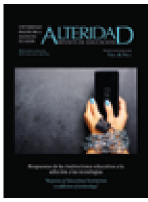
ALTERIDAD
REVISTA DE EDUCACIÓN
p-ISSN:1390-325X / e-ISSN:1390-8642
http://alteridad.ups.edu.ec

January-June 2023
Vol. 18, No. 1, 83-96
https://doi.org/10.17163/alt.v18n1.2023.07



Mathematical and digital competence of future teachers using GeoGebra application

Competencia matemática y digital del futuro docente mediante el uso de GeoGebra

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Received on: 2022-05-03 / **Revised on:** 2022-06-14 / **Accepted on:** 2022-10-18 / **Published on:** 2023-01-01

Abstract

University students use technology daily, so that some digital skills are sometimes taken for granted. However, it is necessary for future teachers to use their digital competence in didactic contexts, which is why they must broaden this competence in their academic training and apply it for didactic purposes. The aim of this work is to ensure that future teachers master the techniques related to geometric transformations while improving their digital, mathematical and didactic competence (TPACK Model). For this, the GeoGebra Classic free software application has been used, with the aim of enabling students in training to acquire skills for teaching geometric concepts such as axial symmetry, central symmetry, inversion, rotation, translation and homothetic symmetry. A mixed methodology with a pre-experimental design is used with a sample of 68 participants belonging to the subject of Mathematics and its didactics III of the Degree in Primary Education at the Universidad Rey Juan Carlos enrolled during the 2021/2022 academic year. The notes, the workshops and the data from the expressly designed questionnaire allow for a quantitative and qualitative analysis of the information. The results are very satisfactory since the application of GeoGebra Classic allows the development of strategies that combine competences simultaneously and it is concluded that it facilitates and improves the acquisition of mathematical and digital competence and simplifies some difficulties that emerge in learning.

Keywords: Mathematics education, competence, communication, digital, teaching, GeoGebra.

Resumen

El alumnado universitario utiliza la tecnología a diario, por lo que, en ocasiones, ciertas competencias digitales se dan por supuestas. Sin embargo, es necesario que el futuro profesorado utilice su competencia digital en contextos de aprendizaje, por lo que deben ampliarla en su formación académica y aplicarla con fines didácticos. El objetivo de este trabajo es conseguir que el futuro profesorado, domine las técnicas relacionadas con las transformaciones geométricas a la vez que mejora la competencia digital, matemática y la didáctica de esta (Modelo TPACK). Para ello, se ha aplicado el Software libre de GeoGebra Classic, con la finalidad de que el alumnado en formación adquiera competencias para la enseñanza de conceptos geométricos como los de simetría axial, central, inversión, rotación, traslación y homotecia. Se ha utilizado una metodología mixta con diseño pre-experimental con una muestra de 68 participantes pertenecientes a la asignatura de Matemáticas y su didáctica III del Grado en Educación Primaria de la Universidad Rey Juan Carlos matriculados durante el curso 2021/2022. La evaluación, los talleres y los datos del cuestionario especialmente diseñado permiten realizar un análisis cuantitativo y cualitativo. Los resultados son muy satisfactorios ya que la aplicación de GeoGebra Classic permite desarrollar estrategias que combinan competencias de forma simultánea y se concluye que facilita y mejora la adquisición de competencia matemática y digital y simplifica algunas dificultades que surgen en el aprendizaje.

Descriptores: Educación matemática, competencia, comunicación, digital, docente, GeoGebra.

Suggested citation: García-Lázaro, D. and Martín-Nieto, R. (2023). Mathematical and digital competence of future teachers using GeoGebra application. *Alteridad, 18*(1), 83-96. https://doi.org/10.17163/alt.v18n1.2023.07

1. Introduction

The acquisition of basic mathematical competence is one of the goals to be achieved in the Sustainable Development Goals of 2030 foreseen by the UN. Specifically, SDG number four (SDG4) reflects the need for quality and universal education. Therefore, in order to achieve this objective, it is important to analyze the relationship between the results obtained in the classroom and the academic training received by teachers and future teachers in training.

To this end, the PISA Report (Program for International Student Assessment) provides educational information on the levels of knowledge and competencies acquired by students at the international level, which allows a glimpse of the areas of knowledge that improve in some areas and those that are still progress (OECD, 2019). The results obtained in Spain in the area of mathematics in recent years are below the OECD average, as well as in neighboring countries.

