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Didactic-mathematical knowledge of some teachers about prime numbers

Conocimiento didáctico-matemático de algunos docentes sobre los números primos

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Abstract

Studying the teacher's knowledge has become one of the most relevant lines of research nowadays, considering that it encompasses different factors that have a direct implication in the teaching and learning processes. Considering the above, the objective was to establish the mathematics teacher's knowledge of prime numbers through the Didactic-Mathematical Knowledge Model (DMK). For this purpose, a case study was carried out in which five teachers who work in different High Schools in Colombia were taken as the unit of analysis. For the design of instruments and the analysis of the information, the indicators of didactic-mathematical knowledge of the DMK were taken and three situations related to epistemic, cognitive, affective, interactional, mediational, and ecological elements of prime numbers were designed. From the analysis made by the teachers to the situations, concrete elements emerged revealing the teachers' knowledge regarding prime numbers and some factors that evidence their difficulty in handling the connection of this type of numbers with other objects of mathematics. In addition, it is concluded that teachers do not manage to give students a broad vision of the meaning of these numbers by not knowing cognitive and epistemic elements that make possible their management in the classroom.

Keywords: Knowledge, didactics, onto-semiotic approach, educational indicator, prime number, mathematical object.

Resumen

El estudio del conocimiento del profesor se ha convertido en una de las líneas de investigación más relevantes en la actualidad, considerando que engloba diferentes factores que tienen una implicación directa en los procesos de enseñanza y aprendizaje. Teniendo en cuenta lo anterior, el objetivo fue establecer el conocimiento del profesor de matemáticas sobre los números primos a través del Modelo del Conocimiento Didáctico-Matemático (CDM). Para esto se realizó un estudio de caso en el que se tomó como unidad de análisis a cinco docentes que desarrollan su labor en la educación básica secundaria de diversas instituciones educativas de Colombia. Para el diseño de instrumentos y el análisis de la información se tomaron los indicadores del conocimiento didáctico-matemático del modelo CDM y se diseñaron tres situaciones relacionadas con aspectos epistémicos, cognitivos, afectivos, interaccionales, mediacionales y ecológicos de los números primos. Del análisis realizado por docentes a las situaciones, emergieron elementos concretos que revelaron los conocimientos de profesores respecto a los números primos y algunos factores que evidencian la dificultad para manejar la conexión de este tipo de números con otros objetos de la matemática. Se concluye que el profesorado no logra dar al estudiantado una visión amplia del significado de estos números al desconocer elementos cognitivos y epistémicos que posibilitan su manejo en el aula.

Descriptores: Conocimiento, didáctica, enfoque ontosemiótico, indicador educativo, número primo, objeto matemático.

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1 Introduction

The didactics of mathematics seeks to understand the different problems that emerge in the teaching and learning of mathematics and how they could be solved (Godino, 2021). In this search, the importance of improving the teacher training process has been highlighted (Ball, 2022; Ball *et al*., 2008; Rowland *et al.,* 2005; Shulman, 1986), understanding that a competent teacher for the teaching and learning of mathematics must, for example, know multiple ways of conceptualizing and representing mathematical content, understand the key aspects of each topic and see connections with other objects in the intra- and extra-mathematical context (Godino *et al*., 2018). It is also essential to know the origins, structure, and curricular developments, as well as content directions (Ball, 2000; Kilpatrick *et al*., 2001).

Given the complexity and breadth of mathematical objects, attention can be focused on learning and teaching concrete objects (D'Amore and Sbaragli, 2019). In this article, the knowledge of mathematics teachers about prime numbers is presented from the view of the Didactic-Mathematical Knowledge Model (DMK). The choice of this mathematical object occurs by considering that there is evidence of the lack of knowledge by some teachers about strategies to show students the importance and usefulness of prime numbers, so they must resort to superficial and anecdotal conceptions that lead to misconceptions (Bernaschini, 2017; Kiss, 2020; Zazkis and Lijedahl, 2004).

1.1. Didactic-Mathematical Knowledge Model (DMK)

DMK has developed in the framework of the Onto-semiotic Approach to Mathematical Knowledge and Instruction (EOS), a theoretical system that has its own epistemological, ontological, and methodological foundations (Godino, 2022). Specifically, in the EOS it is considered that the learning of mathematics occurs through problem solving (Godino *et al.,* 2020). In this sense, teaching is the process through which the teacher establishes a didactic trajectory for the mathematical object(s) of which the student is expected to acquire knowledge (Godino *et al.,* 2008).

