ALTERIDAD REVISTA DE EDUCACIÓN
p-ISSN:1390-325X / e-ISSN:1390-8642
http://alteridad.ups.edu.ec

july-december 2022
Vol. 17, No. 2, 224-242
https://doi.org/10.17163/alt.v17n2.2022.05

The learning of logarithmic functions by 12th-grade students based on modeling tasks

El aprendizaje de las funciones logarítmicas por parte de estudiantes de 12.º grado basado en tareas de modelización

Samuel Araújo is a professor at Lycée Français International de Porto, Portugal (mrsamas17@gmail.com) (https://orcid.org/0000-0001-8407-395X)

Dr. Floriano Viseu is a professor at Universidade do Minho, Portugal (fviseu@ie.uminho.pt) (https://orcid.org/0000-0002-8221-6870)

Dra. Ana Jacinta Soares is a professor at Universidade do Minho, Portugal (ajsoares@math.uminho.pt) (https://orcid.org/0000-0003-4771-9859)

Isabel Leite is a professor at Escola Secundária de Vila Verde, Portugal (isabelleitemat@gmail.com) (https://orcid.org/0000-0001-8656-1703)

Received on: 2022-05-03 / **Revised on:** 2022-06-12 / **Accepted on:** 2022-06-21 / **Published on:** 2022-07-01

Abstract

The relevance that the application of what is learned in Mathematics to everyday situations has in student learning has led us to develop an experiment on teaching the logarithmic function using modelling tasks. Based on this experiment, we intend to characterize the activities of 12th-grade students in the performance of modelling tasks concerning topics of logarithmic functions and identify the difficulties they may present while solving these tasks using a graphing calculator. When adopting a qualitative and interpretative approach, data were collected through students' written records while solving the proposed tasks using the graphing calculator. The results indicate that the students show that the modelling tasks promoted group work and their interest and participation in class. During the exploration of the tasks, students performed activities that result from the accomplishment of modelling phases such as understanding the statement of the task, organizing and analyzing data, building and validating the model that best fits the data, and exploring the model, either in the introduction of the logarithmic function and its derivative or in the consolidation of the acquired knowledge. In such activities, some students presented difficulties regarding the properties and characteristics of the logarithmic function and its graphic and symbolic representation. The phases of the modelling cycle also made it difficult for students to use the graphing calculator, namely in performing statistical regressions and setting the visualization window.

Keywords: Mathematics, logarithmic functions, learning, mathematical modelling.

Resumen

La relevancia de aplicar lo aprendido en matemáticas a situaciones cotidianas en el aprendizaje de alumnos nos ha llevado a desarrollar un experimento sobre la enseñanza de la función logarítmica mediante tareas de modelización. A partir de este experimento, pretendemos caracterizar las actividades de alumnos de 12° curso, en la realización de tareas de modelización relativas a temas de funciones logarítmicas utilizando una calculadora gráfica., a su vez, identificar las dificultades que pueden presentar al resolverlas. Al adoptar un enfoque cualitativo e interpretativo, se recogieron datos a través de los registros escritos del alumnado mientras resolvían las tareas propuestas utilizando la calculadora gráfica. Los resultados indican que las tareas de modelización promovieron el trabajo en grupo y su interés y participación en clase. Durante la exploración de las tareas, el alumnado realizó las actividades que se derivan de la realización de las fases de modelización como son la comprensión del enunciado de la tarea, la organización y el análisis de los datos, la construcción y validación del modelo que mejor se ajusta a los datos y la exploración del modelo, ya sea en la introducción de la función logarítmica y su derivada o en la consolidación de los conocimientos adquiridos. En estas actividades, algunos alumnos presentaron dificultades en cuanto a las propiedades y características de la función logarítmica y su representación gráfica y simbólica. Las fases del ciclo de modelización también dificultaron el uso de la calculadora gráfica por parte de alumnos, concretamente en la realización de regresiones estadísticas y en la configuración de la ventana de visualización.

Descriptores: Matemáticas, funciones logarítmicas, aprendizaje, modelización matemática.

Suggested citation: Araújo, S., Viseu, F., Soares, A. J. and Leite, I. (2022). The learning of logarithmic functions by 12th-grade students based on modeling tasks. *Alteridad, (17)*2, 224-242. https://doi.org/10.17163/alt.v17n2.2022.05

1 Introduction

When thinking about the learning activities of mathematical topics, the characteristics of the tasks are related to challenge learners to engage and reflect on what they do in these activities (Tekkumru-Kisa, 2020). These characteristics distinguish tasks in terms of the degree of challenge, high or low, and the degree of structuring, open or closed (Ponte, 2005). Among the tasks commonly worked in mathematics classes, we can mention exercises (low level of challenge and closed structure), problems (high level of challenge and closed structure) and research tasks (high level of challenge and open structure). Modeling tasks are part of these types, as problems or investigations, depending on the degree of structure of their statement. The activity resulting from solving this type of task is called mathematical modeling, which is currently part of the mathematics academic program in several countries (Chong *et al.*, 2019; Hoe and Dawn, 2015). In Portugal, it is part of the profile of students who graduate from compulsory education (Ministério da Educação, 2017). Dawn (2018) and Stillman *et al.* (2017) argue that modeling should be part of the activities performed at school in science learning.

Regarding mathematics, the Program for International Student Assessment (PISA) contemplates the definition of mathematical literacy activities related to modeling:

> Mathematical literacy is an individual's ability to formulate, apply and interpret mathematics in different contexts. It includes mathematical reasoning and the use of mathematical concepts, processes, facts, and tools to describe, explain, and predict phenomena. (OECD, 2019, p. 75)

There are several authors (e.g., Shahbari and Tabach, 2020; Viseu and Rocha, 2020) and reference works (e.g., NCTM, 2000) that defend the contribution of mathematical modeling in the development of critical skills for students´ engagement in their future activities (Reys-Cabrera, 2022). The NCTM (2000) recommends that students experiment with mathematical modeling of real-world problems, both social and physical, throughout their schooling, and this practice should be more present in later grades.

A favorite topic in mathematics for modeling phenomena is functions (Viseu and Rocha, 2020). From elementary school, students work with relationships between quantities, such as proportionality between quantities, for example, through a problem involving the speed of cars, or quadratic functions, through optimization problems of areas of geometric figures (Rocha, 2019). In high school, far-reaching mathematical concepts are introduced, such as derivation, which allows describing and understanding more complex functions, to illustrate, the logarithmic functions introduced in the 12º grade[1] of high school.