In addition, the National Council of Teacher of Mathematics (NCTM, 2020) collects standards for preparing mathematic teachers for both early childhood education (De Castro Hernández, 2020) and primary education in general. Specifically, for geometry, it is intended to apply transformations and use symmetry to analyze mathematical situations (NCTM, 2020).

The education law that regulates basic knowledge in primary education in Spain (BOE, 2020) reflects the need for students to acquire a mathematical skill sufficient to develop in their daily and formative life. This skill involves the solution of mathematical problems (which as a complex and integral activity requires a specific training coupled with cognitive and metacognitive knowledge) to achieve the constitutive elements of competition (Pistón-Rodríguez and Parejo-Jiménez, 2019) and which should be acquired in the first formative stages.

Mathematics is considered the basis of complex processes of knowledge and requires other skills such as analytical, reflective and critical thinking, i.e., the ability to reason, formulate or solve problems (Fernández Bravo, 2006). Hence, the importance that all people can acquire basic mathematical skills during their training and that they can consolidate more complex mathematical contents. The work of the teacher is very relevant in this task, as it requires didactic and methodological skills to solve possible difficulties and facilitate learning (Saucedo *et al.*, 2014). Likewise, the teacher must "develop argumentative capacity, the use of the appropriate mathematical lexicon, and the appropriate use of different representations of a mathematical object accompanied by manipulative material or technological resources" (Vargas-Díaz and Apablaza, 2019, p. 87).

According to Rivero (2012), teaching mathematics implies also being aware of some difficulties, such as: the abstraction in the teaching of the content, the use of repetition, mechanized or inadequate methodology that does not facilitate the understanding of mathematical concepts; all this coupled with how students feel mathematics, which usually shows a feeling of hatred, rejection or anxiety (Novelo Sánchez *et al.*, 2015).

Once future teachers acquire knowledge and basic mathematical competence, it is also necessary to have sufficient communicative and didactic competence, which facilitates an effective teaching of the subject. It is "a process that gives greater meaning and connotation to the training process" (Jerez Berrio, 2020, p. 13), which requires effective interpersonal communication and the use of technology for teaching purposes in the classroom.

Thus, future teachers are challenged to having skills in digital and communicative competence in addition to mastering the mathematical area (Gràcia *et al.*, 2017; 2020) to effectively perform the teaching process. These competences, as Colás-Bravo *et al.* state (2019, p. 30), "goes beyond the individual training of teachers in ICT, being necessary the development of teaching practices that promote this in students".

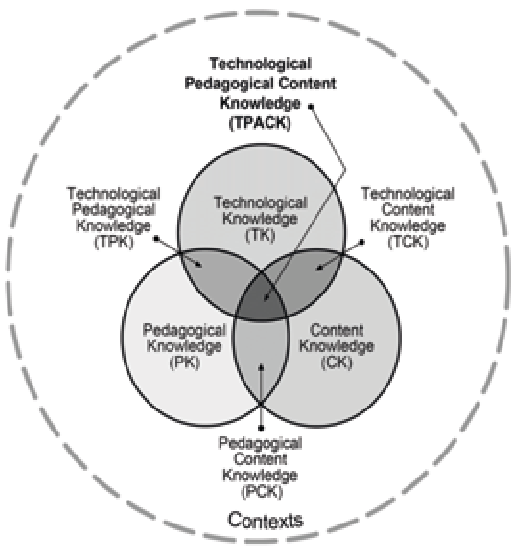
One of the emerging strategies to improve these skills is to integrate technology as a tool to approach mathematical content and achieve greater attractiveness to the subject. For authors such as Cotic (2014), their integration in the classroom depends on the interest of the teacher and his/her ability to stimulate collaborative work of the students and long-life learning or continuous learning. Also, his/her ability to manage technology, requiring his/her digital competence (Ortega-Sánchez *et al.,* 2020).

It is undeniable that digital tools play an important role in the teaching and learning process of mathematics at all educational levels; therefore, the didactic use of technology forces to adapt traditional resources to digital. As stated by Gómez-Gómez (2021), the problem or difficulty is their adaptation for pedagogical purposes.

