estimación por el Consejo Editor, es de 100 días como máximo; los remitidos para *Calls for papers*, sus fechas de revisión científica se inician al cierre de los mismos. Los trabajos que sean evaluados positivamente y requieran modificaciones, deberán ser reenviados con los cambios, dentro de los siguientes 15 días.

3.3. Edición y publicación del manuscrito

El proceso de corrección de estilo y maquetación de los artículos aceptados es realizado por el Consejo Técnico de la Revista en coordinación con la Editorial Abya-Yala. «Alteridad» se reserva el derecho de hacer corrección de estilo y cambios editoriales que considere necesarios para mejorar el trabajo. A los autores de artículos se enviará una prueba de imprenta en formato PDF para su corrección únicamente de tipografía y ortografía en un máximo de tres días.

La Editorial Abya-Yala realizará, gratuitamente para los autores, la traducción profesional de la versión final del manuscrito al idioma inglés (o español, según la versión original), lo que garantizará su consulta y difusión internacional. Los artículos serán publicados en la plataforma de la revista en tiempo y forma. Todos los artículos, en sus dos versiones idiomáticas (español e inglés), son publicados en formato PDF, HTML, EPUB y XML-Jats.



3.4. Promoción y difusión del artículo publicado

Los autores se comprometen a darle la máxima difusión a su artículo publicado en «Alteridad». En este sentido, se les exhorta a compartir y archivar su artículo publicado en las redes académicas (Academia.edu, ResearchGate, Mendeley, Kudos), sociales (Twitter, Facebook, LinkedIn, publicando en estos también el DOI), repositorios institucionales, web o blog personal, entre otras. Asimismo, se anima a los autores a compartir el artículo publicado a través de listas de correo electrónico, grupos de investigación y contactos personales.

«Alteridad» cuenta con sistemas de medición de métricas alternativas (PlumX) que permiten verificar el cumplimiento de este compromiso. Para la postulación de futuros artículos de autores de «Alteridad», se tendrá presente el impacto de los trabajos anteriores.

4. Estructura de los manuscritos

Los trabajos se presentarán en tipo de letra Arial 10, interlineado simple, justificado completo y sin tabuladores ni espacios en blanco entre párrafos. Solo se separarán con un espacio en blanco los grandes bloques (título, autores, resúmenes, descriptores, créditos y epígrafes). La página debe tener 2 centímetros en todos sus márgenes. Los trabajos deben presentarse en documento de Microsoft Word (.doc o .docx), siendo necesario que el archivo esté anonimizado en Propiedades de Archivo, de forma que no aparezca la identificación de autor/es.

4.1. Portada

Título (español) / Title (inglés): Conciso pero informativo, en castellano en primera línea y en inglés en segunda, conformado por el mayor número de términos significativos posibles. El título no solo es responsabilidad de los autores, pudiéndose proponer cambios por parte del Consejo Editorial. Se aceptan como máximo 80 caracteres con espacio.

Resumen (español) / Abstract (inglés): Se describirán de forma concisa y en este orden: Justificación del tema, objetivos, metodología empleada (enfoque y alcance), resultados más relevantes, discusión y principales conclusiones. Ha de estar escrito de manera impersonal “El presente trabajo analiza...”. En el caso del *Abstract* no se admitirá el empleo de traductores automáticos por su pésima calidad. Tendrá como extensión entre 220/230 palabras.

Descriptores (español) / Keywords (inglés): Se deben exponer 6 descriptores por cada versión idiomática relacionados directamente con el tema del trabajo. Será valorado positivamente el uso de las palabras claves expuestas en el Thesaurus de la UNESCO (<http://bit.ly/2kIgn8I>) o del Vocabulario controlado del IRESIE (<http://bit.ly/2mgg4m8>).

4.2. Estructura IMRDC

Para aquellos trabajos que se traten de Investigaciones de carácter empírico, los manuscritos respetarán rigurosamente la estructura IMRDC, siendo opcionales los epígrafes de Apoyos y Notas. Los trabajos que se traten de Estudios y revisiones de la literatura podrán ser más flexibles en sus epígrafes, especialmente en Metodología, Resultados y Discusión. En todas las tipologías de trabajos son obligatorias las Referencias.

1. **Introducción y estado de la cuestión:** Debe incluir los fundamentos teóricos y el propósito del estudio, utilizando citas bibliográficas, así como la revisión de la literatura más significativa del



tema a nivel nacional e internacional. Se valorará positivamente el uso de referencias de alto impacto (JCR y Scopus).

