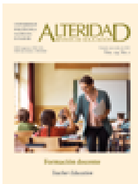
ALTERIDAD
REVISTA DE EDUCACIÓN
p-ISSN:1390-325X / e-ISSN:1390-8642
http://alteridad.ups.edu.ec

January-July 2024
Vol. 19, No. 1, 57-68
https://doi.org/10.17163/alt.v19n1.2024.05



Educational experience on modelling for Panamanian mathematics teachers

Experiencia educativa en modelización para docentes de matemática en Panamá

Carlos Ledezma es doctorando is coursing a PhD at Universidad de Barcelona, Spain (cledezar25@alumnes.ub.edu) (https://orcid.org/0000-0001-9274-7619)
Dra. Luisa Morales-Maure is a professor at Universidad de Panamá, Panamá (luisa.morales@up.ac.pa) (https://orcid.org/0000-0003-3905-9002)
Dr. Vicenç Font is a professor at Universidad de Barcelona, Spain (vfont@ub.edu) (https://orcid.org/0000-0003-1405-0458)

Received on: 2023-09-13 / **Revised on:** 2023-10-23 / **Accepted on:** 2023-11-27 / **Published on:** 2024-01-01

Abstract

Internationally, mathematical modelling has been gaining an important space in educational curricula, this is why the teaching of this process has been included in educational programmes for mathematics teachers. Due to this importance, in this study, an educational experience on modelling for practising secondary education mathematics teachers from the Panamanian school system is reported, whose objective is to analyse the solving procedures of these teachers to modelling problems and classify these solving procedures. The context is the Diploma Course in Mathematics Education Applied to Secondary Education, taught by the University of Panama, which included a submodule on modelling. The "modelling cycle from a cognitive perspective", which was taught in the submodule, was used to analyse the solving procedures for three problems posed to the participating teachers, through the written productions that they uploaded to the virtual platform of the diploma course. Four solving procedures could be identified in the 40 productions analysed, which varied from not totally understanding the wording of the problems to developing a whole modelling cycle. With these results, it is intended to provide a first overview of the teaching and learning of modelling in the Panamanian context and to lay the foundations for possible curricular adaptations to mathematics teaching.

Keywords: diploma, teacher education, mathematical modelling, Panama, mathematics teachers, practising teachers.

Resumen

A nivel internacional, la modelización matemática ha ido ganando un espacio importante en los currículos educativos, razón por la cual la enseñanza de este proceso ha sido incluida en los programas de formación de profesores de matemática. Dada esta importancia, este estudio reporta una experiencia educativa sobre modelización para profesores de matemática de educación secundaria en servicio del sistema escolar panameño, cuyo objetivo es analizar los procedimientos de resolución de estos profesores a problemas de modelización y clasificar dichas resoluciones. El contexto es el Diplomado en Educación Matemática Aplicada a Secundaria, impartido por la Universidad de Panamá, el cual incluyó un submódulo sobre modelización. El "ciclo de modelización desde una perspectiva cognitiva" que se impartió en el submódulo, se utilizó para analizar los procedimientos de resolución a tres problemas planteados a los profesores participantes, mediante las producciones escritas que cargaron a la plataforma virtual del diplomado. Se pudieron identificar cuatro procedimientos de resolución en las 40 producciones analizadas, que fluctuaban desde no comprender totalmente el enunciado de los problemas hasta desarrollar un ciclo completo de modelización. Con estos resultados, se pretende aportar una primera visión general sobre la enseñanza y el aprendizaje de la modelización en el contexto panameño, y sentar las bases para posibles adaptaciones curriculares a la enseñanza de la matemática.

Palabras clave: diplomado, formación de profesores, modelización matemática, Panamá, profesores de matemática, profesores en servicio.

Suggested citation (APA): Ledezma, C., Morales-Maure, L. & Font, V. (2024). Educational experience on modelling for Panamanian mathematics teachers. *Alteridad, 19*(1), 57-68. https://doi.org/10.17163/alt.v19n1.2024.05

1. Introduction and state-of-the-art

One of the competencies that allows individuals to be able to link their mathematical knowledge to the needs and demands of the 21st century is mathematical modeling (Maass et al., 2022). In this sense, it is required to prepare teachers with the right competencies for modeling and, thus, educate students in skills to develop this process (Blum & Borromeo Ferri, 2009). While there is a discussion in the literature on how to define *competence in modeling* (see Kaiser & Brand, 2015), on the one hand, this process is considered as one of the axes for problem solving in the international evaluation PISA (Organization for Economic Cooperation and Development, 2019) and, on the other hand, there is a consensus that working with modeling brings a number of benefits for mathematical learning (Blum, 2011).

Given this growing interest in modeling at the international level, this study aims to provide a first overview of a pioneering educational experience on modeling for mathematics teachers at the Panamanian school system. This continuing education program, developed in 2022, recognizes the importance of integrating modeling as a fundamental part of teaching mathematics in Panama. It is necessary that teachers include, among other relevant processes of mathematical activity, modeling in their classes, for which this program decided to incorporate their teaching to active teachers.