Therefore, some methodological models, such as Technological Pedagogical Content Knowledge (TPACK), point out that in order to obtain optimal results when integrating technology in the training process, it must be done considering three fundamental aspects: the mastery of the content, the application of technology to it and the pedagogical component of it. Hence, TPACK classifies it into three blocks: Technological Knowledge, Content Knowledge and Pedagogical Knowledge (Schmidt *et al.*, 2009).

Figure 1

TPACK model



Note. Mishra and Koehler (2008).

Other models, such as NETS-T Standards, from the ISTE International Society for Technology in Education, analyze technological profiles for teacher preparation and its link with digital competence (Fuller, 2020). In the context of digital competence, one of the best-known models is the European framework for the digital competence of educators: DigCompEdu (Redecker, 2017), which in Spain is the Common Framework of Digital Teaching Competence (IN-TEF, 2017)

The 2012 IEA (National Institute of Educational Evaluation) TEDS-M (Teacher Education Study in Mathematics) report points out that teacher training in Mathematics Education is one of the variables that interfere with the international differences between the academic results of primary and secondary school children. The test distinguishes between Mathematical Content Knowledge (MCK) and Mathematical Pedagogical Content Knowledge (MPCK), which represent the whole range of difficulty.

Specifically, the TEDS-M Report (2012) shows how future teachers know that they have less training in the field of geometry than the rest of the students from other countries, although they are at practically the same level in the other blocks.

Therefore, considering the principles of the TPACK methodological model and the results of the TEDS-M Report (2012), specifically the Geometry module, six items out of 24 corresponding to mathematical content and 2 out of 10 to Pedagogical Content are analyzed for designing this research.

In recent years, early education has witnessed the changes of technology in the way students approach knowledge and leisure, and how a wide range of possibilities is opened for its use in classrooms. Referring to digital competence means going beyond instrumental knowledge of the basic use of equipment (Eshet-Alkalai, 2012); it must be understood as “a combination of a set of technical-procedural, cognitive and socio-emotional skills, necessary to live, learn and work in a digital society” (Fraser *et al.*, 2013). The development of critical thinking as well as other creative and intellectual skills derives from this digital competence. Therefore, the basic use of technology, from which future teachers and their students usually start, is only the precondition for achieving full digital competence (European Commission, 2011). As early as 2012, Ferrari is following the Institute for Prospective Technological Studies (IPTS) definition for the European Commission that defines digital competence as:

> A set of knowledge, skills, attitudes, strategies and values required when using ICT and digital media to perform tasks, solve problems, communicate, manage information, collaborate, create and share content, and build knowledge effectively, efficiently, appropriately, critically, creatively, autonomously, flexibly, ethically and reflexively for work, leisure, participation, learning, socialization, consumption and empowerment.

Thus, there are studies (for example, Esteve Mon, 2015) that understand digital competence as the accumulation of diverse knowledge: communicative, technological, audiovisual or media.

Hence, depending on the starting point of each student, there will be those who have “skills and abilities in the areas of production and transfer of the new (digital) foundations” and others who do not, which can produce a digital gap (Didriksson, 2007, p. 58) if skills are not achieved, according to UNESCO, at four levels: (a) access; (b) use; (c) appropriation; and (d) production.

There is a consensus in believing that technology makes it easier for teachers to include diverse resources to achieve quality training, which is essential in teaching and learning mathematics (Hohenwarter *et al.*, 2009). In addition, the students can acquire many different mathematical skills with multimedia environments, taking advantage of their digital skills. As Rubio *et al.* (2016, p. 91) say “the advantages of integrating digital technology (e.g., dynamic geometry software, CAS, spreadsheet, among others) in Mathematics Education” have been widely reported in the specialized literature (Artigue, 2009; Hoyles, 2010) and facilitate the opening towards processes of pedagogical interaction, collaboration and knowledge sharing (Barahona *et al.*, 2015).

Technological resources can facilitate, activate and develop the acquisition processes of these competences and can be developed effectively when applying mathematic software such as GeoGebra, as determined by previous research (Aldazábal Melgar *et al.*, 2021; Septian *et al.*, 2020; Suryani and Rofiki, 2020; Freyre and Mántica, 2017).

The application, created by Markus Hohenwarter, offers the possibility of learning dynamic geometry (Geo) and algebra (Gebra), as well as many other mathematical contents in primary, secondary or high school curricula (De Albornoz Torres, 2010) in a connected, compact and easy-to-use software environment (Diković, 2009). It is a simple but powerful resource.