The teaching and learning process of mathematics is full of multiple dimensions (personal, social, political, economic, psychological, cognitive, epistemological, instructional, etc.) that generate the need to raise foundations that cover each of them deeply (Burgos *et al*., 2018). The DMK has developed considering that teachers of mathematics must know about the six elements that define a process of study (Godino, 2009; Pino-Fan *et al*., 2014).

The first is the epistemic aspect, which considers the representativeness of the meanings that are developed in the classroom in contrast with the existing reference meaning of each mathematical object (Burgos and Godino, 2021). The cognitive element addresses the proximity of the meanings and their relationship with the cognitive development of the learner; the affective aspect directs to the involvement of the students in the trajectory determined by the teacher (Beltrán-Pellicer and Godino, 2020; Hummes *et al*., 2019).

The interactional and mediational aspects arise when talking about the knowledge of the mathematics teacher about teaching; the first referring to the identification and resolution of cognitive conflicts that arise in learning and the second to the adequacy of material and temporal resources (Giacomone *et al.,* 2019). Finally, the ecological element links the knowledge that teachers should have regarding the program, its implementation and adjustment to the demands of society and the students' environment (Castro and Pino-Fan, 2021).

The previous aspects become operative through the analysis of practices, actions performed in the mathematical activity; configurations, description of mathematical objects and processes that emerge in the practices; norms and consideration of the rules and habits that condition the practices; and finally suitability, the process of identifying possible improvements in the instructional processes. In the DMK, the study and reflection on each aspect is materialized in the indicators presented in Figure 1.

Figure 1

DMK indicators

Knowledge of the content (common and specialization)		
Epistemic aspect	Indicators	Denotation
Common knowledge	Do the task	FEp1
Specialized knowledge:	Stake in the plausible solutions of the task and other related tasks.	FEp2
Type of problems	Identify the variables of the task; generalizes the statement.	FEp3
Languages	Do the task by using different representations	FEp4
Procedures	Do the tasks by using different procedures (intuitive and formal)	FEp5
Concepts / properties	Identify the concepts and properties in the solutions	FEp6
Arguments	Explain and justify the solutions	FEp7
Broad knowledge:		
Connections	Identify possible generalizations of the task and connections with more advanced topics	FEp8
Knowledge of the content in relation to the students		
Cognitive and affective aspect	Indicators	
Cognitive configurations (strategies, representations, statements, argumentations, ...)	Describe the types of cognitive configurations that learners have developed in solving the proposed task (or tasks).	FCA1
Mistakes, difficulties, learning conflicts, conceptions, etc.	Describe the main types of learning conflicts in the solution of this type of tasks.	FCA2
Learning assessment	Explain students' personal meanings in solving this type of task or content.	FCA3
Attitudes, emotions, beliefs and values	Describe strategies that can be implemented to promote student involvement in solving these tasks.	FCA4
Knowledge of the content in relation to teaching		
Interactional and mediational aspect	Indicators	
Roles of the teacher and students in relation to the task or content. Ways of interaction: teacher-student; students-students. Material resources. Allocated time.	Describe the didactic configuration you would implement using the given mathematical task.	FIM1
Didactic trajectory (sequence of didactic configurations)	Describe other tasks related to the one provided and how to manage the corresponding didactic trajectory	FIM2
Knowledge of the program and intra and inter-disciplinaries connections		
Ecological aspect	Indicators	
Program oriented	Identify the elements of the program that are addressed through the performance of the task(s). Proposal (aims, objectives)	FEc1
Intra-disciplinary connections	Explains connections that can be made to other topics in the syllabus through the completion of the assignment	FEc2
Inter-disciplinary connections	Explains connections that can be made to other topics in the syllabus through the completion of the assignment	FEc3
Other conditioning factors	Identifies social, material, or other factors that condition the performance of the task or the development of the intended or implemented educational project.	FEc4

Note. Taken and adapted from Godino (2009).

2 Methodology

The approach assumed is qualitative, that is a flexible inquiry process that intends to understand the meaning of the actions of the subjects who experience the phenomenon of interest (Bejarano, 2016). In addition, the scope of this research is descriptive, since data related to the educational reality of five teachers of Colombian high schools were collected, which are mentioned as: A, B, C, D and E, are collected and described. The selection of these teachers is not probabilistic, the only criterion considered was the acceptance of the invitation made by the researchers through e-mail.