Many everyday phenomena can be modeled using these functions, such as the growth of a bacterial population or the magnitude of an earthquake. Facing a mathematical modeling problem is essential to understand and structure the mathematics involved in the problem. In solving some of these tasks, functions are essential to represent and solve real-life problems (Sawalha, 2018), as is the case with logarithmic functions that translate into numerous everyday situations. According to Kastberg (2002) and Weber (2002), students have difficulty understanding and representing logarithmic functions, because the exploration of logarithmic functions is mainly limited to theoretical aspects. Moreover, the

1 The Portuguese education system includes 12 years of schooling up to higher education. The first nine years correspond to Elementary School (BE) and the last three to High School (SE). The BE consists of three teaching cycles: the first four years (with a single teacher), the second two years and the third three years. In the three years of SE, in which students begin to be oriented towards a group of higher education courses, the mathematics academic program varies depending on whether they follow courses in science, humanities, arts or technology.

symbolic writing of logarithms and, consequently, of logarithmic functions tends to disrupt their understanding (Mulqueeny, 2012). Teaching these functions through real-life problems and situations can improve students' understanding of their topics. Therefore, it is essential to conduct studies that deepen on knowledge by enhancing this understanding (Viseu and Rocha, 2020).

For this, technology is essential to identify and visualize the model that best fits the context of the problem under study. The graphing calculator is essential to conceptualize and understand the different representations of a function (Viseu and Menezes, 2014; Viseu *et al.*, 2020). The relevance of this didactic device increases by allowing to connect analytical and graphical representations of logarithmic function concepts and to compare results. In the performance of modeling tasks in learning logarithmic functions, the graphing calculator is a didactic material that provides students tables, the editing of graphs and the exploration of regression curves that best fit the data resulting from the tasks proposed in class (Viseu and Menezes, 2014).

The combination of solving mathematical modeling tasks using the graphing calculator shows the valuation of what the student learns in solving problematic situations, his/her involvement in the activities proposed in the mathematics class and gives meaning to what he/she learns (Viseu and Rocha, 2020).

Based on these assumptions, this paper aims to characterize the activities that 12th grade students perform with mathematical modeling tasks in learning logarithmic functions and to identify the difficulties they have using the graphing calculator.

1.1 Mathematical modeling

The use of mathematical models in the teaching and learning of science subjects in recent years has been the subject of several studies (Anhalt *et al.*, 2018; Stillmann *et al.*, 2017). The connections between mathematics and the natural world condition the teaching and learning of mathematics (Barbosa, 2006, 2009; Blum, 2002). In this research, the impact of mathematical modeling in different grades has drawn attention (Stillman *et al.*, 2007). It is critical to analyze how mathematical modeling supports skill development in students' education (Kaiser and Sriraman, 2006; Kaiser and Maaß, 2007; Karawitz and Schukajlow, 2018). The first thing is to define mathematical modeling, distinguishing it from applied mathematics. According to Blum (2002), Blum *et al.* (2007) and Stillman *et al.* (2007), applied mathematics focuses on the use of mathematical tools in a real-world situation. Mathematical modeling focuses on the identification and development of mathematical tools to help solve that problem (Verschaffel, 2000). Mathematical modeling is a process of solving real-world problems (Carreira and Blum, 2021).

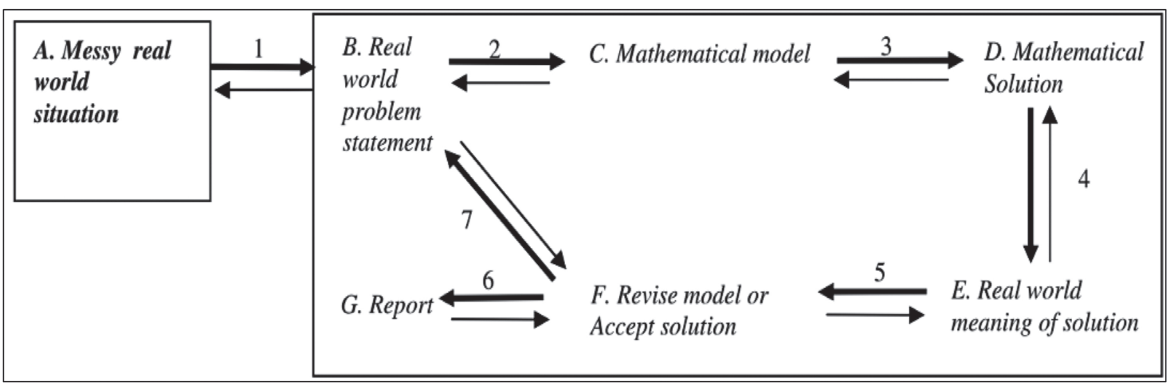
Mathematical modeling can be interpreted in different ways. Some authors interpret it as a motivational factor to introduce, develop and consolidate mathematical knowledge and learning (Barbosa, 2009). Others see it as a purely educational approach to develop students' ability to solve concrete problems (Galbraith and Stillman, 2006). Although these are different interpretations, they are not dissociated. By approaching a mathematical modeling problem focused on the solution processes, the learning objectives can be achieved (Barbosa, 2009; Galbraith and Stillman, 2006).

Solving a mathematical modeling problem requires understanding and interpreting the context of the situation under study and identifying the main aspects of the problem. To do this, data can be collected and the relationships and patterns between quantities can be described and understood in order to translate this information into mathematical language (Blum, 2002). This mathematical structure is called a mathematical model (Blum, 2002; Lesh and Fennewwald, 2010). The development of a model can be done through various representations to encompass the possible potential of responses to a given situation (Lesh and Fennewwald, 2010).

Thus, models run along different dimensions, from the concrete to the abstract, from the specific to the general, from the global to the analytical, from the simple to the complex, from the situational to the decontextualized, and from the intuitive to the formal (Lesh and Fennewwald, 2010). The application of a mathematical model to study a problem requires the use of known mathematical tools and methods to extract mathematical results from that model. In turn, these results must be interpreted and validated by analyzing their relevance to the context and purpose of the real problem. This process can be repeated, from scratch, in the definition of the model, depending on its effectiveness as a solution to the problem (Blum, 2002; Greefrath, 2019; Stillman *et al.*, 2007). This process is called mathematical modeling, which translates into the identification and definition of its phases and transitions in various diagrams or schemes. Stillman *et al.* (2007) adapted a diagram from Galbraith and Stillman (2006) to represent the mathematical modeling cycle, incorporating the reasoning that occurs between the different phases (Figure 1).