2. **Metodología:** Debe ser redactado de forma que el lector pueda comprender con facilidad el desarrollo de la investigación. Deberá contener la explicación sobre el enfoque (cuantitativo, cualitativo o mixto) y el alcance (exploratorio, descriptivo, correlacional o explicativo). En su caso, describirá la muestra y la forma de muestreo, así como se hará referencia al tipo de análisis estadístico aplicado. Si se trata de una metodología original, es necesario exponer las razones que han conducido a su empleo y describir sus posibles limitaciones.
3. **Resultados:** Se procurará resaltar los resultados y las observaciones más relevantes de la investigación, describiéndose, sin hacer juicios de valor, el material y métodos empleados para el análisis. Los resultados se expondrán en figuras o/y tablas según las normas de la revista (Ver epígrafe 4.4). Aparecerán en una secuencia lógica en el texto, las tablas o figuras imprescindibles, evitando la redundancia de datos.
4. **Discusión y conclusiones:** Resumirá los hallazgos más importantes, relacionando las propias observaciones con estudios de interés, señalando aportaciones y limitaciones, sin redundar datos ya comentados en otros apartados. Asimismo, el apartado de discusión y conclusiones debe incluir las deducciones y líneas para futuras investigaciones.

4.3. Apoyos y Notas

Apoyos (opcionales): El *Council Science Editors* recomienda a los autor/es especificar la fuente de financiación de la investigación. Se considerarán prioritarios los trabajos con aval de proyectos competitivos nacionales e internacionales. En todo caso, para la valoración científica del manuscrito, este debe ir anonimizado con XXXX solo para su evaluación inicial, a fin de no identificar autores y equipos de investigación, que deben ser explicitados en la Carta de Presentación y posteriormente en el manuscrito final.

Las notas (opcionales) irán, solo en caso necesario, al final del artículo (antes de las referencias). Deben ser utilizadas para aclarar términos o hacer anotaciones marginales. Los números de notas se colocan en superíndice, tanto en el texto como en la nota final. No se permiten notas que recojan citas bibliográficas simples (sin comentarios), pues éstas deben ir en las referencias. En caso de contener alguna cita, su referencia deberá encontrarse también en la sección de References.

4.4. References

Las citas bibliográficas deben reseñarse en forma de referencias al texto. No debe incluirse bibliografía no citada en el texto. Su número ha de ser suficiente y necesario para contextualizar el marco teórico, la metodología usada y los resultados de investigación en un espacio de investigación internacional: Mínimo 35 para los manuscritos de investigaciones de carácter empírico, y alrededor de 70 para los estudios y revisiones de literatura.

Se presentarán alfabéticamente por el apellido primero del autor (agregando el segundo solo en caso de que el primero sea de uso muy común, y unido con guion). Las citas deberán extraerse de los documentos originales preferentemente revistas y en menor medida libros. Dada la trascendencia para los índices de citas y los cálculos de los factores de impacto, se valorarán positivamente el uso de referencias provenientes de publicaciones indexadas en JCR y/o Scopus y la correcta citación conforme a la Norma APA 6 (<http://bit.ly/2meVQcs>).



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Los artículos de revistas deben ser expuestos en idioma inglés, a excepción de aquellos que se encuentren en español e inglés, caso en el que se expondrá en ambos idiomas utilizando corchetes.

Normas para las referencias

a) Publicaciones periódicas

- **Artículo de revista (un autor):** Ochoa, A. (2019). The type of participation promoted in schools is a constraint factor for inclusive education. [El tipo de participación que promueve la escuela, una limitante para la inclusión]. *Alteridad*, 14(2), 184-194. <https://doi.org/10.17163/alt.v14n2.2019.03>
- **Artículo de revista (hasta veinte autores):** Guarderas, P., Larrea, M., Cuvi, J., Vega, C., Reyes, C., Bichara, T., Ramírez, G., Paula, Ch., Pesantez, L., Íñiguez, A., Ullauri, K., Aguirre, A., Almeida, M., & Arteaga, E. (2018). Sexual harassment in Ecuadorian universities: Content validation for instrument development. [Acoso sexual en las universidades ecuatorianas: Validez de contenido de un instrumento de medición]. *Alteridad*, 13(2), 214-226. <https://doi.org/10.17163/alt.v13n2.2018.05>
- **Artículo de revista (sin DOI):** López, L., & Ramírez-García, A. (2014). Medidas disciplinarias en los centros educativos: ¿Suficientes contra el acoso escolar? *Perfiles Educativos*, 36(145), 32-50. <https://bit.ly/37Xd5mw>

b) Libros y capítulos de libro

- **Libros completos:** Cuéllar, J.C., & Moncada-Paredes, M.C. (2014). *El peso de la deuda externa ecuatoriana*. Abya-Yala.
- **Capítulos de libro:** Padilla-Verdugo, J. (2014). La Historia de la Educación desde los enfoques del conocimiento. In E. Loyola (Ed.), *Ciencia, Tecnología y Sociedad (CTS). Miradas desde la Educación Superior en Ecuador* (pp. 107-128). Abya-Yala. <https://bit.ly/3etRnZH>

c) Medios electrónicos

- Aunión, J. (2011, marzo 12). La pérdida de autoridad es un problema de toda la sociedad, no es específico del aula. *El País*. <https://bit.ly/2NlM9Dp>

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Los epígrafes del cuerpo del artículo se numerarán en arábigo. Irán sin caja completa de mayúsculas, ni subrayados, ni negritas. La numeración ha de ser como máximo de tres niveles: 1. / 1.1. / 1.1.1. Al final de cada epígrafe numerado se establecerá un retorno de carro.

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