Among the features of GeoGebra are design, through free software that facilitates its continuous development; simplicity of use; the possibility of using it in different platforms and multiple languages, as well as learning by experimental and guided discovery (Diković, 2009). In addition to having a simple interface and a variety of geometric and algebraic tools to perform numerous constructions, it is possible to "generalize and even infer which forms can be obtained under certain conditions" (Bolaños and Ruiz, 2018, p. 156). Meanwhile, Barahona *et al.* (2015) highlight that GeoGebra facilitates abstraction processes to show relationships between a geometric and an algebraic model of a real-life situation. It has a spreadsheet, and its numerous views allow to alternate the use of arithmetic, algebraic representations, symbolic calculation and statistical and probabilistic calculation (Del Pino Ruiz, 2013), which facilitates solving a problem, not only mathematically but also visually (Aktümen and Kabaca, 2012). In addition, GeoGebra favors collaborative and constructivist work by interacting with different working groups and interlearning (Prodromou, 2014).

When assessing the mathematical competence of the future teacher in Spain, the seven specific skills established by the PISA Report can be considered (OECD, 2019): communication, mathematization, representation, reasoning and argumentation, design of strategies to solve problems, use of operations and symbolic, formal and technical language, and use of mathematical tools, as well as NCTM learning standards. This concretion facilitates the evaluation and acquisition of mathematical competence determined by the curriculum in each educational stage.

Among the knowledge and skills that are prominently confined to mathematical competence, as noted above, are those related with geometry and algebra and that also encompass SDG4.

Therefore, the aim of this research is to work from the competency point of view the learning achievements of geometry established by NCTM combining the didactic, digital and mathematical competence (TPACK Model) with the application of GeoGebra Classic Software in the university education received by the future teachers, as well as to know their perception of using these in the classroom.

2. Methodology

The research presented arises from the need to incorporate to the traditional didactics of mathematics a digital tool that allows university students in training know how to solve mathematical problems applying the Computer Software GeoGebra Classic. Not only is the solution of exercises limited to obtaining the optimal final result (acquired mathematical knowledge), but the use of this digital tool (digital competence) is part of the training process complementary to the mere use of pencil and paper and simulate their application in a real classroom (didactic competence), in short, it is intended to meet the foundations established by the TPACK Methodological Model.

To this end, a pilot test has been carried out in the subject Mathematics and its Didactics III, belonging to the third course of the Training Plan of the Degrees in Primary Education of the King Juan Carlos University during the 2021-2022 academic year, using a mixed methodology with pre-experimental design with a sample of 68 participants.

The course begins classes in hybrid format due to Covid-19, in which, in turns, face-to-face and telematic attendance to classes is combined.

2.1 Objetives

The general objective is to connect technology, mathematics and didactics to incorporate them into teacher training using the GeoGebra Classic tool.

The specific objectives are:

- OE1 (Content Awareness): Work on geometry as learning content for the future teacher.
- OE2 (Technological Knowledge): work geometry with digital tools for the incorporation of digital competence into mathematics and vice versa.
- OE3 (Pedagogical Knowledge): to work with the contents related to the geometry of primary education with the GeoGebra Classic program.

2.2 Sample

This research is carried out during the first semester of the 2021-2022 academic year with students belonging to the Primary Education Degree (85.29% of the sample) and to the Double Degree in Early Childhood and Primary Education (14.70% of the sample), both in-person degrees and with a population of 68 enrolled, of which 58.82% are men and 29.41% are women. The sampling carried out is non-probabilistic for convenience and also belong to a single group, so it represents 100 percent of the population under study.

2.3 Phases of the research

A brainstorming was done in the first class using the Wooclap tool, in which the students were asked about their perception of mathematics in general and geometry in particular. Most of the students participating in the online modality, through the Teams platform, as well as those who were in the classroom, agreed that they found them difficult and boring. In addition, they were asked how they would feel about "teaching math" as future teachers, and most acknowledged that it made them feel nervous and unprepared. After this first analysis, they were asked if they knew computer programs for teaching geometry, and although many did not know their existence for didactic purposes, they did know the application of GeoGebra in solving problems, and some also indicated that they had worked with it on previous courses.