Figure 1

The modeling process (Stillman et al., 2007)



In this diagram, letters A-G represent the phases of the modeling process, and the arrows represent the transitions between the phases: (1) Understand, structure, simplify, interpret the context; (2) Assume, formulate, mathematize; (3) Develop and explore mathematically; (4) Interpret mathematical results; (5) Compare, criticize, validate; (6) Communicate, justify (if the model is considered satisfactory); (7) Go through the modeling process again (if the model is considered unsatisfactory).

For Borromeo Ferri (2006), the student is not always motivated to go through all the phases of the modeling cycle when learning mathematical content, and it is necessary to try to adapt this cycle to the classroom context. The authors identify, based on studies and empirical data, the critical aspects that can block students in a given phase. These data were collected in the context of learning mathematical content by modeling problems and using technological tools (Galbraith and Stillman, 2006; Stillman *et al.*, 2007). From the analysis of these data, it is clear the main difficulties of students are the understanding of the statement and its context (Galbraith and Stillman, 2006; Stillman *et al.*, 2007), the identification of dependent and independent variables, the elaboration of the model, as well as the mathematical structure of the real problem (Dede, 2016; Galbraith and Stillman, 2006; Shahbari and Tabach, 2020).

1.2 Logarithmic functions

Understanding the concept of logarithmic function is essential to prepare students to face concrete problems (Kenney and Kastberg, 2013). Formally, the logarithmic function is defined as the inverse function of the exponential function. More precisely, a logarithmic function of base is defined by $y=\log_a(x)$, $x>0$, $a>0$, $a\neq 1$, with $y=\log_a(x)$, if and only if $x=a^y$. Similarly, understanding logarithmic functions involves the ability to interpret the notation used (Kenney and Kastberg, 2013; Weber, 2017). In the expression $y=\log_a(x)$, perceiving the duality between the object x as an element of the domain of the function is a y, and the number obtained in the form $x=a^y$ is an obstacle for most students (Weber, 2002). Since the $\log_a x$ notation is relatively atypical compared to algebraic functions, it is confusing for most students to associate specific properties and characteristics (Kenney and Kastberg, 2013). Lack of understanding the notation of logarithmic functions, specifically the bases and the different nomenclatures $\log_a x$, $\log x$ e $\ln x$, can make it difficult to learn their algebraic properties. Students may be able to interpret linear, quadratic and even exponential functions because their algebraic expressions indicate to some extent what the underlying process is. However, the symbolic notation of logarithmic functions is more complex and ambiguous (Mulqueeny, 2012). Faced with a logarithmic equation, students tend to cancel logarithms of the same base as if they were polynomial functions (Kenney and Kastberg, 2013).

This shows that students have difficulty perceiving the role of logarithmic functions as inverse functions of exponential functions. This difficulty is also perceived in the graphical representation, as they match the graphical representation of logarithmic functions with that of exponential functions (Weber, 2017). A reinforcement of the duality between a bijective function and its inverse through, for example, the square root and the quadratic function, can improve the learning of logarithmic functions (Kenney and Kastberg, 2013). According to Sawalha (2018), providing students with modeling tasks in learning exponential and logarithmic functions promotes the structuring and understanding of these concepts.

Current recommendations for mathematics teaching point to the development of competencies that students should acquire in their schooling (e.g., NCTM, 2007). According to Niss and Højgaard (2019), mathematical competence is the ability to respond promptly and insightfully to any mathematical challenge. The authors emphasize the difference between mathematical competence and a mathematical competence, the latter being the ability to respond promptly and insightfully to a specific mathematical challenge. On the other hand, mathematical competence is the combination of mathematical skills (Niss and Højgaard, 2019). In defining these subsets of mathematical competencies, the authors emphasize that these must satisfy the fulfillment of mathematical activities. A mathematical activity is the action aimed at asking and answering questions in or through mathematics (Niss and Højgaard, 2019, p. 14). Based on these assumptions, Niss and Højgaard (2019) define four competencies that are essential to participate efficiently in a mathematical activity: fundamental mathematical thinking; mathematical problem development and solving; the use of models and mathematical modeling; and mathematical reasoning. The mastery of mathematical language, constructs and tools allows in the conduction of mathematical activities, mentioned by the authors as other competencies, skills: the management of mathematical representations; the management of mathematical symbols and formalisms; mathematical communication; the management of material resources and mathematical tools. The authors emphasize that all these competencies are different, but not dissociated.

For this reason, we have chosen to give special relevance to three competencies that we consider fundamental in mathematical modeling: the treatment of models and mathematical modeling; the treatment of mathematical representations; the treatment of material resources and mathematical tools. The first focuses on the ability to construct mathematical models and to analyze and evaluate models already defined, considering external factors such as data, facts, properties, and the context of the situation. The ability to articulate the different phases of the modeling process is included in this competence (Stillman *et al.*, 2007).

Competence in handling different mathematical representations consists on the ability to interpret and translate mathematical objects, phenomena, correspondences, and processes into a variety of equivalent representations, considering the advantages and limitations in the performance of a mathematical activity. The development, analysis and interpretation of models are directly related to the manipulation of mathematical representations (NCTM, 2000; Shahbari and Tabach, 2020). Finally, technology is essential as a constructive and critical resource in the performance of a mathematical activity. This competence also considers the advantages and limitations of these tools to use them accordingly depending on the mathematical activity (Niss and Højgaard, 2019).

2 Methodology

This study aims to characterize the activities of 12th grade students with mathematical modeling tasks in learning logarithmic functions and to identify the difficulties they have in solving these tasks using the graphing calculator. These difficulties highlight the fact that students do not perform adequately in the solution of these tasks or that result in erroneous solutions (Heyd-Metzuyanim, 2013) and in an inadequate use of the graphing calculator in such solution. Given these objectives, one of the authors carried out a teaching experience based on the use of modeling tasks in the teaching of logarithmic function topics using the graphing calculator as part of his pedagogical practices in the last course of the master's degree in teacher training, which he integrated into his teaching strategies. This device is part of the didactic material that High School students in Portugal use when doing their activities. The study of logarithmic functions is performed for the first time in the students' school in the 12th grade. This teaching experience occurred during four lessons and included modeling tasks, from which we selected three (Annexes).