1.1 Mathematical modeling

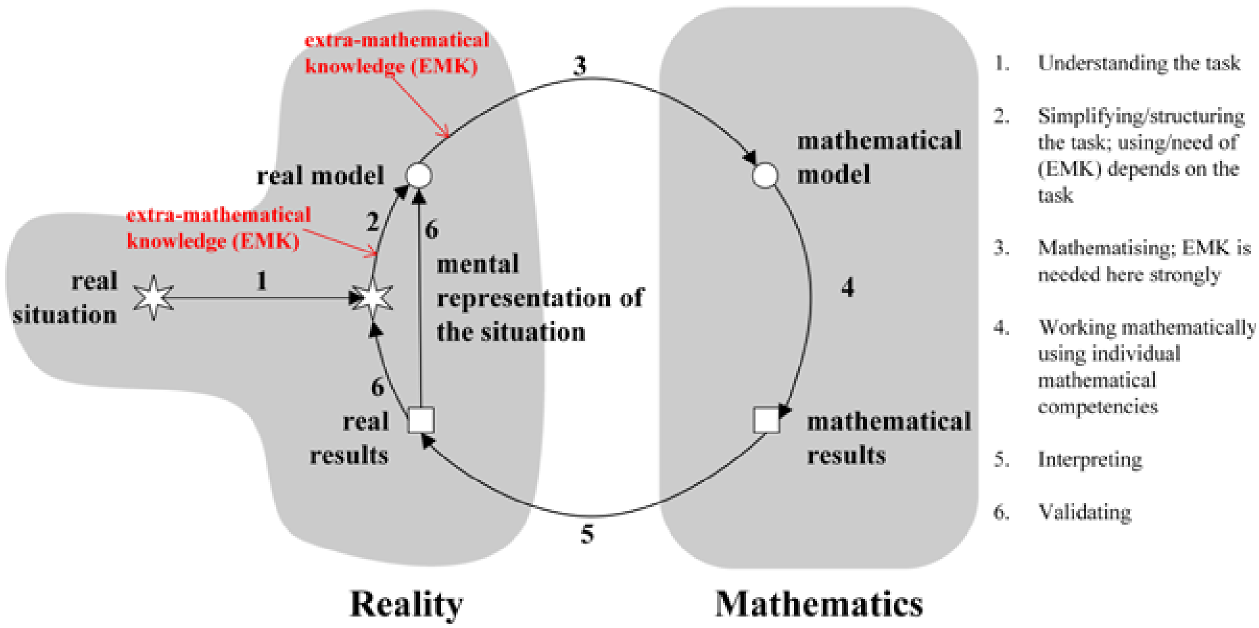
Mathematical modeling briefly describes the translation of a real problem into mathematics and its results back to reality (Pollak, 2007). In the specialized literature, different cycles have been proposed to analyze the modeling process (Borromeo Ferri, 2006) and different perspectives have emerged on its implementation (Preciado et al., 2023). This study uses the "modeling cycle from a cognitive perspective" (see figure 1), proposed by Borromeo Ferri (2018), which is framed in the realistic perspective of working with modeling (Abassian et al., 2020).

The choice of this cycle for this research is justified by (a) the authors' previous experience in theoretical works (see Ledezma et al., 2023) and (b) its use is part of the educational experience reported in this article. To explain the operation of the cycle of Figure 1, the *Problem Sugar Loaf*, which was presented during this educational experience, is used as an example.

> **Sugarloaf:** The Sugarloaf Cable Car takes approximately three minutes to travel from the station in the valley to the foot of Sugarloaf Mountain in Rio de Janeiro. It moves at a speed of 30 km/h and covers a height difference of approximately 180 meters. The chief engineer, Giuseppe Pelligrini, preferred to walk better, as he did in the past, when he was a mountaineer. First, he ran from the valley station across the vast plain to the mountain, and then climbed it in 12 minutes. What is the distance, roughly, that Giuseppe had to run from the valley station to the foot of the mountain? (adapted from Blum & Leiß, 2007, p. 224)

The *real situation* is given by the statement of the *Problem Sugarloaf*, in the form of a text with an image. Through their understanding, there can be a *mental representation of the situation* which involves, for example, establishing relationships with holidays and tourist sites (*extra-mathematical knowledge*), in order to understand what the problem requests (the distance between the valley station and the foot of the mountain). This *mental representation* must be simplified and structured in order to obtain a *real model* that represents the *real situation* posed; in this case you can simplify the mountain and cable as segments, and the cable car as a point, and then structure these simplifications in a drawing. The *mathematical model* takes into account the mathematical objects that allow explaining the *real situation* posed; in this case, the Pythagorean theorem can be applied. From the work with this *mathematical model* are obtained *mathematical results*, which must be interpreted in the context of the *real situation* to obtain real *results*; in this case, approximately 1.49 kilometers are obtained. But does this answer make sense in the context of the *Sugarloaf Problem*? One way to validate these *real results* would be by using a mapping application to measure distances at the actual location of Sugarloaf in Brazil.

Figure 1. *Modeling cycle from a cognitive perspective*



Note. Adapted from Borromeo Ferri (2018, p. 15).

The modeling process should not be understood in linear terms, but as a cycle, because both the context of the *real situation* and the mathematical aspects involved in its resolution can affect the *mathematical model* and the mathematical work with it (Blomhøj, 2004; Borromeo Ferri, 2007). The work with classroom modeling is usually carried out in small groups of students, who are asked a situation-problem that must be mathematized (Doerr & English, 2003; Shahbari & Tabach, 2019). This problem-situation, known as modeling problem, must meet certain characteristics (Borromeo Ferri, 2018): it must be open and complex, whose resolution is not limited to a specific answer or procedure, and where students must search for the relevant data; it must also be realistic and authentic, adding elements taken from the real world and presenting a situation consistent with an event that has occurred or that can occur in reality (in Palm terms, 2007); finally, it must be a *problem* (in Schoenfeld terms, 1994) that is resolvable through of a modeling cycle, which implies the development of all the phases that make up this cycle. Along with the above, modeling problems tend to have different paths to obtain a plausible and coherent solution in the context of the *actual situation* posed (English, 2003; Lesh & Doerr, 2003).

1.2 Mathematical modeling in teacher education

Given the relevance of this process, several studies have addressed the teaching of modeling for teachers, both in training and active.