After the first contact with the participation in Wooclap, the students were proposed to hold a virtual workshop with the use of GeoGebra Classic to work the mathematical contents that are included in this module. To do this, they had to create working groups with assigned roles specific to cooperative work (Johnson and Johnson, 1999).

To carry out the workshop, they are told to analyze the content of two documents, on the one hand, the teaching guide of the subject, and on the other, the analysis of Decree 89/2014 regulating mathematical content for the Primary Education Cycle for the Community of Madrid, Spain. The objective is to know the standards of learning required in the subject and the correspondence between these and the curricular contents regulated in the schools where they will be working as teachers when they graduate.

In the second phase, they are asked to work in a didactic way on transformations, in particular rigid movements and homothetic. To do this, they developed teaching materials based on manipulative materials adapted as if they were to be applied in the real classroom.

Once this activity was completed, they were told to solve exercises on the same subject using tools such as the compass and the rule, dynamics that at first had to develop at an individual level.

Finally, they were asked to solve these exercises using the GeoGebra Classic program.

2.4 Use of GeoGebra Classic in the teaching of geometrical transformations

Then, a case analysis is performed to better understand the application of GeoGebra Classic in the virtual workshop designed for it.

In the first exercise presented to them, the student must make a turn in an anti-clockwise direction of the next circular sector, taking point D as center of rotation and α as angle of rotation.

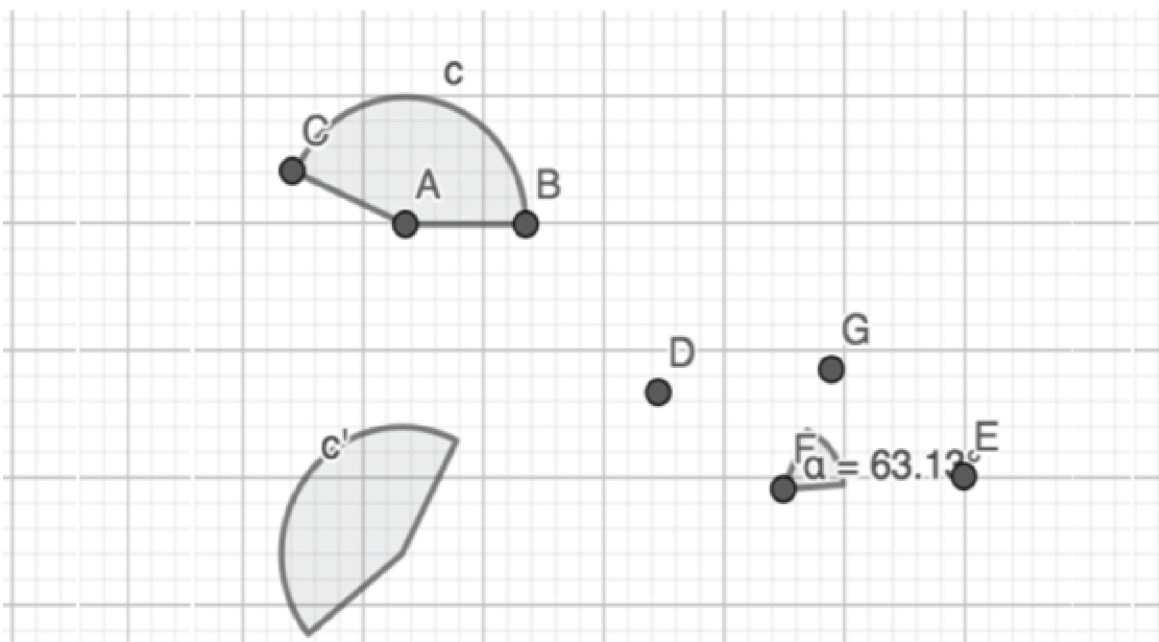
The aspects included in the rubric to solve the problems are:

- It does not provide the requested exercise.
- It provides the exercise but does not do the required movement.
- It provides the exercise and performs the required movement, but it fails to consider the direction, or the center or angle requested.
- It provides the exercise and performs the required movement well considering the direction, center and angle requested.

Figure 2 shows one of the problems successfully solved.