To optimize student learning in solving the modeling tasks, the teaching experience developed outlined strategies that assessed student activity through an exploratory teaching format (Ponte, 2005). In the first phase of the lessons, the task was presented to the class to determine whether the students understood the task statement, the data, and what was intended to be determined. In the second phase, students performed the task autonomously, in pairs or in groups. At this point in the lesson, the teacher supported the students, but without interfering with their solving strategies. Once the students' autonomous work was completed, their solutions were collected so that they would not be modified according to the discussion of the class group. In the penultimate phase, the teacher carefully guided this discussion to manage the students' interventions and compare the different solutions. In the last phase, based on what the students did or said, new concepts or procedures were set as a result of the exploration and discussion of the task.

The class selected was a 12th grade Science and Technology class with 31 students, 14 boys and 17 girls between 16 and 18 years old. The class had no repeaters and was composed of students who, for the most part, considered Mathematics and Physics-Chemistry as their favorite subjects. As for performance in Mathematics, the students' grades at the end of the course ranged from 6 to 20, with a mean of about 14.4 (DP=4.21). As for the solution of modeling tasks, the students had already performed tasks of this nature in previous courses with the class teacher in the study of polynomial functions and rational functions.

Given the nature of the objective, a qualitative and interpretative approach was used to understand the students' activity in solving the proposed tasks in the classroom context (Bogdan and Biklen, 1994). For this purpose, data were collected through the written records of the students in the solution of the proposed tasks, using the graphing calculator to study logarithmic function topics prior to their discussion in the class group. These tasks were developed with the intention that, from their solution, the students would acquire the notion of logarithm of a number in each base, carry out the study of a logarithmic function, determine the derivative of the logarithmic function and systematize the study of the logarithmic function.

The data analysis was based on the content analysis of the students' solutions of the modeling tasks proposed to them, translated according to the topics taught: (i) Introduction to the study of the logarithmic function; (ii) Introduction to the derivative of the logarithmic function; and (iii) Systematization of the knowledge of logarithmic functions. In each of these topics, the modeling phases considered from the theoretical framework are used: (1) understanding of the task statement; (2) data analysis/model construction and validation; (3) model exploration. The analysis of the students' solutions allows to identify the activities they performed in each of these modeling phases and the difficulties they had in their solutions. This analysis focuses on the solutions made in pairs in the modeling tasks related to the introduction of the logarithmic function and its derivative and in groups in the modeling task related to the systematization knowledge of logarithmic functions.

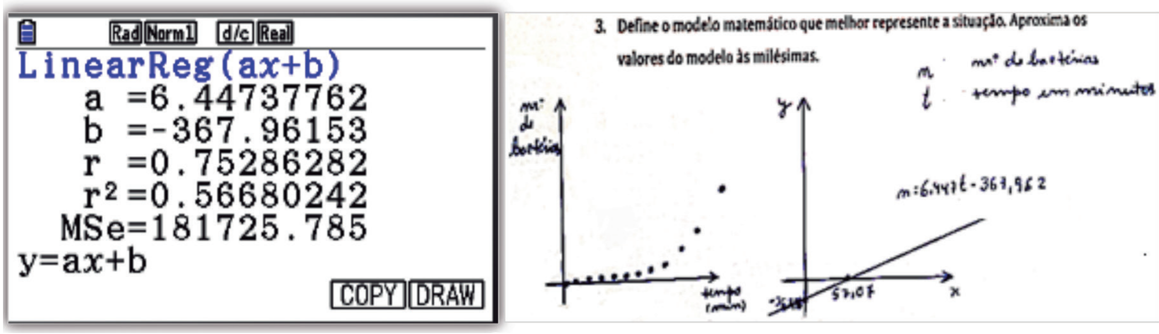
3 Results

3.1 Introduction to the study of the logarithmic function

In the introduction of logarithmic function, the students began by solving a modeling task in pairs involving knowledge of the exponential function previously studied. The choice of this function as a model arising from the solution of the proposed task was due to the relationship between the exponential function, with a positive real base other than one, and its inverse function, the logarithmic function with the same base (task 1, appendices). When exploring this task, students indicated that they interpreted what the task asked. In the data analysis, model building, and validation phases, students aimed to develop a mathematical model that represented the task situation. Questions 2 and 3 of the task complete and illustrate these phases. Students used diagrams to organize the data on the graphing calculator, identifying the dependent and independent variables. The P3 pair resorted to instrumented action diagrams from the graphing calculator to obtain a linear model and represented a sketch of the point cloud of the number of bacteria as a function of time. Although this sketch shows an exponential curve, it defined a linear model. This shows that they did not perceive well the role of the determination coefficient or that students did not try to find out other models (Figure 2).

Figure 2

P3 Pair's answers to Task 1, items 2 and 3



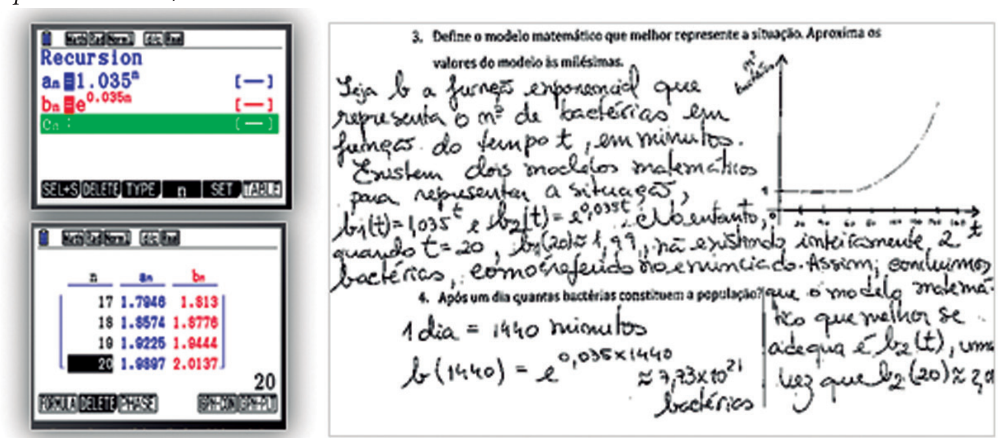
Likewise, when defining the best model that fits the data, pairs P11 and P13 did not resort to the graphing calculator and defined a mathematical model by means of algebraic processes from the data in the table. These students recognized a geometric progression of ratio 2 and general term 2^n, where *n* represents the number of binary division processes, which allowed them to define the following functions: $f(x)=2^{\frac{x}{20}}$where x is the time in minutes (P13); $N(t)=2^{\frac{t}{20}}$, where t is the time in minutes (P11). These models correspond perfectly to the data in the table and the binary splitting process if one works with discrete rather than continuous models.