In order to obtain the information, a two-hour meeting was held where the analysis of the didactic-mathematical activity was implemented based on DMK. In this type of analysis, teachers are provided learning situations so that they can analyze and reflect on each of the DMK indicators (Godino *et al.,* 2007). This research presents three situation-problems that include didactic-mathematical knowledge about prime numbers, as they are related to three elements of the historical and epistemic development of these numbers: the search for algorithms for determining prime numbers, the inclusion or exclusion of 1 in this set, and the use of prime numbers in problems related to the greatest common divisor and the least common multiple.

The data were structured through the analysis of categories, in this case, the six aspects of the DMK model. Textual data were taken from the answers given by the teachers in each of the activities, identifying those manifestations that reveal the thoughts, ideas, and knowledge of the participants regarding the categories (Strauss and Corbin, 2002).

3 Results

This section presents in detail the three situations analyzed and their relationship with the aspects of the DMK and the level of didactic-mathematical knowledge of the participating teachers.

3.1. Didactic-mathematical analysis of situation 1

The first situation was proposed with the objective of analyzing the conception teachers have of what is a prime number, considering its relationship with odd numbers and the inclusion or not of the number 1 as a prime number. This situation is as follows:

Situation 1. Students at an educational institution were asked to write down the prime numbers lower than 10 and the following answers were obtained.

Type 1: 1, 2, 3, 5, 7.
Type 2: 2, 3, 5, 7.
Type 3: 3, 5, 7.

3.1.1. Epistemic aspect in situation 1

The teachers were asked to answer: Which answer or answers should the teacher accept as correct? Which mathematical concepts and/or properties should the students use to give a correct solution? These allow addressing the common and specialized knowledge components. The expected solutions, respectively, were:

- Answers 1 and 2 are correct. This is considering that 1 can be considered prime or not.
- Division, divisibility, divisors, divisibility criteria, prime number, decomposition, multiple, even and odd number, order relation, etc.

All the participant group responded that the Type 2 answer was correct, only E indicated the Type 3 answer as valid. For question 2, A expressed that the concepts to be used were divisors, multiples, and divisibility criteria; B indicated odd and even numbers, divisors, and divisibility criteria; C prime numbers; D chose prime and even numbers; while E answered that

natural numbers, multiples, divisors, factor, division, and decomposition were necessary.

From these answers, it stands out that no teacher considers number 1 as prime, only 2 of them recognize that the concept of prime number is necessary to answer the situation and none of them identifies that the use of the expression "lower than" should be considered. Finally, the teacher who indicated Type 3 as the correct answer reveals that he/she does not differentiate prime numbers from odd numbers.

3.1.2. Cognitive and affective aspects in situation 1

The idea regarding the aspects was to describe the possible difficulties present in the incorrect answers, which led the student to answer incorrectly. The expected solution was: if the teacher indicated that the Type 1 answer was correct, then it was expected to highlight that the student failed to recognize 1 as a prime number and that there was difficulty in differentiating prime numbers from odd numbers. If the teacher had indicated that 2 was correct, it was expected to express the difficulty in recognizing that 1 is not prime. Finally, if he or she indicated that 3 was correct, he or she was expected to identify that the student did not recognize that even numbers are not prime. The responses were:

- A: Perhaps the student is confused as to whether number 1 is a prime number; he/she might be confused that it is a divisor of itself and a divisor of one. For type 3 response, perhaps he/she is confused with the fact that the divisors of two are 1 and 2 and would believe that because it is the first number it should not be included.
- B: Not differentiating odd and even numbers. Not identifying the divisors of a number. Not being clear on the concept or characteristics of prime number.
- C: Confuse odd numbers with prime numbers or are unclear on the definition.
- D: Failure to complete the activity in class and therefore do not have the concepts defined. They forget the concept of prime number. Multiplication tables and therefore does not determine the divisors.
- E: Handling of concepts, difficulty in dividing, difficulty in decomposing numbers, difficulty in reading comprehension.

The aspect that stands out in the answers is that all the teachers say they know the concept of prime number; however, they had not contemplated it in question 1 of the epistemic aspect as something necessary to solve the situation. In addition, B raises the need to differentiate between odd and even numbers, i.e., again confusing odd and prime numbers. Finally, A in trying to explain errors in solution type 3, did not identify that the numbers being presented are odd.