One line has focused on the knowledge and competencies of mathematics teachers. In the Austrian context, Kuntze et al. (2013) study teachers' self-perceptions about their *pedagogical content knowledge* (PCK [Shulman, 1986]) related to modeling, considering both the PCK needed to help their students during the modeling process in the classroom, and what they think about their own professional development at the university level. By applying a questionnaire to 38 teachers in training and 48 teachers in service, the results showed a need for professional development that not only covers the PCK on modeling, but also the teaching of strategies for the pedagogical self-efficacy of teachers when implementing this process, for example, using technological tools. Likewise, in the German context, Greefrath et al. (2022) study the facets of professional competences for teaching modeling (see Blum, 2015), specifically those related to knowledge about modeling tasks and classroom interventions. These authors report the results of a 12-session seminar taught to three groups of future professors from different German universities, in which the improvement of the PCK on modeling of the participating subjects was evidenced. In the Spanish context, Ledezma et al. (2022) study the knowledge and beliefs about modeling of a future teacher from the analysis of the argumentation. In this study, the authors infer these knowledge and beliefs using the model of Knowledge and Didactic-Mathematics Competencies of the Professor of Mathematics (Godino et al., 2017), which they apply to the reflection made by the future teacher in his final work of master (see other studies in this line in Batista et al., 2022).

In the Singaporean context, Ng (2013) tackles the problem of the scarce efforts to incorporate modeling tasks in schools, even though the national curriculum introduced this process in mathematics teaching in 2003. To this end, we compare the results of the implementation of two modeling courses for primary school teachers with no previous experience with this process: one with 48 teachers in service (from a previous study [Ng, 2010]) and another with 57 teachers in training (the current study [Ng, 2013]). In both courses, teachers had to solve the *task Youth Olympic Games* (adapted from English, 2013). The results show the similarities and differences between both groups of teachers when solving the proposed task, suggesting a working method for the Singaporean context, that includes teachers in training and in service when addressing the teaching of modeling.

In the American context, Manouchehri (2017) reports on efforts to assist a group of active mathematics teachers to develop knowledge on modeling and its implementation in the school curriculum. The implementation context was a 25-hour professional development course, where 85 teachers worked on modeling tasks and discussed their implementation. This study reports the results of 25 teachers who participated in the course, where it was evident a growth in their knowledge about modeling from the mathematical challenges (construction and work with the mathematical model), pedagogical (strategies to develop this process in the classroom), and epistemological (obstacles during the modeling process) that they had to face.

In the case of this study, we report a pioneering educational experience with active mathematics teachers in Panama. Panama's school system is formed by the levels of Basic General Education (students aged 4-15 years) and Middle Education (students aged 15-18 years). In this context, the University of Panama, in collaboration with other foreign universities, implemented two diplomas for active professors of both educational levels during 2022: "Didactic Strategies for Teaching Mathematics" (EDEM Diploma) for Basic General Education, and "Mathematics Applied to Secondary Education" (EMAS Diploma) for Secondary Education. The objective of both graduates was to expand the pedagogical skills of mathematics teachers. This research focused on the EMAS Diploma, where one of the topics addressed was mathematical modeling.

1.3 Objective and research question

The approach of the research question on the results of this pioneering educational experience developed in the Panamanian context is the following: What are the procedures for solving modeling problems by active mathematics teachers participating in the EMAS Diploma? To answer this question, the modeling cycle represented in Figure 1 was used to analyze the resolution procedures to three problems posed to teachers during a sub-module of this diploma. These resolutions were classified according to the phases and transitions identified in their written productions. Finally, we reflect on these results and their possible implications for future implementations of this diploma in the Panamanian context.

The relevance of this study lies in two areas. On the one hand, it addresses a topic that has not been explored enough in the Panamanian context, such as the education of active mathematics teachers on modeling. Although there are different investigations on education of mathematics teachers in Panama, mainly based on the experience of the EDEM Diploma (see García-Marimón et al., 2021; Morales et al., 2019), these focus on the development of teacher reflection using the construct Criteria of Didactic Suitability (Breda & Lima, 2016). Although the national curriculum documents of Panama do not include a systematic work with modeling for the teaching of mathematics (see Ministry of Education of Panama, 2014a, 2014b, 2014c), the EMAS Diploma does include this process as a relevant subject to teach, since current trends that promote the inclusion of modeling in the processes of teaching and learning mathematics are assumed. On the other hand, the University of Panama, responsible for teaching these diplomas, is considered as a reference in teaching and research at the Central American level (García-Marimón, 2023; Morales-Maure, 2019). Therefore, this educational experience can be replicated in other countries of the region due to the existing sociocultural similarities.

2. Methodology

For this study, a qualitative research methodology was followed from an interpretative paradigm (Cohen et al., 2018). This section describes the methodological aspects.

2.1 Context of the investigation

This research was developed in the context of the EMAS Diploma, taught by the University of Panama during the period May-October 2022, with a total duration of 320 hours. The objective of this diploma is to contribute to the continuous professional development of mathematics teachers in Panamanian secondary education, which includes the design, implementation, evaluation, and improvement of mathematical teaching and learning processes, with theoretical support in the construct Didactic Suitability Criteria. In the 2022 EMAS Diploma course, 113 teachers from different areas of the country participated, who were grouped in the four virtual rooms of the online platform designed by the University of Panama by a teacher trainer. This program consisted of six modules taught in hybrid mode: (a) Introduction to Mathematics Education; (b, c, d) Didactics of Mathematics I, II, and III; (e) Social, family, and educational contexts; and (f) Reflection on the own practice. At the end of this program, the participating teachers received a certificate of completion of the course. The EDEM and EMAS Diplomas are pioneering educational experiences in the Panamanian context, which are not only supported by a government research project awarded by the University of Panama, but also by academics from foreign universities (especially from Spain and Hispanic America).

2.2 Sub-module on mathematical modeling and a priori analysis of modeling problems

Within the module "Introduction to Mathematical Education" is the submodule "Mathematical modeling". Due to the hybrid modality of the diploma, the participating professors could access a general explanation on the virtual platform about what modeling is and four problems of this type that they had to solve. The follow-up of this remote work was developed by the teacher trainer in charge of each virtual room. Along with this explanation, a lecture was given at the beginning of this sub-module to the participating teachers and trainers of the diploma, where the explanation on modeling available on the platform was expanded. In this lecture (which lasted 90 minutes), the speaker (the first author) began by explaining what is meant by mathematical modeling, what characterizes these types of problems, what strategies of working with this process are suggested to be followed in the classroom, and how the resolution of a modeling problem (the *Sugar Loaf Problem* in subsection 1.1) can be analyzed using the cycle of Figure 1. These problems are presented in Table 1, together with the *a priori* analysis of each one, in terms of the modeling cycle considered as the theoretical benchmark of the study.