The P7 pair identified the exponential model as the best fit, based on the determination coefficient r^2, but did not recognize that both models shown on the screen ($y=ae^{bx}$ ou $y=ab^x$) represented the same model. To determine which fit better, these students used the recursion menu of the graphing calculator and plotted the two models as a function of in a table to compare their values.

Some students were critical with the models obtained with the graphing calculator and tried to check the results using another function of $n \in \mathbb{N}$ raphing calculator (Figure 3).

Figure 3

P7 Pair's response to Task 1, items 2 and 3



In exploring the model to answer the remaining items, it is worth noting that the pairs P11 and P13, defined with paper and pencil the same exponential model $y=2^{\frac{x}{20}}$, and tried to solve this question analytically. They decomposed the number 8000 into prime factors to solve the equation $=8002^{\frac{x}{20}}$However, the obtained decomposition 2^6 x 5^3 was a dead end for these students. Pair P13 used the graphing calculator to solve the equation graphically.

Solving questions 5 and 6 of the task led to the inverse process underlying the exponential function to determine the value of x by solving the equation. The application of the properties of inverse functions, i.e., the graphical representation of a function compared to its inverse function, allowed the graphicng calculator to represent the symmetric curve of the graph of the exponential function determined by the line of equation $y=x$.. Thus, the reciprocal relationship between the exponential function and the logarithmic function with the same base was established $x=a^y \Longleftrightarrow y=log_a x$, $a \in IR^+ \backslash \{1\}$. To systematize this knowledge, the students solved tasks on the notion of logarithm of a number in a given base and on the study of logarithmic functions.

3.2 Introduction to the derivative of logarithmic function

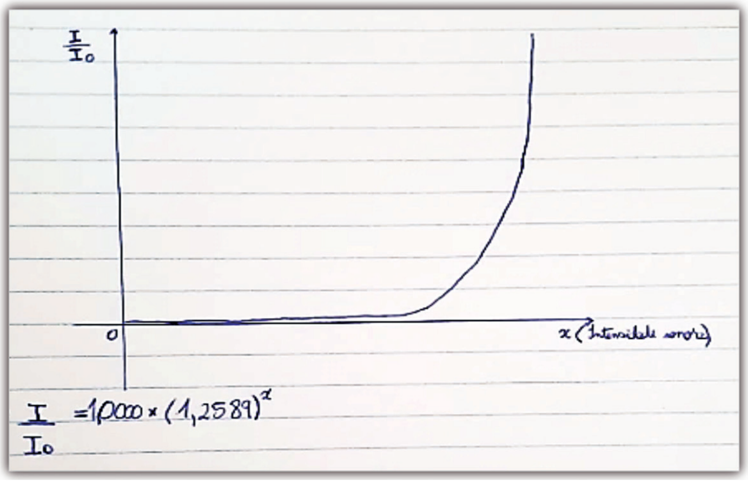
The introduction to the derivative of the logarithmic function resulted from solving a modeling task on sound intensity in decibels (Task 2, Appendix). From the data of the table and the graphing calculator, students had to identify the model that best fit the data presented and answer the remaining questions based on that model. This task was solved in pairs, and the conclusions were discussed and analyzed in class.

The interpretation of the task statement was clear for students; no pair questioned the meaning of the linear scale expression in the question. Most of the pairs found no difficulty in verifying the truth of Bell's statement, understanding that for the data to verify a linear scale, there must be a relationship of direct proportionality between them, as illustrated by P1's statement: the statement is true, since it is not possible to verify a relationship of direct proportionality between the two variables present in the table.

Although the students had already completed a mathematical modeling activity, the notion of a mathematical model was not clear to all, nor was the use of the graphing calculator to determine this model. Eight pairs of students (out of fifteen) correctly identified the role of each variable in the task, correctly organized the data on the graphing calculator, and found the logarithmic model to be the best fit to the data. Some students were unable to find the correct model because they reversed the variables or the lists in the table in the statistics menu. By default, list 1 is assigned to independent variable and list 2 is assigned to dependent variable . In the case of this task, the first column of the data corresponds to the dependent variable and the second to the independent variable. By reversing the role of the variables, these students naturally obtained an exponential model. However, other students did not understand the role of the quantities and plotted the point cloud of the $\frac{I}{I_0}$ relationship as a function of sound intensity in dB, resulting in an exponential model, as exemplified by the answer given by the P13 pair (Figure 4).

Figure 4

P13 pair response to Task 2



Regarding the exploration of the model, the pairs that defined an exponential model found it difficult to answer the remaining items. In item 3, the idea is to calculate the variation in the interval [1010;1030]. However, the graphing calculator does not allow this type of calculation, indicating that these students faced some limitations of the graphing calculator.

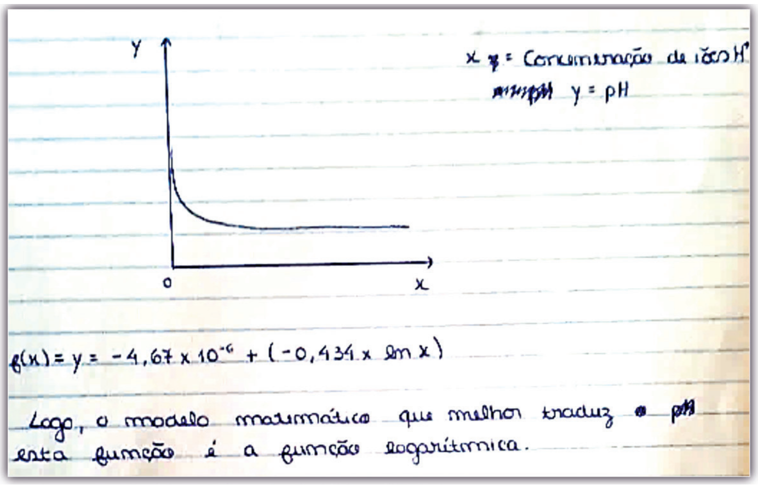
In addition to the technical limitations, there are also constrains due to certain mathematical concepts, such as the definition of instantaneous rate of change at a point. As for section 4, since the first derivative of logarithmic functions had not yet been introduced, the students did not have the possibility to solve it analytically. Five pairs of students understood that they could use the graphing calculator to determine the instantaneous rate of change of the model at the abscissa of point 15, as shown by the answer of pair P1: using the derivative of the logarithmic function (on the graph of my calculator) it is possible to determine the instantaneous rate of change of the sound intensity when $\frac{I}{I_0} = 15$. The response of this pair of students allowed to introduce the derivative of the logarithmic function that modeled the task situation by formally defining the first derivative of a function: $f\left(\frac{I}{I_0}\right) = 1{,}03 \times 10^{-4} + 4{,}3429\, ln\left(\frac{I}{I_0}\right)$. Next, the first derivative of any logarithmic basis function was defined by means of the algebraic properties of logarithms and the corresponding derivative rules.