3.1.3. Mediational and interactional aspects in situation 1

To address these aspects, the question was posed: What strategies would you use as a teacher to guide those students who have not been able to solve the problem? Explain in detail your answer. As a solution, strategies such as problem solving, collaborative work, explanation by the teacher, use of didactic material, development of playful activities, etc., were expected to be mentioned, as well as explanations of how the problematic aspects in student learning would be solved. The responses were as follow:

- A: I would ask the student to divide each number into one, itself and the numbers before it, since the divisions that are exact are the values that work. I would reinforce the concepts of prime and composite number.
- B: Recall the concept or characteristics of a prime number. Recall and identify odd and even numbers. Practice the concept of divisor of a number using simple exercises.
- C: First make them see the difference between odd number and prime number based on the definition and examples.

- D: Extra work with multiplication tables, division process and divisors of a number. Define and clarify the properties of numbers 2 and 1 with the definition of prime number. Create a bulletin board listing the first prime numbers.
- E: Didactic games. Videos. Practical exercises knowing concepts that will help them better understand the topic, by carrying out sequenced activities where they follow instructions.

In these answers, four teachers keep saying that the solution to the difficulties is achieved with teacher strategies such as clarifying the concept, reviewing the algorithms of division and multiplication, and differentiating even, odd, and prime numbers. Only E considers activities of a different nature, proposing playful strategies; however, his response focuses on stating strategies and does not clarify the relationship with the mathematical object, i.e., he considers the strategies as something independent of the object.

3.1.4. Ecological aspect in situation 1

To conclude with situation 1, the following question was asked: For which course do you consider this problem suitable according to the current program? In this aspect, it was expected that this mathematical topic could be worked in any course above the fourth grade of elementary school, since this is what it is indicated in the mathematics program in Colombia. The answers were: A answered third grade of elementary school; B stated that from fourth grade onwards; C expressed that in all grades of elementary and in sixth grade: D chose third, fourth and fifth grade of elementary school; and E answered that in fifth grade of elementary and sixth grade of High school.

Thus, only B mentions the grades in which prime numbers can be studied according to the national program. Only E indicates some appropriate grades, while A, C and D indicate grades in which, according to the academic program and previous mathematical concepts, it is not coherent to work with prime numbers.

3.2. Didactic-mathematical analysis of situation 2

The second situation was proposed with the objective of analyzing what strategies and procedures teachers use to determine a list of prime numbers lower than a given number and how they present it to their students. This situation was:

Situation 2. Write prime numbers lower than 20

3.2.1. Epistemic aspect in situation 2

- The teachers were asked: Solve the problem posed. What are the limitations of the solution presented by you to the students? The expected answers, respectively, were:
- Solution 1: 2, 3, 5, 7, 11, 13, 17, 19.
- Solution 2: 1, 2, 3, 5, 7, 11, 13, 17, 19.
- The method becomes inefficient as the upper increasing the upper limit.

The responses are presented in Figure 2.

Figure 2

Responses in epistemic aspect of situation 2

Professor	Question 1	Question 2
A	2, 3, 5, 7, 11, 13, 17, 19 are the only ones that can be divided in one and in themselves.	I am not giving enough didactic tools to make it easier for them to solve the problem.
B	The answer to the problem is 2, 3, 5, 7, 11, 13, 17, 19 because these are the numbers lower than 20 that can only be divided by themselves and by the unit.	Does not respond
C	2, 3, 5, 7, 11, 13, 17, 19 only have two dividers	Odd numbers to be confused with primes
D	Does not respond	To solve this problem, I would consider the activities, analysis and strategies of the previous topics. If necessary, the activity developed in class would be performed again, but individually to guarantee that the processes are performed and understood.
E	Does not respond	As I present it, students will find it impossible if they do not know prime numbers.

It is seen that only teachers A, B and C solved the situation posed, evidencing the uncertainty of D and E about the correct way to answer. Regarding the question, only A was able to identify a possible difficulty with what he does by explaining that he lacks strategies to present the solution; while C, D and E responded evasively, focusing on the mistakes that students can make and not on their own actions. Finally, B failed to express the limitations of his response.