While these modeling problems were discussed during the conference, the participating teachers had to solve them autonomously and upload their resolutions and responses to the virtual platform designed for the EMAS Diploma. Subsequently, the teacher-trainer in each virtual room provided feedback on the resolutions for the three problems.

Table 1. *Modeling issues raised during the EMAS Diploma*

Statements of problems		
Hay bales problem: To the end of summer, you can see mountains of hay bales in the countryside like the ones in the picture. The bales are arranged so that five are placed at the base, four in the next row, then three, two, and finally a ball of hay on the cusp. Try to find the height of the hay-bale mountain. (Adapted from Borromeo Ferri, 2018, p, 14)	Meanders Problem: In the Yamal Peninsula, northwest of Siberia, a series of active and abandoned meanders can be seen from the air in big rivers. The most recent sediments deposited in the convex parts of the meanders are shown in class color. What is the approximate length of the river with sediment? (Authors' Archives)	Boston Light Problem: In Massachusetts Bay there is a lighthouse called Boston Light, which was built in 1716 at a height of 31 meters. Their beacon was intended to warn ships approaching the coast. How far, approximately, was a ship when it first saw the light of the lighthouse? (Adapted from Borromeo Ferri, 2018, p. 106)
A priori analysis		
A *mental representation of the situation* involves, for example, establishing relationships with the field and hay bales (*extra-mathematical knowledge*), in order to understand what the problem requires (the height of the hay bales mountain). To build a *real model,* you can simplify the hay bales as circumferences of 1.5 meters in diameter and the woman as a segment of 1.7 meters straight (both by estimate), and then structure these simplifications in a drawing. In this case it is possible to apply the addition of hay bale heights as a *mathematical model*, which would give the *real result* a mountain of 6.75 meters high.	A *mental representation of the situation* involves evoking images of rivers and their sinuous behavior (*extra-mathematical knowledge*), in order to understand what the problem requires (the approximate length of the river with sediments). To build a *real model,* meanders can be simplified as semi-circumferences on a straight segment that crosses the river image with sediments, estimating a linear length of 30 kilometers (from the observation of maps), and then structuring these simplifications in a drawing. In this case, the addition of semi-circumference perimeters can be applied as a *mathematical model*, which would give as *real result* an approximate length of the river of 47 kilometers.	A *mental representation of the situation* involves, for example, establishing relationships with the coast, the lighthouses, the ships, and the horizon (*extra-mathematical knowledge*), in order to understand what the problem requires (the distance from where a ship first saw the light of the lighthouse). To build a *real model,* you can simplify the Earth as a radius circumference 6,371 kilometers, the lighthouse as a 31-meter segment of line, and the ship as a point, and then structure these simplifications into a drawing. In this case the Pythagorean theorem can be applied as a *mathematical model*, which would give as a *real result* an approximate distance of vision of 20 kilometers.

2.3 Collection and post-hoc analysis of modeling problems

Since the second and third authors of this article are academics involved in the design and coordination of the implementation of the EMAS Diploma, they had full access to the virtual platform where the participating professors uploaded their resolutions and answers to the three applied modeling problems (instruments). To this end, the participating teachers were asked to record, in the most orderly and explicit way possible, all their resolution procedures and not only their responses to these problems (data). In this way, the participating teachers could upload their written productions to the EMAS Diploma virtual platform in the form of scanned documents or prepared with a text processor.

Once the written productions were collected, they were labeled according to the group to which they belonged (G1 to G4) with a number for each one (P01 to P29). For example, production G2.P07 corresponds to number 7 in group 2. Out of these collected productions, two considerations must be taken into account: (a) of the 113 participating teachers, 40 of them uploaded their written productions to the virtual platform; (b) of these 40 participating teachers, not all solved the three modeling problems posed. Therefore, the *ex post* analysis of the resolution procedures of the participating teachers was carried out from the 40 productions collected and consisted of: first, identifying the phases of the modeling process in the resolution procedures to each of the three problems raised, from the *a priori* analyzes of Table 1; second, classifying these productions into four categories that could be established from the resolution procedures identified, based on the phases of the cycle of Figure 1, which are described and exemplified in the following section. Thus, it is possible to have a first general look at the results of this pioneering educational experience in the Panamanian context.

3. Results

This section presents the results of the study according to the four resolution procedures identified in the productions of the participating teachers.

3.1 Resolution procedure 1 (PR1)

The first resolution procedure identified corresponds to those productions where the participating teachers did not show a complete understanding of the problem statement and/or only provided a description of how it could be solved. The following examples of PR1 are given:

> d = Diameter of the plate. The height of hay piles would be A = 5d. (*Hay Bales Problem*; G1.P22 Production)