3.3 Knowledge systematization of logarithmic functions

After introducing the logarithm function and its derivative with two modeling tasks in the first two lessons, it was proceeded to apply the knowledge acquired in the study of the logarithm function by solving tasks, as exemplified by task 3 on the pH of a substance, in groups of 3 or 4 students. All students determined a logarithmic model as the best fit to the data in the table. However, only half of the groups determined the best logarithmic model, as illustrated by the response of group G2 (Figure 5).

Figure 5

Group G2 answer t



The groups that answered incorrectly determined a logarithmic model that does not correspond to the best fit to the data in the statement. These groups probably made a mistake in defining and organizing the data on the graphing calculator. Group G6 determined the correct coefficients of the expected logarithmic model but found it difficult to translate these values to the model suggested by the graphing calculator. When reading the coefficients of the obtained logarithmic model, these students did not identify that only coefficient b multiplied the Napierian logarithm of x.

Regarding section 3, it is observed that most of the groups tried to solve it only with analytical methods. However, no group reached the expected result because the students did not determine the domain of the logarithmic model of pH, considering that it could only take values between 0 and 14. Only one group restricted the domain of the logarithmic model, considering the situation of the task. Group G4 previously solved the equation f(x)=0, where f represents the model obtained to determine the domain of the model. Based on the data in the table, the values of x for which a solution has an acidic pH were determined. The logarithmic model translates into a strictly decreasing function, which will have an impact in the order of the inequality that translates the statement. Groups that answered incorrectly had difficulty solving the inequality and defining the domain, as shown by the solution of group G8: $-4{,}7410^{-6}-0{,}434\mathit{lnx}<7 \Longleftrightarrow -0{,}434\mathit{lnx}<7 \Longleftrightarrow \mathit{lnx}<-16{,}13 \Leftrightarrow x<e^{-16{,}13} \Leftrightarrow x<9{,}891\times10^{-8}$.

There are several limitations when solving this using the graphing calculator, namely the definition of the display window. When plotting the logarithmic model and the line of equation in a display window adapted to the task situation, it is impossible to observe the intersection of the two curves; since the values are too small, it is impossible to see the intersection even by enlarging the graph. These limitations stem from the limited technological capabilities of the graphing calculator to represent infinitely small or large quantities. The fact that the solution cannot be read graphically may have disturbed the students in their use of the graphing calculator in this section.

Regarding section 5, very few groups had time to explore it. Only two groups responded, identifying that one of the differences between the theoretical pH model and the model obtained through the task was the base of the logarithm, being a decimal logarithm for the theoretical model and a Napierian logarithm for the model, as illustrated by the response of group G3: the standard pH formula is presented through the decimal logarithm. In contrast, the model we present is given as a function of the Napierian logarithm.

When presenting in the class the results of group G3, the procedure for changing the base of the logarithms to any base $a \in \mathbb{R} \backslash \{1\}$ to work with logarithms of equal bases was reviewed. In this way, the accuracy of the obtained model can be compared with the theoretical model of the statement.

3.4 Activities and difficulties experienced by the students

Based on the empirical study by Stillman *et al.* (2007), Table 1 illustrates the performance of students when conducting key cognitive activities to solve the proposed modeling tasks.

Table 1

Success rate of student activities when exploring modeling tasks (%)

Modeling phases	Understanding the task statement			Data analysis and model creation			Exploration of the model		
Cognitive activities	Clarify the context of the problem	Identify magnitudes	Developing hypotheses and assumptions	Identification of variables	Develop relevant hypotheses	Choosing the technology to build the best model	Apply appropriate mathematical knowledge	Use technology to make calculations and graphs	Mathematize the statement and
Task 1 (15 pairs)	100 %	100 %	100 %	100 %	86,7 %	66,7 %	73,3 %	53 %	100 %
Task 2 (15 pairs)	80 %	93,3 %	60 %	93,3 %	53,3 %	53,3 %	20 %	33,3 %	0 %
Task 3 (8 groups)	100 %	100 %	100 %	100 %	100 %	50 %	56,25 %	75 %	25 %

The analysis of the table shows that all students, in some way, performed the expected cognitive activities while exploring the tasks. Some modeling skills, mathematical and technological, are identified, since are essential to task exploration and are in line with the literature (Niss and Højgaard, 2019). One of these is statement interpretation. Most of the students identified, when reading the statement, relevant and non-relevant information, and dependent and independent variables.

In developing the model, it was essential to place and organize the data on the graphing calculator, to know how to work with lists, to represent the point cloud associated with the data, and to define the list that corresponds to each variable. Students tested which model best matched the points representing the data in the task from the point cloud. It should be noted that some students showed difficulties in organizing the data in a table according to the task situation and in knowing how to use and read the regressions available on the graphing calculator.

For analyzing questions, it was essential to apply mathematical knowledge. From the statement, the students translated and related the information to the model obtained. As the complexity of the tasks increased, students showed difficulties in correctly applying this mathematical knowledge. Recognizing the need to use the graphing calculator to perform calculations, represent graphs, solve equations, and check results is inevitable when exploring these types of tasks. The need for students to know how to graph a function and define a display window appropriate to the situation is also highlighted, although this can lead to confusing representations when dealing with disproportionate magnitudes (Campos *et al.*, 2015; Consciência, 2013). However, students cannot identify if what appears on the graphing calculator display is mathematically valid and whether it is consistent with the task situation without a solid theoretical basis (Consciência, 2013).