3.2.2. Cognitive and affective aspects in situation 2

Teachers were asked to: describe the possible difficulties that may lead students to answer wrongly. From which, they were expected to raise difficulties such as the confusion of prime numbers with other sets such as odd numbers, the inclusion or not of 1 as a prime number, the lack of an efficient algorithm to generate prime numbers, the lack of knowledge of the concept of prime number or the impossibility of calculating the requested numbers due to the lack of mastery of previous mathematical objects. The answers obtained are:

- A: Are not clear about the concept of prime number, do not know divisors and multiples of a number. Do not know how to divide.
- B: Some of the possible difficulties could be not differentiating odd and even numbers, not identifying the divisors of a number, not being clear on the concept or characteristics of prime number.
- C: They confuse odd numbers with prime numbers or are unclear on the definition.
- D: Do not respond.
- E: Do not have clear concepts.

It is observed that three teachers (A, B and C) agree that the previous concepts such as multiples, divisors, even and odd numbers must be clear. However, only A recognizes the need for understanding the concept of prime number. In addition, D failed to establish possible difficulties in the solution of the situation and E gives a non-specific answer, which does not show an

understanding of what can happen to the students when facing this situation.

3.2.3. *Mediational and interactional aspects in situation 2*

The following questions were asked to address these aspects: What type of resource would you use to present the solution to the problem; what questions would you ask your students after presenting the solution; how would you evaluate the learning achieved? The expected answers, respectively, were:

- Sieve of Eratosthenes, virtual applications, ludic activities, the board, multiplication tables, etc.
- Questions such as: what difficulties did they observe in their development? What solution strategies would they use? Was the development of any of them not clear? What can be concluded with the above? What relationships and differences do they find with other numerical sets such as natural numbers, even, odd, multiples of a number, etc.? Among others.
- The explicit enunciation of strategies such as continuous assessment, self-assessment, co-assessment, or the implementation of questionnaires, tasks, problems, and exercises, among others.

The responses are presented in Figure 3

Figure 3

Responses in mediational and interactional aspects in situation 2

Professor	Question 1	Question 2
A	2, 3, 5, 7, 11, 13, 17, 19 are the only ones that can be divided in one and in themselves.	I am not giving enough didactic tools to make it easier for them to solve the problem.
B	The answer to the problem is 2, 3, 5, 7, 11, 13, 17, 19 because these are the numbers lower than 20 that can only be divided by themselves and by the unit.	Does not respond
C	2, 3, 5, 7, 11, 13, 17, 19 only have two dividers	Odd numbers to be confused with primes
D	Does not respond	To solve this problem, I would consider the activities, analysis and strategies of the previous topics. If necessary, the activity developed in class would be performed again, but individually to guarantee that the processes are performed and understood.
E	Does not respond	As I present it, students will find it impossible if they do not know prime numbers.

Regarding question 1, it is observed that B and E mention the Sieve of Eratosthenes (although E does not remember the name), while A would use the multiplication tables and C does not manage to make any instrument. From the above, a limited knowledge of instruments related to the learning and teaching of prime numbers is observed.

Regarding question 2, A and B emphasize the need to address the difficulties faced by the students, while C expresses that he would resort to questioning conceptual aspects of prime numbers without specifying how he would do it. On the other hand, D would emphasize normative aspects such as students' emotions and needs.

Finally, E would delve into the strategies used by the students when they solved the situation.

In the final question, A, C and E fail to mention how they would evaluate, providing generic answers, while B and D state in detail the strategies and instruments they would use, but omit which aspects of the mathematical object they would address..

3.2.4. Ecological aspect in situation 2

To conclude the analysis of situation 2, the following question was asked: For which course do you consider this problem suitable according to the current program? As an answer, it was expected that they would express that the situation can be worked in grades higher than fourth grade, since it is a mathematical problem that is related to topics of higher levels of education.

The answers are: A says it is suitable for third grade of elementary school; B expresses that any grade from fourth grade of elementary school; C chooses all grades of elementary school and sixth grade of high secondary school; D does not answer; and E indicates sixth grade of high school. In these answers it is seen that only B knows the grades in which prime numbers can be worked according to the national program and the nature of the mathematical problem. Only E indicates an appropriate grade but omits that it can be taught in other levels, while A and C indicate grades in which it is not coherent to approach this mathematical topic, according to the program and the previous mathematical concepts necessary when teaching prime numbers.

3.3. Didactic-mathematical analysis of situation 3

Situation number three was proposed with the objective of analyzing whether the teachers recognized the connection of prime numbers with other mathematical objects, specifically the least common objects such as multiple, greatest common divisor, decomposition into prime factors and divisibility. The situation is:

Situation 3. The students of an institution were asked to solve the following problem: Maria has decided to make bracelets and wants to decorate them with pearls. If she has 24 white pearls and 36 blue pearls, and she wants to make as many bracelets as possible she needs to: use all the pearls, all the bracelets have the same number of pearls, all the handles have pearls of the two colors. How many bracelets will she be able to make?