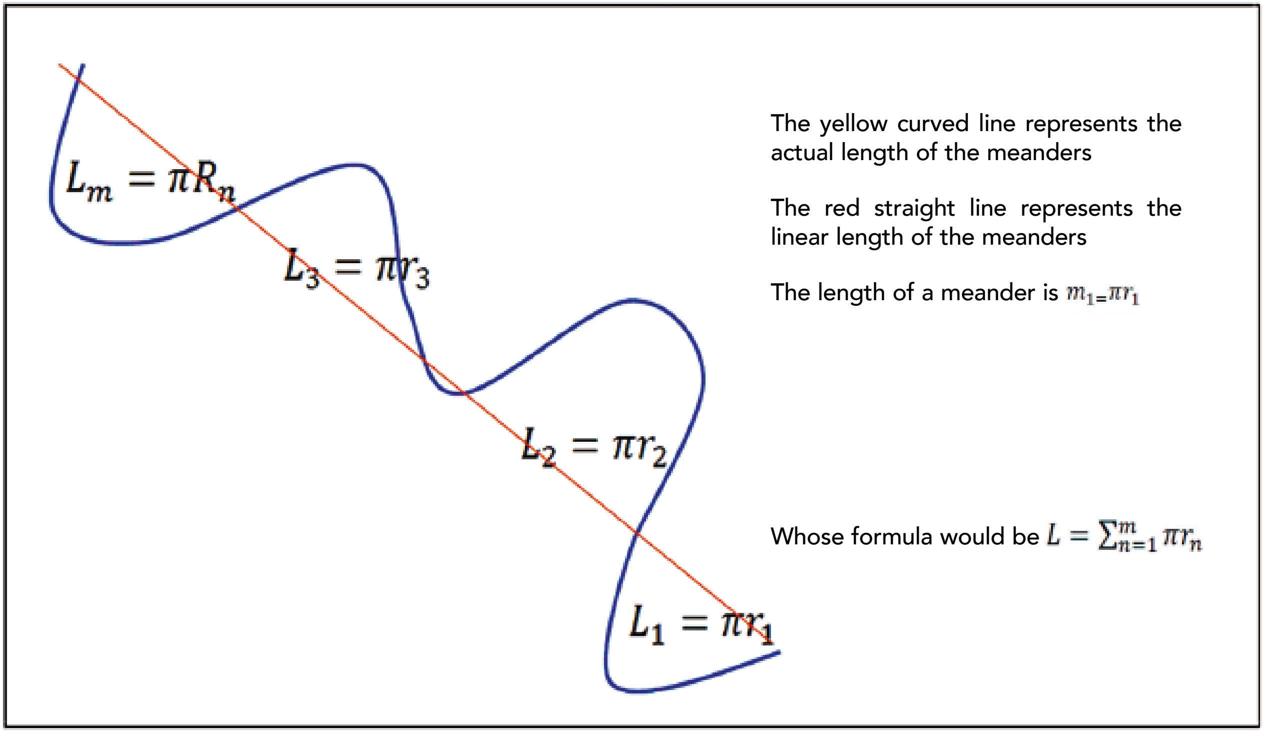
> The distance between two points d = Square root of the first point squared plus the square of the second [*sic*] is calculated with a formula. (*Meanders Problem*; G3.P23)

> The ship must be at a distance from the base of the headlamp equal to the height of the headlamp for the first time, so that the line of sight of the observer, when viewing the headlamp, is at a horizontal angle of 45°, taking into account the curvature of the planet. (*Boston Light Problem*; G2.P09 Production)

3.2 Resolution procedure 2 (PR2)

The second resolution procedure identified corresponds to those productions where the participating teachers developed the phases *real model → mathematical model*. In these productions, it was considered sufficient to formulate a *mathematical model* to solve the problem, without even working mathematically with it; in other words, the problem was mathematized to give an answer. The following example of PR2 for the *Meanders Problem* is presented in Figure 2:

Figure 2. *Production G4.P01 of Meandros Problem (PR2)*

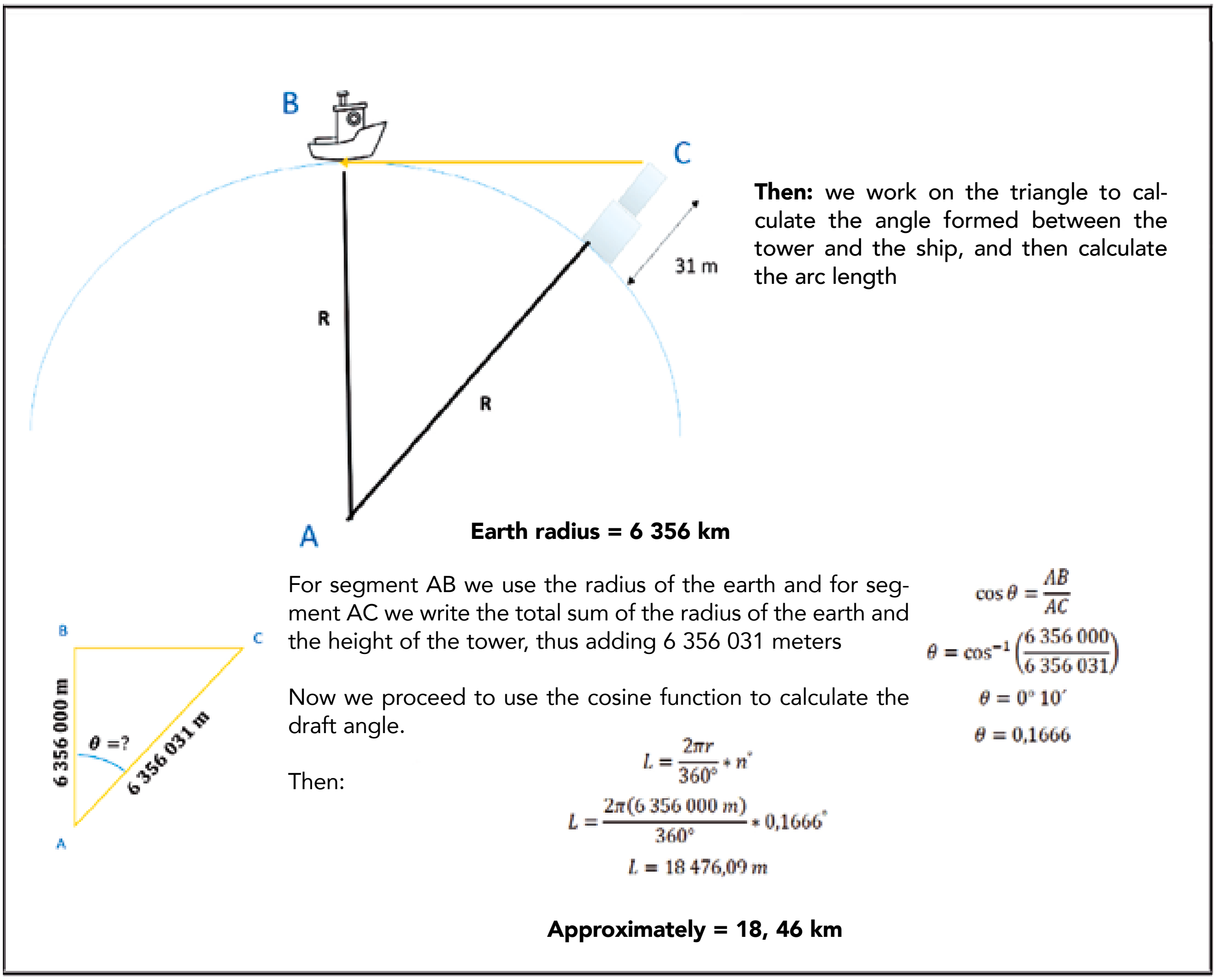


Note. Authors' archives.

3.3 Resolution procedure 3 (PR3)

The third resolution procedure identified corresponds to those productions where the participating teachers developed the phases *real model* → *mathematical model* → *mathematical results*. In these two productions, it was considered sufficient to obtain *mathematical results* from the *mathematical model* to solve the problems, without interpreting them as *real results* and, much less, validate them in the context of the proposed *real situation*; in other words, the problem was worked mathematically to give an answer. The following example of PR3 for the *Boston Light Problem is* in Figure 3:

Figure 3. *G1.P25 Production of the Boston Light Problem (PR3)*

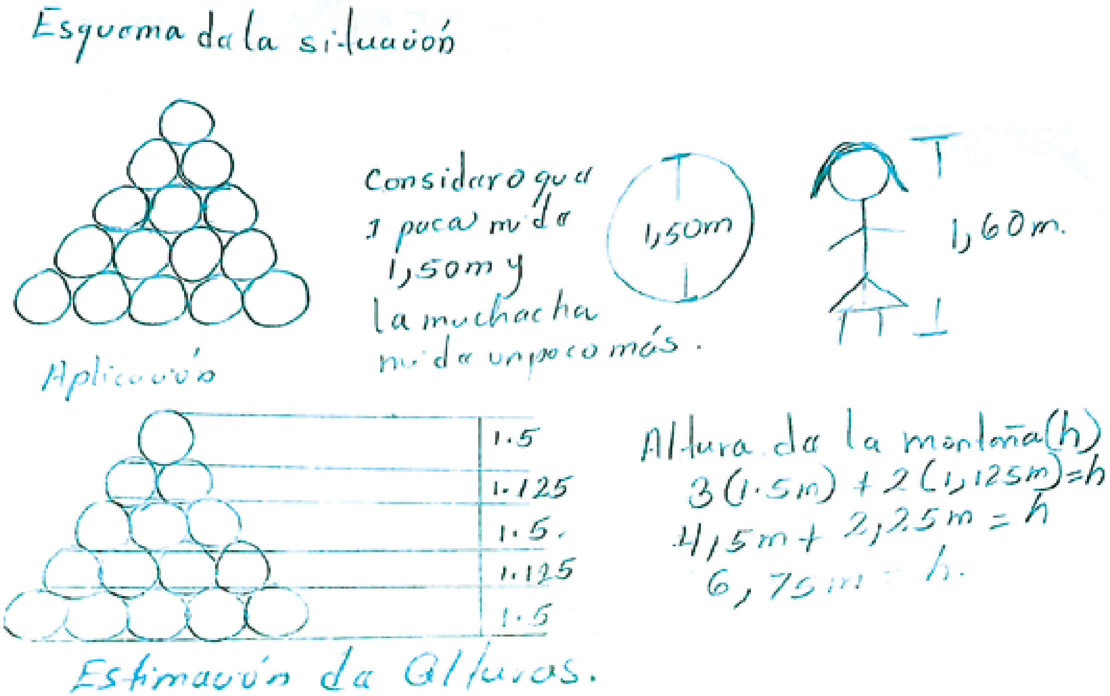


Note. Authors' Archives.

3.4 Resolution procedure 4 (PR4)

The fourth resolution procedure identified corresponds to those productions where the participating teachers developed a complete modeling cycle. In these productions a real model of the real situation was built, a mathematical model was worked with, and the *mathematical results* were interpreted as *real results in the context of the* actual situation from the extra-mathematical considerations made by the resolver. The following example of PR4 for the *Hay Bale Problem* is given in Figure 4:

Figure 4. *Production G1.P20 of the Hay Bales Problem (PR4)*



Note. Authors' archives.

3.5 Synthesis of results

Table 2 shows the number of productions by the participating teachers according to the resolution procedures (PR1 to PR4) that they used for each of the three modeling problems raised during the relevant sub-module.

Table 2. *Synthesis of results*

Problems	PR1	PR2	PR3	PR4
Hay Bales	19	5	9	4
Meanders	9	21	5	0
The Light of Boston	5	9	6	1

4. Discussion and conclusions

The results of Table 2 show that most of the resolution procedures that could be evidenced in the productions of the participating teachers were PR1 and PR2. Regarding these results, a plausible conclusion that may explain this situation would be that, for the case of PR1, the participating teachers did not receive modeling training beyond that given in this diploma and, for the case of PR2, the modeling knowledge they may have had could be interpreted as an attempt to mathematize reality instead of developing a complete modeling cycle. As mentioned at the beginning, working with modeling for mathematics teaching is not part of the national curriculum documents of Panama; therefore, it is likely that these teachers have not had a very broad knowledge about the resolution strategies for this type of problems, or experiences of implementing modeling in their educational practice.

The results of Table 2 also show that there were teachers who evidenced PR3, which is consistent with part of the results reported by Ledezma et al. (2023), in which future teachers of mathematics of secondary education were not interested in returning to the “real world” to interpret or validate the results obtained from the *mathematical model* used, focusing their attention on the sub-competences of mathematization and mathematical work with the proposed problems. In addition, there were five productions of the participating teachers that showed the development of a complete modeling cycle (PR4), however, they are not a representative result of the total of collected productions.