Figure 2

Geometric Transformation: The Rotation

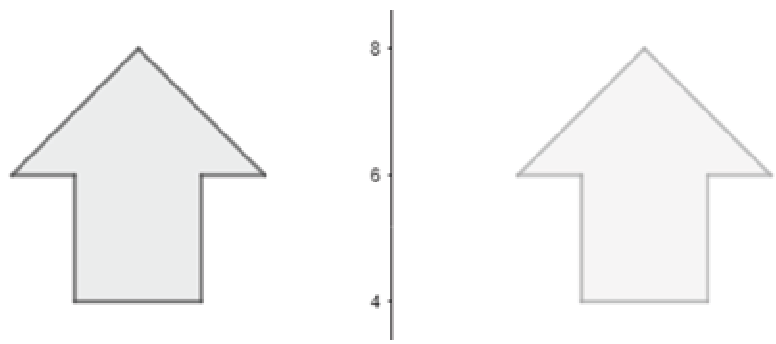


Peer-to-peer evaluation criteria have been based on a rubric provided by the faculty and a feedback section. The coincident assessments given by two colleagues based on the rubric and supervised by the faculty determine the correct performance of the exercise.

On the other hand, two solutions are shown on another planned statements, a correct solution and a partially correct solution. In this case, students are asked to perform a symmetry based on the axis provided. Figure 3 shows one of the productions that is considered not entirely correct, receiving the assessment of another colleague among the available options as "the student contributes to the exercise, but does not perform the required movement".

Figure 3

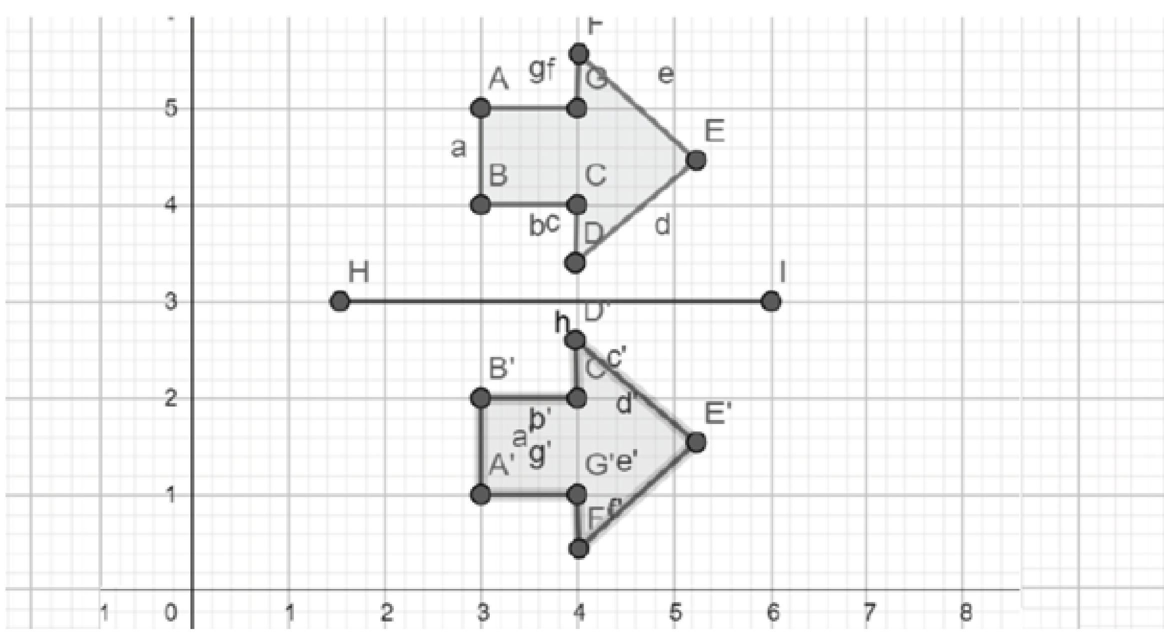
Partially successful geometric transformation: symmetry



However, in Figure 4, there is a positive evaluation by correctly providing the exercise and perform-ing the required movement.

Figure 4

Correct geometric transformation: symmetry



In short, the participation in the workshops with the delivery and peer evaluation allowed students to know how to develop the same exercises by other peers, as well as the possibility of self-evaluation and identify if what they had done was correct or not.