3.3.1. Epistemic aspect in situation 3

For this aspect, the question asked was: With which previous and more advanced concepts of the school program do you relate the content involved in the solution of this problem? The answers expected were: with the previous concepts, natural numbers, division, multiplication, divisors of a number, divisibility criteria, multiples of a number, even and odd number, simplification of fractions; for the more advanced ones, the greatest common divisor, least common multiple, factoring, rational numbers, among others. The answers were:

- A: Multiples, divisors, prime numbers and composite numbers, decomposition into prime factors and least common multiple.
- B: Odd and even numbers, divisor of a number, prime numbers, composite numbers, decomposition into prime factors, divisibility criteria, greatest common divisor.
- C: prime numbers, decomposition, and mathematical logic.
- D: M.C.D. and divisibility criteria.
- E: Factorial decomposition and least common multiple.

It is possible to observe in the responses several of the expected prior concepts, which is not the case with more advanced knowledge. In fact, teachers were not able to explain which ones they considered as previous and which ones as more advanced. Among the previous concepts,

it stands out that no teacher identified any relationship between prime numbers and fractions, mathematical topics taught from elementary school. In addition, although in the analysis of situation 1 and the answers to the other questions they emphasize difficulties related to the handling of the different numerical sets, only B mentions this aspect.

3.3.2. Cognitive and affective aspects in situation 3

The following instructions were provided for these aspects: point out the possible difficulties present in the incorrect answers. The expected solution was: not considering that prime numbers are used and not composite numbers to perform the decomposition into factors; not knowing the algorithm to calculate the greatest common divisor; lack of knowledge about prime numbers; among others. The answers

- A: Decides to do greatest common divisor separately and does not consider the order of the decomposition. He is not clear on the definition of prime factor decomposition and looks for a divisor that is easy for him.
- B: Some of the possible difficulties could be: Not clear on the concept or characteristics of prime number, not clear that the decomposition process should be done only using prime numbers and from smallest to largest.
- C: Not being clear that decompositions should be done only with prime numbers.
- D: Not clear on the concepts of M.C.D. Not remembering the list of prime numbers or how to find them. Reading comprehension.
- E: Handling elementary operations.

Regarding the answers of teachers, none of them managed to identify that there were different errors and difficulties in each type of solution, so they present a global view of difficulties related to the problem, but not with the particularities of each solution; moreover, none of them found that there was a correct solution. Additionally, except for E, who gives an answer without specifying, all of them emphasize that there was no clarity in the management of the decomposition into prime factors, but they did not express which aspect of that decomposition is the one that fails in each solution.

3.3.3 Mediational and interactional aspects in situation 3

To conclude the analysis of situation 3, the following question was asked: What strategy would you use to correct the errors found? Solutions were expected, such as: take up the situation by developing it step by step; clarify the need to decompose into prime numbers; address the concept of prime number; ask the students the explanation of correct answers, among others. The answers were:

- A: Decides to do greatest common divisor separately and does not consider the order of the decomposition. He is not clear on the definition of prime factor decomposition and looks for a divisor that is easy for him.
- B: Some of the possible difficulties could be: Not clear on the concept or characteristics of prime number, not clear that the decomposition process should be done only using prime numbers and from smallest to largest.
- C: Not being clear that decompositions should be done only with prime numbers.
- D: Not clear on the concepts of M.C.D. Not remembering the list of prime numbers or how to find them. Reading comprehension.
- E: Handling elementary operations. Regarding the answers of teachers, none of them managed to identify that there were different errors and difficulties in each type of solution, so they present a global view of difficulties related to the problem, but not with the particularities of each solution; moreover, none of them found that there was a correct solution. Additionally, except for E, who gives an

answer without specifying, all of them emphasize that there was no clarity in the