These results provide a first overview of the teaching and learning of modeling in the EMAS Diploma and for the authors, as professors involved in this context, allow them to question: What should be improved/modified in the modeling sub-module for future implementations? Based on previous research on this type of educational experiences (see Borromeo Ferri, 2018; Wess et al., 2021) and the results reported here, it can be concluded that:

- First, the time spent on the modeling sub-module is not enough for teachers to acquire modeling competencies themselves and also to think about how they could teach/implement this process in their educational practice. Given this situation, a reformulation is proposed that includes a minimum of ten sessions (similar to Greefrath et al., 2022) to address the teaching of modeling.
- Second, the above conclusion leads to reformulate the didactic aspects of the sub-module, starting from the knowledge and previous competencies of teachers on modeling, dividing this process into sets of phases that form the cycle, and then raising tasks that work specific transitions (or sub-competencies of modeling) (see Maaß, 2010).
- Third, as the final stage of the sub-module, it would be interesting for teachers to design a modeling class, including solving and creating their own problems, that promotes reflection on the practice itself after its implementation (see Ledezma et al., 2022).

Modeling is considered a process that enriches the quality of teaching and learning mathematics, as evidenced in the educational experience of the EMAS Diploma. However, this experience could have an even greater impact if curricular adjustments are implemented in Panama, which should include, among other aspects, the integration of modeling and give it a significant weight in the teaching and learning of mathematics in this country. Finally, it is emphasized that this study is seminal in a line of research on modeling in the Panamanian context for greater professional development of teachers.

Support

The study was carried out in the framework of the FIED21-002 R&D Project, financed by SENACYT/Universidad de Panamá (Panama); of Project No. 72200458 financed by ANID/PFCHA (Chile); Project PID2021-127104NB-I00 financed by MCIN/AEI/10.13039/501100011033 and “FEDER A way to make Europe”.

References

Abassian, A., Safi, F., Bush, S. & Bostic, J. (2020). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning, 12*(1), 53-65. https://doi.org/10.1080/19477503.2019.1595360

Batista, L., Crisóstomo, E. & Macêdo, J. A. (2022). Conocimiento didáctico-matemático movilizado por futuros profesores de matemáticas. *Alteridad, 17*(2), 194-207. https://doi.org/10.17163/alt.v17n2.2022.03

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. En G. Kaiser, W. Blum, R. Borromeo Ferri & G. Stillman (eds.), *Trends in Teaching and Learning of Mathematical Modelling: ICTMA 14* (pp. 15-30). Springer. https://doi.org/10.1007/978-94-007-0910-2_3

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? En S. J. Cho (ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and Attitudinal Challenges* (pp. 73-96). Springer. https://doi.org/10.1007/978-3-319-12688-3_9

Blum, W. & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45-58. https://bit.ly/3IEeWA0

Blum, W. & Leiß, D. (2007). How do students and teachers deal with modelling problems? En C. Haines, P. Galbraith, W. Blum y S. Khan (eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 222-231). Horwood. https://doi.org/10.1533/9780857099419.5.221

Blomhøj, M. (2004). Mathematical modelling: A theory for practice. En B. A. Clarke, D. M. Clarke, G. Emanuelsson, B. Johansson, D. V. Lambdin, F. Lester, A. Wallby y K. Wallby (eds.), *International Perspectives on Learning and Teaching Mathematics* (pp. 145-159). National Center for Mathematics Education.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik, 38*(2), 86-95. https://doi.org/10.1007/bf02655883

Borromeo Ferri, R. (2007). Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils. En D. Pitta-Pantazi y C. Philippou (eds.), *European Research in Mathematics Education V: Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2080-2089). University of Cyprus, ERME.

Borromeo Ferri, R. (2018). *Learning How to Teach Mathematical Modeling in School and Teacher Education*. Springer. https://doi.org/10.1007/978-3-319-68072-9

Breda, A. & Lima, V. M. d. R. (2016). Estudio de caso sobre el análisis didáctico realizado en un trabajo final de un máster para profesores de matemáticas en servicio. *REDIMAT: Journal of Research in Mathematics Education, 5*(1), 74-103. https://doi.org/10.17583/redimat.2016.1955

Cohen, L., Manion, L. & Morrison, K. (2018). *Research Methods in Education* (8ª ed.). Routledge.

Doerr, H. M. & English, L. D. (2003). A modeling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34*(2), 110-136. https://doi.org/10.2307/30034902

English, L. (2003). Mathematical modelling with young learners. En S. J. Lamon, W. A. Parker y K. Houston (eds.), *Mathematical Modelling: A Way of Life – ICTMA 11* (pp. 3-17). Horwood.

English, L. D. (2013). Modeling with complex data in the primary school. En R. Lesh, P. L. Galbraith, C. Haines y A. Hurford (eds.), *Modeling Students' Mathematical Modeling Competencies: ICTMA 13* (pp. 287-299). Springer. https://doi.org/10.1007/978-94-007-6271-8_25

García-Marimón, O. (2023). *Evaluando la idoneidad didáctica de participantes en el diplomado semipresencial de Estrategias para la Enseñanza Idónea de la Matemática para maestros en ejercicio en la República de Panamá* [Tesis doctoral no publicada]. Universitat de Barcelona.