2.5 Data collection tools

In order to know the perception of this initiative among future teachers and see their association established by implementing the activities proposed in a real classroom, an *ad hoc* questionnaire was designed using the Forms tool of Office 365 Package, as the main tool for collecting information. The questionnaire has eight items consisting of dichotomous questions and Likert scale with values between 1 and 5, 1 being the worst value and 5 the maximum value, with some open questions. The average response time is less than four minutes. Questions are oriented to assessing the experience with the participation through the workshops, the use of GeoGebra Classic and the knowledge of the agenda. They are also asked about their perception of their training in mathematics education, as well as their training in mathematical content at the levels prior to the Degree in Education.

The voluntary participation in the questionnaire is 40% (28 students), participation lower than expected but it allows analyzing the situation that facilitates the use of appropriate teaching and pedagogical strategies to achieve the teaching objectives successfully (Hernández Ávila and Carpio Escobar, 2019). On the other hand, the participation in the workshops and the scores obtained in them were analyzed, as well as the impact on the final grade of the subject.

3. Results

The quantitative analysis of the answers provided in the questionnaire is carried out using the statistical package Microsoft Excel, performing a descriptive analysis of the results. The qualitative analysis is performed considering the contributions and opinions of the participants from answers obtained with Wooclap and with open questions of the questionnaire, thus completing the information obtained in the quantitative analysis.

Thus, a block classification of the most relevant information is presented:

Prior formation: they were asked about the branch of knowledge prior to entering university, and more than half of the class identified with "humanities" in 54% of cases while only 48% identified with "sciences".

Content: Regarding the difficulty of geometry-related content, they were asked to assess using a dichotomous question whether it was difficult or easy or incomprehensible. Mode indicates that they perceive mathematical content difficult or very difficult. It is true that in this question they were also asked if they thought they had to study more, being that 74.2% of respondents rated 4 points out of 5 to the statement that they have to study more, at least at the time they answered the survey, and 29% said they did not understand anything.

Perception of training: more than half of respondents believe that their Degree training plan needs more dedication hours to mathematics teaching

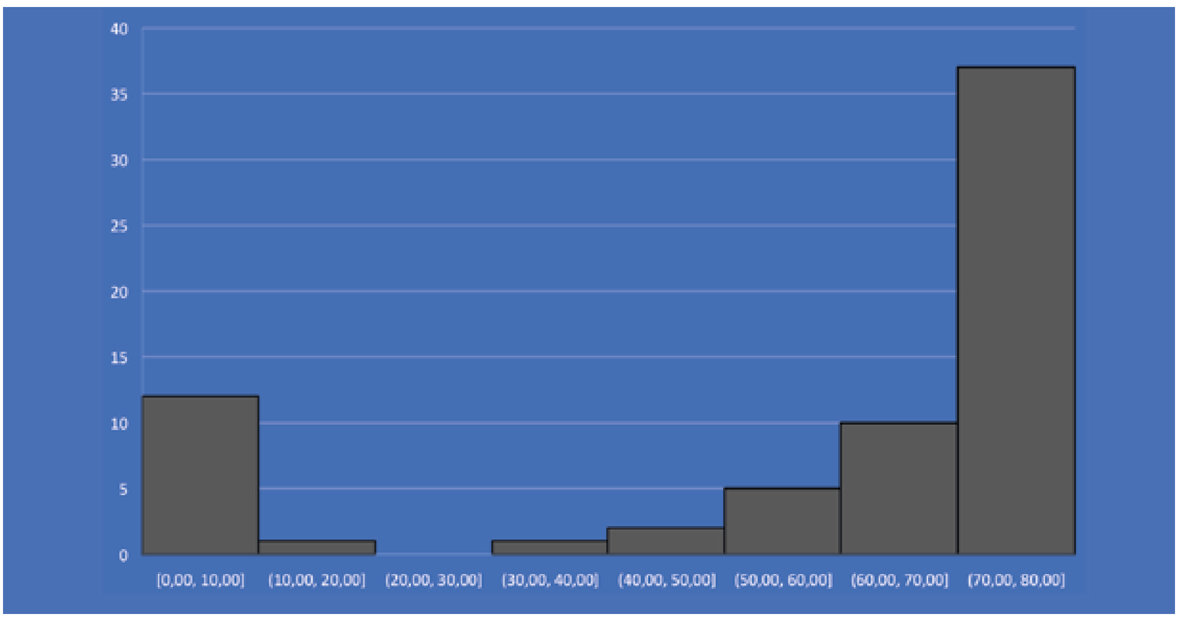
Workshop rating: the workshop rating is generally positive. To find out what they thought, this question was left open to collect their opinions. Some of the comments to note are: "GeoGebra has helped me visualize what I could not see by myself" "the fact of doing exercises in GeoGebra seems to me a good resource". However, one of the negative aspects is that it has taken them a long time to carry out exercises with the application.