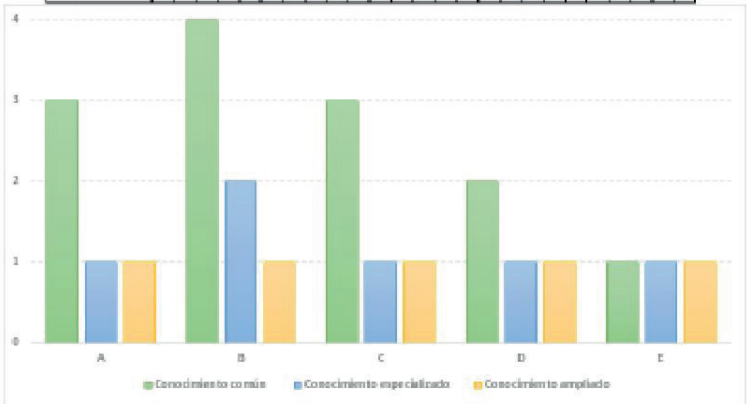
3.4 Classification ok knowledge

Based on the indicators presented in Figure 1 and the answers to the questions posed regarding prime numbers, the knowledge of didactic-mathematical of each teacher is classified as zero level (0), low level (1), medium level (2), high level (3) and advanced level (4) in each of the DMK aspects. To begin with, the classification of the level of knowledge in the epistemic aspect is presented in Figure 4.

Figure 4

Classification of knowledge in the epistemic aspect

Epistemic phase	Knowledge of the content (common, specialize)																								
Indicator	Professor																								
	A					B					C					D					E				
	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
FEp1				x						x				x				x				x			
FEp2			x					x				x					x					x			
FEp3		x						x				x					x						x		
FEp4		x					x					x					x					x			
FEp5		x					x					x					x					x			
FEp6			x					x				x					x					x			
FEp7		x						x				x					x					x			
FEp8		x					x					x					x					x			



It is found that A and C have a high common knowledge, B advanced, D medium and E low. This implies that, except for E, the participants have the necessary knowledge to solve situations related to the management of prime numbers in the school context corresponding to the sixth grade of elementary school in Colombia.

However, in the assessment of specialized knowledge, B obtained a medium level and the others a low level, because the type of solutions they can offer to the situations lacks the management of a diversity of representations, strategies, and connections with another mathematical topic. This low level is also observed in the extended knowledge, where no teacher was able to relate prime numbers to more advanced mathematical topics in the school program.

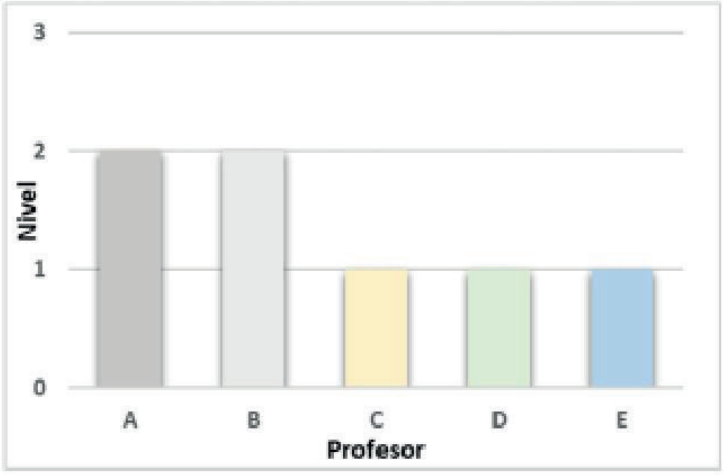
Regarding the cognitive and affective aspects, the evaluation presented in Figure 5 shows that A and B reach a medium level of knowledge and the others a low level. The main shortcoming found is the difficulty in presenting the students' personal meanings when

solving situations that deal with prime numbers. This aspect relates to another problematic indicator: not promoting actions that involve students in the solution of such situations

Figure 5

Classification of knowledge into cognitive and affective aspects

Cognitive and affective aspects	Knowledge of the content of students																								
Indicator	Professor																								
	A					B					C					D					E				
	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
FC1			x					x					x				x						x		
FC2			x					x				x					x					x			
FC3			x				x					x					x					x			
FC4		x					x					x					x					x			