4 Discussion and conclusions

The students' activity was carried out in small groups to assess the students' actions in an exploratory teaching perspective conducive to the exploration of modeling tasks (Blum and Borromeo Ferri, 2009; Borromeo Ferri *et al.*, 2017; Ponte, 2005). This organization of the students gave them the opportunity to learn to work in groups, develop mathematical communication skills and the attitude of being critical with the results obtained (Rodríguez-García and Arias-Gago, 2020). Gradually, with the students' participation in the activities carried out in their group and in the class, the group improved in the solving of the proposed tasks, corroborating the results obtained by Blum and Borromeo Ferri (2009) and Sawalha (2018). Such participation indicates that it is due to the nature of the modeling tasks, which motivated students to share ideas and strategies and to confront knowledge, processes, and results.

The implementation of the mathematical modeling tasks was based on the modeling cycle defined by Stillman *et al.* (2007), adapting the phases of this cycle to the objectives in the lessons taught in the study of the logarithmic function. Using the graphing calculator, the students carried out the phases of understanding the statement, data analysis, model construction and validation, and model exploration. These phases allow to characterize the sequence of activities they perform when exploring the tasks proposed in the learning of the topics studied.

After reading the proposed tasks, most students can interpret the statements by identifying the independent and dependent variables, due to the development of their functional reasoning, a skill that is expected to be developed at the end of high school (Ambrus *et al.*, 2018; Ministerio de Educación, 2017). It is a skill that allows students to establish relationships between the data they extract from the interpretation of task statements and the values of variables. The perception of the behavior of values of the variables is evident when these values are organized in tables or scatter plots. The meaning drawn from this analysis highlights the relevance of the connection between the different mathematical representations of function concepts (Viseu *et al.*, 2022). The numerical representation aligns with the symbolic representation (an algebraic expression that translates the model), which in turn is articulated with the graphical representation.

By building the best model that fits the data collected, students make sense to the mathematical model they obtain and the procedures necessary to perform statistical regressions on the graphing calculator. This is part of the instrumented performance schemes that are developed when taking advantage of this technological device (Teixeira *et al.*, 2016), which translates into the identification of the parameter roles that integrate the different models provided by the graphing calculator, the meaning of the value of the determination coefficient and the definition of the display window.

Regarding the exploration of the model, students followed different solution strategies, either by analytical processes or by graphing calculator. When exploring this technological device, students initially presented some limitations to integrate it into their activities. These limitations are due to the lack of familiarity with its use, which influences in the way the teacher guides them: exploring the statistics menu, editing algebraic expressions in the function editing menu and defining the display window. It is observed that the tasks that do not always allow solving problems with paper and pencil make the students feel more familiar with the tool, having implications in the meaning they give to the topics under study.

The exploration of the statistical menu of the graphing calculator in the solution of the modeling tasks makes it possible to establish the determination coefficient, which informs about the reasonableness or not of the model found to fit the data resulting from the task being solved. The model translates the function that is the object of study, which, in the case of this work, directed the students to learn the notion of logarithm of a number in each base, logarithmic functions and their properties, as well as the derivative of a logarithmic function. These results conclude that the solution of modeling tasks is based on the value of conceptual understanding of the topics under study and of the computational procedures needed to solve a problem (Kenney and Kastberg, 2013). It is about the development of skills that are intrinsic to the solution of modeling tasks, among which are the understanding of different mathematical representations, the sense of mathematical symbology and the ability to communicate mathematically which, in today's world, is mostly done using technological devices (Niss and Højgaard, 2019).

Regarding the difficulties reported by students in solving the modeling tasks, the interpretation of the statement turned out to be a challenge when exploring the model, as mentioned by Stillman *et al.* (2007). Some students found it difficult to connect the context of the task with the mathematical model obtained, which translated into difficulty in understanding the mathematical model they were working with, the functions in general and their properties.

Regarding the definition of the best model that fits the data in the statement, students were not too familiar with the use of the functions of the graphing calculator to organize data, plot point clouds and the preparation of statistical regressions. It is paramount that they know the features of the graphing calculator to be able to organize the data in a table, plot the associated point cloud, build a mathematical model that fits the task situation and, consequently, solve this type of tasks (Blum, 2002; Campos *et al.*, 2015). These types of activities are reinforced by solving tasks that encourage students to collect and organize data and to model the problem situation under study from a functional perspective. Gradually, students are aware of the usefulness of what they learn at school to understand situations and solve problems they find on a daily basis. This is a teaching perspective that promotes dialogue between two worlds, the real and the mathematical, very different from the perspective that motivates students to reproduce facts and procedures.

In the model exploration phase, some students showed some difficulties when using the graphing calculator to overcome the limitations of specific analytical processes. The definition of the display window was difficult for some students, as it depends on the domain of the model and the context of the task situation. A poorly adjusted display window can lead to misinterpret the graphical representation and become an obstacle when exploring this type of tasks (Arcavi, 2003; Viseu and Menezes, 2014). The proper use of the graphing calculator intrinsically implies having solid mathematical knowledge to relate the concepts and properties of functions to their graphical representation (Consciência, 2013). To fully take advantage of the potential of the calculator, it is essential to know the mathematical concepts and understand the notations displayed on the screen (Galbraith and Stillman, 2006).

In general terms, it is inferred from the results of this study that the solution of modeling tasks promotes in students the acquisition of knowledge (in this case, of facts and procedures of the logarithmic function) and the development of skills (reasoning and problem solving) and attitudes, including autonomy, critical thinking, and the search for scientific, technical, and technological deepening.

There were some limitations when conducting this paper. One of them was due to the use of the graphing calculator. Even though the students have had a graphing calculator since 10th grade, they were unaware of certain features of the graphing calculator, especially the ability to organize data and perform statistical regressions from these data. Defining a display window to suit the graphical representation or part of it, depending on the objectives of the task, was a challenge for the students. Without a suitable display window, wrong conclusions about the characteristics of the function under consideration could be drawn from reading and interpreting the graphing calculator display. In addition, logarithmic models establish relationships between quantities that are often unreasonable in the graphical representation. Due to the technological limitations of the graphing calculator, particularly the screen resolution (384×216 pixels), it cannot represent specific parts of the graph of the functions underlying these models.