García-Marimón, O., Diez-Palomar, J., Morales-Maure, L. & Durán-González, R. E. (2021). Evaluación de secuencias de aprendizaje de matemáticas usando la herramienta criterios de idoneidad didáctica. *BOLEMA: Boletim de Educação Matemática, 35*(70), 1047-1072. http://doi.org/10.1590/1980-4415v35n70a23

Godino, J. D., Giacomone, B., Batanero, C. & Font, V. (2017). Enfoque ontosemiótico de los conocimientos y competencias del profesor de matemáticas. *BOLEMA: Boletim de Educação Matemática, 31*(57), 90-113. https://doi.org/10.1590/1980-4415v31n57a05

Greefrath, G., Siller, H.-S., Klock, H. & Wess, R. (2022). Pre-service secondary teachers' pedagogical content knowledge for the teaching of mathematical modeling. *Educational Studies in Mathematics, 109*(2), 383-407. https://doi.org/10.1007/s10649-021-10038-z

Kaiser, G. & Brand, S. (2015). Modelling competencies: Past development and further perspectives. In G. Stillman, W. Blum & M. S. Biembengut (eds.), *Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences* (pp. 129-149). Springer. https://doi.org/10.1007/978-3-319-18272-8_10

Kuntze, S., Siller, H.-S. & Vogl, C. (2013). Teachers' self-perceptions of their pedagogical content knowledge related to modelling – An empirical study with Austrian teachers. In G. A. Stillman, G. Kaiser, W. Blum & J. P. Brown (eds.), *Teaching Mathematical Modelling: Connecting to Research and Practice* (pp. 317-326). Springer. https://doi.org/10.1007/978-94-007-6540-5_26

Ledezma, C., Font, V. & Sala, G. (2023). Analysing the mathematical activity in a modelling process from the cognitive and onto-semiotic perspectives. *Mathematics Education Research Journal, 35*(4), 715-741. https://doi.org/10.1007/s13394-022-00411-3

Ledezma, C., Sol, T., Sala-Sebastià, G. & Font, V. (2022). Knowledge and beliefs on mathematical modelling inferred in the argumentation of a prospective teacher when reflecting on the incorporation of this process in his lessons. *Mathematics, 10*(18), Artículo 3339. https://doi.org/10.3390/math10183339

Lesh, R. & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (pp. 3-33). Lawrence Erlbaum.

Maaß, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik, 31*(2), 285-311. https://doi.org/10.1007/s13138-010-0010-2

Maass, K., Artigue, M., Burkhardt, H., Doorman, M., English, L. D., Geiger, V., Krainer, K., Potari, D. & Schoenfeld, A. (2022). Mathematical modeling – A key to citizenship education. In N. Buchholtz, B. Schwar. & K. Vorhölter (eds.), *Initiationen mathematikdidaktischer Forschung: Festschrift zum 70. Geburtstag von Gabriele Kaiser* (pp. 31-50). Springer. https://doi.org/10.1007/978-3-658-36766-4_2

Manouchehri, A. (2017). Implementing mathematical modelling: The challenge of teacher educating. En G. Stillman, W. Blum y G. Kaiser (eds.), *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education* (pp. 421-432). Springer. https://doi.org/10.1007/978-3-319-62968-1_35

Ministerio de Educación de Panamá. (2014a). *Programa de Matemática – Décimo Grado*. Autor. https://bit.ly/3sZPa4D

Ministerio de Educación de Panamá. (2014b). *Programa de Matemática – Undécimo Grado*. Autor. https://bit.ly/3T1n6Ze

Ministerio de Educación de Panamá. (2014c). *Programa de Matemática – Duodécimo Grado*. Autor. https://bit.ly/47N4d0w

Morales-Maure, L. (2019). *Competencia de análisis e intervención didáctica del docente de primaria en Panamá.* [Tesis doctoral, Universitat de Barcelona]. Dipòsit Digital de la Universitat de Barcelona. https://bit.ly/3QXvq9U

Morales, L, Durán, R., Pérez, C. & Bustamante, M. (2019). Hallazgos en la formación de profesores para la enseñanza de la matemática desde la idoneidad didáctica: Experiencia de cinco regiones educativas de Panamá. *Revista Inclusiones, 6*(Número especial), 142-169. https://bit.ly/3R6PPtg

Ng, K. E. D. (2010). Initial experiences of primary school teachers with mathematical modelling. In B. Kaur & J. Dindyal (eds.), *Mathematical Applications and Modelling: Yearbook 2010, Association of Mathematics Educators* (pp. 129-147). World Scientific Publishing. https://doi.org/10.1142/7798_0008

Ng, K. E. D. (2013). Teacher readiness in mathematical modelling: Are there differences between pre-service and in-service teachers? In G. A. Stillman, G. Kaiser, W. Blum & J. P. Brown (eds.), *Teaching Mathematical Modelling: Connecting to Research and Practice* (pp. 339-348). Springer. https://doi.org/10.1007/978-94-007-6540-5_28

Organisation for Economic Co-operation and Development. (2019). *PISA 2018 Assessment and Analytical Framework*. OECD Publishing. https://doi.org/10.1787/b25efab8-en

Palm, T. (2007). Features and impact of the authenticity of applied mathematical school tasks. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 201-208). Springer. https://doi.org/10.1007/978-0-387-29822-1_20

Preciado, A. P., Peña, F., Ortiz, Y. A. & Solares, A. (2023). Diversity of perspectives on mathematical modelling: A review of the international landscape. In G. Greefrath, S. Carreira & G. A. Stillman (eds.), *Advancing and Consolidating Mathematical Modelling: Research from ICME-14* (pp. 43-57). Springer. https://doi.org/10.1007/978-3-031-27115-1_3

Pollak, H. (2007). Mathematical modelling – A conversation with Henry Pollak. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 109-120). Springer. https://doi.org/10.1007/978-0-387-29822-1_9

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. En A. H. Schoenfeld (ed.), *Mathematical Thinking and Problem Solving* (pp. 53-70). Erlbaum.

Shahbari, J. A. & Tabach, M. (2019). Adopting the modelling cycle for representing prospective and practising teachers' interpretations of students' modelling activities. In G. A. Stillman & J. P. Brown (eds.), *Lines of Inquiry in Mathematical Modelling Research in Education* (pp. 179-196). Springer. https://doi.org/10.1007/978-3-030-14931-4_10

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Wess, R., Klock, H., Siller, H.-S. & Greefrath, G. (2021). Measuring professional competence for the teaching of mathematical modelling. In F. K. S. Leung, G. A. Stillman, G. Kaiser & K. L. Wong