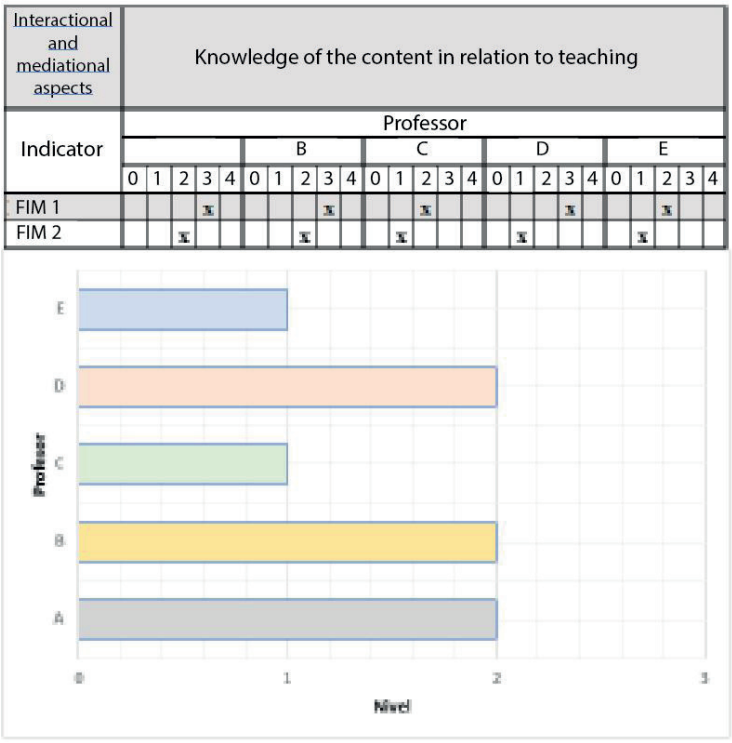


On the other hand, in the interactional and mediational aspects presented in Figure 6, it is explicit that A, B and D have a medium level of knowledge, since they manage to describe the way in which they would conduct their teaching actions to address situations associated with prime numbers. This aspect was not achieved by C and E, who are located at the low level.

The aspect that does not allow participants to reach an advanced level of knowledge is that they were not able to relate their teaching actions with different situations; on the contrary, all of them emphasized the need to solve the difficulties of the students from the same situation that originated.

Figure 6

Classification of knowledge into interactional and mediational aspects



Finally, when analyzing the ecological aspect, it is observed that all participating teachers have a low level of knowledge, since they are unable to identify the academic levels or grades in which prime numbers are present, either as a central object of study or as an object that connects with others. In addition, none of them manages to explain the factors that condition the relationship of prime numbers with their properties, problem solving, nor with the social, cultural, and scientific dynamics around them.

4 Discussion and conclusions

Regarding the epistemic aspect, it was found that the lack of advanced knowledge lies, as explained by D'Amore and Fandiño (2005), in the fact of not knowing the historical and epistemological development of prime numbers, which implies not assuming committed and meaningful positions regarding them. In turn, the difficulty in establishing a connection with other mathematical objects may be due to a conception of mathematics in which teachers are unaware that mathematical objects are not isolated entities; on the contrary, they are constantly related (Bagni, 2006; Bagni and D'Amore, 2005). In this case, the lack of knowledge on the connection of prime numbers with other objects generates a weak awareness in teachers of what these numbers offer mathematically and didactically (Grugnetti and Rogers, 2000).

In the cognitive and affective aspects, the main obstacle identified was the inability to establish mechanisms to involve students in the solution of situations related to prime numbers. Radford (2020, 2021) argues that this is because the beliefs and conceptions of teachers regarding the role of students and the teacher place them as

opposites, in which learning is an exclusive attribute of the student and, therefore, teacher does not have to get involved in arousing the student's interest in learning.

Regarding the mediational and interactional aspects, it was observed in teachers the difficulty to manage different didactic trajectories according to the needs, difficulties, mistakes, and obstacles that the students may face. This aspect emerges from the strong need to describe what they observe using qualifiers such as good or bad, right, or wrong, among others, but they do not consider necessary to give space for the recognition of descriptions and deeper reasoning that allow establishing possibilities for improvement (Breda *et al.,* 2018; Font, 2011; Oyarzún and Soto, 2020).

In the ecological aspect, it is seen that the main difficulty for teachers is recognizing the connections between prime numbers and the students' environment. De Gamboa *et al.* (2015) explain that this is due to two types of beliefs; the first one, that the mathematics teacher's role obeys exclusively to teaching the discipline, thus omitting the need to offer the student relations with other disciplines; and the second one, that mathematical relations are complex for students and therefore it is not recommended to present them (D'Amore and Radford, 2017; Llinares, 2016).

The results of each aspect enable to conclude that the DMK model allows identifying the mathematical didactic knowledge of teachers regarding prime numbers, which is essential to establish criteria to improve the processes of teacher training. Furthermore, the possibility of establishing knowledge enables educators and researchers to develop processes of reflection and improvement of didactic practices.

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