Consequently, the reading and interpretation of the graphical representation obtained with the calculator, as well as the graphical solution of the problem, are limited. Another limitation was related to the design of the tasks involving logarithmic models. It was a great challenge to propose modeling tasks that matched the learning objectives set out in the curriculum and that were associated with significant natural phenomena. The design of real context modeling tasks, focused on the properties of the problem situation and not semi-real and only focused on its mathematical properties (Ponte and Lent, 2012), proved to be a difficult task to perform. Such limitation highlights the use of modeling tasks in teaching strategies in the training of future mathematics teachers, promoting the development of their professional knowledge about the characteristics of the tasks that current mathematics teaching programs recommend integrating in teaching strategies.

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Annex

Homework 01. The growth of bacteria population

Tarea: **El crecimiento de una población de bacterias**

1. **Las bacterias se reproducen asexualmente mediante un proceso llamado división binaria. La división binaria se produce cuando una bacteria duplica su material genético y se divide inmediatamente, dando lugar a dos bacterias idénticas a ella. En condiciones ideales de temperatura y nutrientes, una bacteria tarda aproximadamente veinte minutos en completar el proceso de división.**
2. **Completa la siguiente tabla.**

Tiempo (minutos)	0										
Número de bacterias	1										

3. **Utilizando la calculadora gráfica, grafica la nube de puntos del número de bacterias en función del tiempo t.**
4. **Define el modelo matemático que mejor represente la situación.**
5. **Después de un día, ¿cuántas bacterias componen la población?**
6. **¿Cuánto tiempo tarda la población en alcanzar las 8000 bacterias?**
7. **Dado que el número de células bacterianas en el cuerpo humano es de aproximadamente 40 X 1012, ¿cuánto tiempo tardará la población de bacterias en alcanzar este valor?**

Homework 2. What is a decibel?

Tarea: ¿Qué es un decibelio? En sus primeros estudios sobre acústica, Bell, inventor del teléfono, se dio cuenta de que la variación del sonido que el oído humano puede sentir no sigue una escala lineal. Si duplicamos la amplitud de la señal, nuestro oído no capta esta transformación. Basándose en sus experimentos, Bell decidió utilizar una escala logarítmica para representar la amplificación o atenuación de un sistema para cuantificar la reducción a nivel acústico en un cable telefónico estándar de 1 milla de longitud. Bell creó la unidad de medida TU (Transmission Unit). Esta unidad de medida pasó a llamarse Bel. Con la práctica, se vio que la unidad era demasiado grande y se decidió dividir la unidad Bel en diez, creando así el Decibelio (dB).

Fuente Sonora	Intensidad Sonora (dB)	Relación entre la intensidade sonora percibida *I* y el umbral audibilidad *Io*
Murmullo	10	10
Conversación normal	20	100
Habitación Tranquila	25	316,2
Biblioteca	30	1000
Vía residencial	40	10000
Lavavajilla	45	31622,8
Oficina	50	1000000
Sala de Aula	55	316 227,8
Motor de un carro	70	1000000
Aspiradora	65	3162277,7
Tránsito congestionado	70	10000000
Cantina escolar	80	100000000

1. A partir de los valores de la tabla, comente la afirmación: "Bell se dio cuenta de que la variación del sonido que puede sentir el oído humano no sigue una escala lineal".
2. Utilizando una calculadora gráfica, represente gráficamente la nube de puntos de intensidad sonora (dB) en función de la relación I/Io. ¿Qué modelo matemático representa mejor esta situación?
3. Con respecto al modelo que has definido, compara la variación de la intensidad sonora en los intervalos [10; 30] y [1010; 1030].
4. Determine la tasa de variación instantánea de la intensidad sonora para I/Io = 15
5. Determine la tasa de variación instantánea de la intensidad sonora en función de la relación I/Io
6. Describa el comportamiento de la variación de la función que modela la situación en estudio, indicando dónde se acentúa este crecimiento.

Homework 3. The pH of a substance

Tarea. El pH de una sustancia. La acidez de una sustancia se mide por la concentración (en moles por litro) de iones de hidrógeno (H+) en la sustancia. La forma estándar de describir esta concentración es definir el pH (potencial de hidrógeno) de una sustancia, que indica si una sustancia es ácida, neutra o básica. La escala de pH va de 0 a 14 a una temperatura de 25°C. Si el valor del pH es igual a 7, el medio de la sustancia es neutro, pero si el pH es inferior a 7, es ácido, y si es superior a 7, básico La siguiente tabla muestra las concentraciones de iones H+ y los respectivos valores de pH de las sustancias presentes en nuestra vida cotidiana.

Substancias	Concentração de iões H *	PH
Zumo de limón	3.981×10^{-3}	2.4
Vinagre	1.2589×10^{-3}	2.9
Zumo de manzana	$3.1623 \mathrm{X} 10^{-4}$	3.5
Cerveza	$3.1623 \mathrm{X} 10^{-5}$	4.5
Café	$1.0 \mathrm{X} 10^{-5}$	5.0
Té	$3.1623 \mathrm{X} 10^{-6}$	5.5
Leche	$3.1623 \mathrm{X} 10^{-7}$	6.5
Água pura	$1.0 \mathrm{X} 10^{-7}$	7.0
Sangre	$3.981 \mathrm{X} 10^{-8}$	7.4
Agua potable	$1.0 \mathrm{X} 10^{-8}$	8.0
Jabón de Manos	6.3096×10^{-10}	9.2
Amoníaco casero	3.1623×10^{-12}	11.5
Agua del baño	3.1623×10^{-13}	12.5

1. A partir del análisis de los datos presentados en la tabla, ¿qué modelo crees que traduce mejor el pH de cualquier sustancia en función de su concentración de iones? Utilizando la calculadora gráfica, comprueba si el modelo idealizado traduce esa representación.
2. Tomando como referencia el modelo que has definido, determina la concentración de iones H^+ de un zumo de uva cuyo valor de pH es 3,2.
3. ¿Qué valores expresan las concentraciones de iones H^+ para que el medio de una sustancia sea ácido?
4. A partir de la gráfica de tu modelo, puedes ver que la función representada es estrictamente decreciente en su dominio de validez y que su gráfica tiene una concavidad hacia arriba.
5. Formalmente, el pH viene dado por pH = - log[H^+]. ¿Cuál es la diferencia entre el modelo que has establecido y este modelo formal de pH?