In addition, to know the participation in the workshop, two activities were set in the virtual classroom, on the one hand, the delivery of the scheduled task in which each coordinator of the group had to deliver the activities designed with manipulative material in time and format established. In this case, the participation was 100% of the enrollees, which was relevant since participation was a voluntary decision. On the other hand, a virtual workshop was set up for the individual delivery of the proposed exercise solution. In this case, it was divided into two sessions, on the one hand the delivery and on the other hand the evaluation of the work of another partner to promote peer evaluation. In the first phase, 89.65% of the enrollees participated; however, 83.82% participated in the evaluation of the work of a peer, this data must take into account that students who do not present their work cannot evaluate it.

The overall average score for workshop participation is satisfactory, ranging from 70 to 80 points. Specifically, if the average result of the first planned exercise is analyzed as an example, a score of 78 points out of 80 is obtained. In general, as can be seen in Figure 5, most students obtained this grade because they correctly solved the exercise proposed by applying GeoGebra Classic.

Figure 5

Evaluation results of the geometric transformation exercise with rotation



4. Discussion and conclusions

The results presented indicate that the application of technology allows to achieve optimal results if combined with knowledge and didactics in the teaching and learning process. The inclusion of GeoGebra Classic in the training process of future teachers and the introduction of mathematical concepts in learning have been well accepted among students, while, as stated by Gómez-Gómez (2021), the application of different technological resources for pedagogical purposes allows achieving better results. Likewise, as stated by Rubio *et al.* (2016), ICTs improve technological competence, among other things, because they develop abstract thinking. During the process, it has been determined that, as in previous research such as that of Ruiz López (2012), the didactic competence using GeoGebra Classic may be conditioned by the previous digital competence that the students have, so it is necessary to implement activities in the classroom that facilitate the competence during the training processes of future teachers, so that they are more in line with the current situation of education in Spain.

Students can acquire mathematical competence according to the TPACK model, and as stated in the study of Mishra and Koehler (2008), if the technological dimension and skills of the students are combined with the acquired mathematical knowledge and its application in the classroom. On the other hand, other research, such as that of Pistón-Rodríguez and Parejo-Jiménez (2019), highlight that mathematical competence is acquired in the first formative stages; however, the empirical evidence of this work shows that the application of GeoGebra Classic allows students to recognize the abstract component of mathematics, as most respondents say; which is related, in turn, to the research carried out by Fernández-Bravo (2006) in which the author concludes that basic mathematical competences can be acquired at any formative stage, or the Systematic analysis by Yohannes and Chen (2021) that highlight the importance of integrating GeoGebra for mathematical education.

Regarding acquired mathematical competence, the results show an improvement in the grades of those students who participated individually in the workshops with GeoGebra Classic. It is especially remarkable the difference of previous knowledge that some students had when assimilating and applying mathematical concepts and that was conditioned by their branch of previous knowledge identified with the field of humanities and not science. In addition, it has allowed them to demand more training in this regard. Therefore, some rejection to mathematics and geometry are removed as also concluded by Novelo Sánchez *et al.* (2015)

On the other hand, it is of special interest the teaching and media competence of the teacher to facilitate the access of the student to knowledge, as well as to favor the access to digital resources that facilitate the approach to basic mathematical knowledge while developing the competence. The need to provide digital teaching skills to future teachers through the appropriate inclusion of ICT is highlighted, where the TPACK model is shown as an effective teaching model.

In short, the application of GeoGebra Classic facilitates and improves the acquisition of mathematical and digital competence and reduces certain deficiencies or difficulties that arise in learning. The main contribution when applying the mixed pre-experimental methodology designed for this research with the use of workshops and when using GeoGebra Classic is that there is replicability in the procedure and allows developing strategies that combine, at the same time, the three competencies foreseen in the TPACK model.

Undoubtedly, a good starting point to complete this research would be to carry out a comparative study between the results of future promotions with those obtained in this work, observing the existing deviations to adopt strategies of didactic improvement to achieve quality teaching and effectively comply with the SDGs by verifying that the use of GeoGebra improves the digital mathematical competence of the future teacher.

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